Polarization of Time and Income – A Multidimensional Analysis for Germany

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Abstract

A growing polarization of society accompanied by an erosion of the middle class is receiving increasing attention in recent German economic and social policy discussion. Our study contributes to this discussion in two ways: First, on a theoretical level we propose extended multidimensional polarization indices based on a CES-type well-being function and present a new measure to multidimensional polarization, the mean minimum polarization gap 2DGAP. This polarization intensity measure provides transparency with regard to each single attribute, which is important for targeted policies, while at the same time respecting their interdependent relations. Second, in an empirical application time is incorporated, in addition to the traditional income measure, as a fundamental resource for any activity. In particular, genuine personal leisure time will account for social participation in the sense of social inclusion/exclusion and Amartya Sen’s capability approach.

Instead of arbitrarily choosing the attribute parameters in the CES well-being function, the interdependent relations of time and income are evaluated by the German population. With the German Socio-Economic Panel (GSOEP) and detailed time use diary data from the German Time Use Survey (GTUS) 1991/92 and 2001/02, we quantify available and extended multidimensional polarization measures as well as our new approach to measuring the polarization of the working poor and affluent in Germany.

There are three prominent empirical results: Genuine personal leisure time in addition to income is an important and significant polarization attribute. Compensation is of economic and statistical significance. The new minimum 2DGAP approach reveals that multidimensional polarization increased in the 1990s in Germany.

JEL: I32, D31, J22

Keywords: Multidimensional polarization, intensity of time and income poverty and affluence, interdependent multidimensional time and income poverty and affluence, minimum multidimensional polarization gap (2DGAP), extended economic well-being, satisfaction/happiness, working poor and affluent, CES well-being function, German Socio-Economic Panel, German Time Use Surveys 1991/92 and 2001/02
1 Introduction
A growing polarization in society accompanied by an erosion of the middle class is receiving greater attention in recent economic and social policy discussion in Germany. This drifting apart has many far reaching consequences for the economy and for the quality of life in general and requires engagement on many levels. If this complex topic is examined from an economic perspective, the question is about a growing income gap, which means that “the rich get richer and the poor get poorer” (Grabka and Frick 2008). Though the case is important and there is a large literature about inequality and in particular income inequality with a focus on the poor, there are only a few theoretical and empirical studies which explicitly investigate both poles of income distribution (but see the approaches discussed below). Even less empirical evidence and fewer theoretical approaches about polarization can be found when not only income but a multidimensional approach to economic well-being is considered, though multidimensional approaches have proven to be important for extended poverty studies (Alkire and Foster 2011, Chakravarty 2009, Chakravarty and Silber 2008, Bourguignon and Chakravarty 2003, Atkinson 2003).

Our study contributes to the polarization discussion with respect to interdependent multidimensional (IMD) theoretical measurement and its empirical application in two ways:

First, we propose a methodology using extended multidimensional polarization indices based on a CES-type well-being function, and we present new well-being measures of multidimensional polarization, and in particular the new mean minimum polarization gap $2DGAP$, a shortest way to escape poverty or to leave affluence. This unique polarization intensity measure provides transparency with regard to each single attribute and ensures at the same time their interdependent relations.

Second, since polarization has both an economic and social dimension, in the empirical application we respect both and argue that – in addition to the traditional income measure – time as a fundamental resource for any activity should be incorporated into multidimensional analysis. Genuine personal leisure time in particular will account for the social participation aspect in the sense of social inclusion/exclusion and Amartya Sen’s capability approach (e.g. Sen 1999, 1985). The interdependent relations of time and income via the polarization attribute parameters in the CES well-being function will be estimated and evaluated by the German population instead of arbitrarily assigning them values (as in Bourguignon and Chakravarty 2003 or Lugo and Maasoumi 2009).

This empirical application is based on the German Socio-Economic Panel (GSOEP) with additional detailed time use diary data from two available German Time Use Surveys (GTUS) 1991/92 and 2001/02. We quantify available and extended polarization measures as well as our new approaches to polarization development over that decade in Germany according to time and income. In addition to one-value multidimensional polarization indices quantified and discussed in our study, we argue that our new transparent multidimensional polarization $2DGAP$ components has empirical significance for Germany and is important for targeted policies aiming to overcome polarization.

Current approaches to polarization measurement can be divided into two strands, regardless of whether they are unidimensional or multidimensional. One strand, based on Foster and Wolfson 2010, considers bipolarity with two income groups, one above and one below the income median. This family of measures includes the unidimensional indices of Foster and Wolfson 2010 and Wang and Tsui 2000 as well as the multidimensional extension of Scheicher 2010. The second strand, which is based on the work of Esteban and Ray 1994, relies on a separation of the income distribution into several groups and defines polarization
by inequality within the groups and the distance between the groups. This family of polarization measurements includes the unidimensional measure by Esteban and Ray 1994, Esteban et al. 2007 and Duclos et al. 2005, as well as the multidimensional approach of Gigliarano and Mosler 2009. In the following we only briefly characterize these multidimensional approaches; the respective formulas can be found in the Appendix.

Gigliarano and Mosler 2009 construct a class of multidimensional polarization measures by decomposing different inequality measures and measuring the relative sizes of various groups. Polarization is then described as inequality within groups and as inequality between groups. This measure is a multidimensional extension of the earlier group approach of Esteban and Ray 1994. Scheicher’s 2010 multidimensional polarization index is based on individual middle class attribute distances to middle class thresholds. For each individual the single attribute distances are summed up so as to provide individual mean distances of the poor and of the affluent, which are then aggregated. An advantage is certainly the incorporation of multiple attributes and this was an inspiration for our following proposal. However, since the attributes could have different dimensions (euros for income and minutes for time, say), it is difficult to provide a convincing and comprehensive argument in favor just of adding up. While the approach of Gigliarano and Mosler allows a substitution of attributes, the distance measure by Scheicher does not include a substitution parameter.\(^1\)

Though the multidimensional approach is obviously the virtue of these approaches, because of the differences in the attribute dimensionalities, it is necessary to find a combined “umbrella” which aggregates the single polarization attributes and respects their interdependence. The contribution of this paper will be to fill that gap.

This study is the multidimensional polarization extension of our minimum 2DGAP approach and application recently proposed for multidimensional poverty studies (Merz and Rathjen 2014b).

The remainder of this paper is organized in a theoretical and an application part as follows: Section 2 is about the identification and aggregation of multidimensional polarization (2.1), and the specification of the multidimensional CES well-being function (2.2). Section 3 proposes our new well-being measures of interdependent multidimensional polarization. Section 4 introduces the minimum multidimensional polarization gap (2DGAP) with transparent attribute contributions in the compensation context.

The entire application part consists of different sections which build on each other. Section 5 justifies time and income as the two most important MDP attributes, describes their expected interdependence and discusses the polarization threshold lines concepts. Section 6 describes the data base and the empirical polarization threshold lines. Section 7 provides polarization results based on measures reported in the literature and in particular on the results of our new (CES) well-being gap and minimum 2DGAP interdependent multidimensional polarization indices. Section 8 is about the importance of polarization for various socio-demographic groups. Section 9 provides a conclusion.

There are three prominent empirical results for Germany: First, genuine personal leisure time, in addition to income, appears to be an important and significant polarization attribute. Second, compensation between these two attributes, evaluated by the German population, is found to be of economic and statistical significance. Third, as shown by the new minimum 2DGAP approach, multidimensional polarization significantly increased in Germany over the decade 1991/92-2001/02.

\(^1\) For a more in-depth discussion of uni- and multidimensional polarization indices see Merz and Scherg 2013.
2 Multidimensional Polarization: Identification. Aggregation and Multidimensional CES Well-Being Function

2.1 Multidimensional Polarization: Identification and Aggregation

To define multidimensional polarization, we mainly build on the multidimensional poverty discussion (Kakwani and Silber 2008, Deutsch and Silber 2005). In multidimensional poverty identification (Bourguignon and Chakravarty 2003), there are two approaches. In the so-called union approach (strong focus axiom) a person is judged to be multidimensional poor as soon she or he is deprived in at least one dimension (see Figure 1b for the two-dimension case, the shaded area with the corresponding poverty thresholds \( z_1 \) and \( z_2 \)). The intersection approach, by contrast, judges an individual to be multidimensional poor when she is deprived in all dimensions (Figure 1a). Intermediate concepts are conceivable as well.

Figure 1: Identification of Multidimensional Polarization

Multidimensional Poverty

Intersection Approach

Union Approach (Strong Focus)

Compensation Approach (Weak Focus)

Multidimensional Affluence

Intersection Approach

Union Approach (Strong Focus)

Compensation Approach (Weak Focus)

- Multidimensional Poverty
- Multidimensional Affluence

Note: \( x_1 \) and \( x_2 \) are the quantities of the first and second dimension, \( z_1 \) and \( z_2 \) are the corresponding poverty and \( r_1 \) and \( r_2 \) the corresponding affluence dimension thresholds.

Source: Own figure.
The selection of the union, intersection or intermediate approach ultimately depends on the relationship between poverty dimensions. Given a substitutive situation the intersection approach is preferable. If a complementary relation is given, then the union approach would be appropriate.

Since the union and intersection approaches seem to be too rigid as identification strategies for most cases, an intermediate approach would be most appropriate (Lugo and Maasoumi 2009). It allows compensation (weak focus axiom) for all ranges in one dimension given poverty in the other (Figure 1c). In the compensation approach thus, besides being poor in both dimensions (intersection), a person is multidimensional poor when she cannot compensate poverty in one dimension by the other non-poverty dimension.

The empirical question is whether and to which extent a poverty gap in one dimension might be compensated for by higher quantities in the other. If a gap in one dimension can be compensated by another’s dimension quantity above the dimension threshold, then a person is off poverty (Figure 1c, unshaded area). Thus, the multidimensional poverty line in the two dimensional case runs through the intersection of the dimension thresholds \( z = (z_1, z_2) \) dividing the poor (shaded areas in Figure 1a,b,c) and the non-poor (unshaded areas in Figure 1a,b,c).

We extend multidimensional poverty identification to multidimensional polarization in a similar though not identical way to poverty, and then we identify the other pole of the distribution, the affluent, by mirroring the poverty areas along the single affluence thresholds \( r = (r_1, r_2) \) (Figure 1d,e,f). The kind of relationships between dimensions does not change with respect to the intermediate, union and compensation ranges. Figure 1 also allows the identification of unidimensional polarization with regard to the \( x_1 \) dimension (poor to the left of \( z_1 \), affluent to the right of \( r_1 \)) and to the \( x_2 \) dimension (poor below \( z_2 \), affluent above \( r_2 \)).

In the following we focus on the compensation approach (weak focus axiom), where the grade of compensation will be evaluated empirically by the German population. For the compensation approach, Figure 2 illustrates the poverty and the affluence situation (two-dimensional case). Again, the shaded areas in Figure 2 describe the poor under the poverty line and the affluent above the affluence line. Take for example the affluence compensation above the income threshold (right of \( r_1 \) in

**Figure 2: Multidimensional Isopolarization Contours – Compensation Approach (Weak Focus Axiom) in the Two-Dimension Case**

![Multidimensional Isopolarization Contours](image-url)
The time deficit (being below the time affluence line \( r_t \)) is considered to be compensated by a high income above the multidimensional affluence line to be assigned IMD affluent. The time deficit, however, is considered to be not compensated by high income below the multidimensional affluence line; there is no more multidimensional affluence.

The poverty and affluence lines (in the two-dimension case) can be interpreted as *multidimensional isopolarization contours*, which are isoquants of an underlying well-being function, a function which comprises all polarization attributes and evaluates their interdependent relation. This well-being function is specified in the next section.

To measure then multidimensional polarization for a population, the possible interdependence of attributes has to be specified by the aggregation across the dimensions for each individual well-being index as well as the aggregation across individuals obtaining a poverty measure; see the development for our approach in Section 3.

### 2.2 Multidimensional Polarization: Multidimensional CES Well-Being Function

Following the compensation approach, this section specifies a particular multidimensional well-being function, a Constant Elasticity of Substitution (CES) function, whose multidimensional isopolarization contours, as described above, identifies the individuals in the polarization poles, both the poor and the affluent. The CES well-being function accounts for the interdependence of the polarization attributes and will be the key element in our new multidimensional polarization indices (described in the next section) and the foundation for our new minimum 2DGAP polarization approach.

Our CES-type well-being function with its individual well-being indicator \( V_i \) (weak focus axiom) is based on the multidimensional poverty approach by Merz and Rathjen 2014a and evaluates the interdependence of dimensions by

\[
V_i = \gamma \left[ w_1 (x_{i1})^{-\rho} + w_2 (x_{i2})^{-\rho} \right]^{\frac{\nu}{\rho}}
\]

with the substitution elasticity \( \sigma = 1/(1+\rho) \) measuring the curvature of the isoquants, \( \rho \) as a substitution parameter of the isopolarization contours with \( \rho \neq 0 \), \( \gamma \) as a constant, \( \nu \) as returns to scale, \( x_{i1} \) and \( x_{i2} \) as the polarization attribute quantities, and the coefficients \( w_1 \) and \( w_2 = 1 - w_1 \) as distribution and weighting parameters describing the skewness of the isopolarization contours.

The degree of substitution between genuine personal leisure time and income is measured by the Hicks’ elasticity of substitution as the relative change in the proportion of the two attributes dependent on the relative change of the corresponding marginal rate of substitution. With the CES function, the intersections of all isoquants with a ray from the origin have the same marginal rate of substitution. Substitution/compensation, however, is different between different rays from the origin, which allows different degrees of substitution with different time and income ratios.

Compared to other specifications the CES function has the virtue that the elasticity of substitution/compensation can be estimated empirically and is not restricted to a certain value, like a value of one as with the Cobb-Douglas function (a special case of the more general CES function). Since in our empirical application we estimate the CES by a log-Taylor...
approximation, the results can even be interpreted as being a more flexible specification like a translog one.²

Following the CES well-being compensation approach to quantifying the interdependence of the polarization attributes, the aggregated multidimensional poverty line and the aggregated multidimensional affluence line will be defined at their respective thresholds by

\[
V_i = \gamma \left[ w_1 \left( z_i \right) - \rho + w_2 \left( z_i \right)^{-\rho} \right]^{\mu} \quad \text{and} \quad V_r = \gamma \left[ w_1 \left( r_i \right) - \rho + w_2 \left( r_i \right)^{-\rho} \right]^{\mu}
\]

resulting in the two isopolynomial contours, the isopoverty and the isoaffluence contours, which cross the poverty threshold intersection at \(z = (z_1, z_2)\), or respectively the affluence threshold intersection at \(r = (r_1, r_2)\). All individuals with their calculated multidimensional well-being \(V_i = V(x_1, x_2)\) below the isopoverty line are assigned to be poor, above the isoaffluence line to be affluent, and together assigned to be polarized (see again Figure 2).

3 Multidimensional Polarization: New Indices Based on a Multidimensional (CES) Well-Being Function

We propose a new straightforward measurement approach to multidimensional polarization which respects the interdependence of the attributes based on the above compensation perspective by a multidimensional well-being function like our CES function. With regard to the aggregation of individual well-beings to a multidimensional polarization index for a population, we follow the so-called “shortfall of well-being” aggregate approach used also by Lugo and Maasoumi (2008) for the multidimensional poverty aggregation of well-being levels. In our empirical application in particular we follow this well-being level approach rather than the “well-being of shortfalls” component approach based on the relative differences between individual attributes and their thresholds.

Multidimensional Well-Being Polarization (Median)

The first multidimensional polarization measure is the multidimensional well-being extension of the unidimensional Wang and Tsui 2000 polarization measure which considers polarization poles below and above the respective medians. Our multidimensional well-being polarization index (median) (weak focus axiom) measures the polarization intensity as a mean relative well-being gap with respect to the median, and is defined (for the two dimensional case) by

\[
P_{\text{median}} = \frac{1}{n} \sum_{i=1}^{n} \left[ \frac{V(x_{1i}, x_{2i}) - V(m_1, m_2)}{V(m_1, m_2)} \right]^{\alpha}
\]

where \(V(\cdot)\) is a (CES) well-being function as in Equation 1, \(m_i\) is the median value of the polarization attributes, and \(n\) is the population size. The greater the distance from the median well-being to the individual well-being, the greater is this index. In contrast to Wang and Tsui 2000, who relate \(\alpha\) to the interval \([0,1]\), we follow the Foster-Greer-Thorbecke (FGT) 1984 idea of \(\alpha\) describing here a polarization aversion index, with \(\alpha = 1\) as the relative well-being distance to the median, and \(\alpha = 2\) (or \(\alpha \geq 1\)) for a greater polarization sensitivity with greater weights for larger gaps.

² For a further discussion, reasoning and justification of the CES well-being function with multidimensional poverty application, the reader is referred to Merz and Rathjen 2014a.
Multidimensional Well-Being Polarization (Poverty and Affluence Threshold Lines)

Whereas the last index comprises all gaps below and above the well-being median, the next well-being measure considers the individual gaps with respect to distinct poverty and affluence threshold lines.\(^3\)

Our multidimensional well-being polarization index (poverty and affluence threshold lines) \(P_{\text{all}}\) is the sum of a mean relative poverty and a mean relative affluence well-being gap under the weak focus axiom as defined by

\[
P_{\text{all}} = \frac{1}{n} \sum_{i=1}^{n} \left[ \max \left( \frac{V(z_i, z_2) - V(x_{i1}, x_{i2})}{V(z_1, z_2)}, 0 \right) \right]^{\alpha} + \frac{1}{n} \sum_{i=1}^{n} \left[ \max \left( \frac{V(x_{i1}, x_{i2}) - V(r_1, r_2)}{V(x_1, x_2)}, 0 \right) \right]^{\beta}
\]

The exponents \(\alpha\) and \(\beta\) serve as polarization aversion coefficients, with \(\alpha = 0\) and \(\beta = 0\) delivering the multidimensional polarization headcount ratios. With \(\alpha = 1\) and \(\beta = 1\) an average relative polarization gap in well-being units is measured, and with \(\alpha > 1\) and \(\beta > 1\) a higher aversion against strong polarization (which may be different for the poor and the affluent) is reflected. The proposed polarization index (4) has its origins in a multidimensional Foster-Greer-Thorbecke 1984 poverty index under the weak focus axiom according to well-being units (Lugo and Maasoumi 2009, Merz and Rathjen 2014a,b).

The construction principle of this index – which transfers gap measures from poverty analysis to the analysis of the affluent – reveals a general problem of measuring any gap for the affluent. Whereas a poverty gap is restricted to the maximum interval \(z\), the affluence gap would be unbounded. If the affluence part were specified in the same manner as the poverty part, then a reference to the affluence threshold \(V(r_1, r_2)\) might deliver values greater than one (further implications are discussed e.g. in Peichl et al. 2010). Thus the second part of our multidimensional polarization index refers to the individual situation \(V(x_{i1}, x_{i2})\) ensuring affluence percentage ratios in the interval \([0,1]\).

Though both parts have a different reference, for both parts a larger index characterizes an increasing polarization as increasing mean distances within the multidimensional poles.

Since in both parts of the multidimensional polarization index (Equation 4), the average is related to the whole population \((n)\), relatively small values have to be expected in an empirical application. A more intuitively appropriate average would be related to only the poor or affluent population numbers. The multidimensional well-being polarization index (poverty and affluence threshold lines) \(P_{\text{poles}}\) then is defined by

\[
P_{\text{poles}} = \frac{1}{n_{\text{poor}}} \sum_{i=\text{poor}}^{n_{\text{poor}}} \left[ \max \left( \frac{V(z_i, z_2) - V(x_{i1}, x_{i2})}{V(z_1, z_2)}, 0 \right) \right]^{\alpha} + \frac{1}{n_{\text{rich}}} \sum_{i=\text{rich}}^{n_{\text{rich}}} \left[ \max \left( \frac{V(x_{i1}, x_{i2}) - V(r_1, r_2)}{V(x_1, x_2)}, 0 \right) \right]^{\beta}
\]

with exponents \(\alpha\) and \(\beta\) greater or equal one. Our polarization index (Equation 5) respects well-being units, whereas the unidimensional Scheicher 2010 polarization index refers to income units.

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\(^3\) It is an extension of the Scheicher 2010 index, which for each individual gap only sums up attribute values without any compensation possibilities.
With regard not to relative but absolute deviations, the respective multidimensional well-being polarization index (poverty and affluence contour) in absolute well-being deviations then is

\[
P_{\text{all, abs}} = \frac{1}{n} \sum_{i = 1}^{n} \left[ \max \left( V(z_i, z_j) - V(x_{i1}, x_{i2}) , 0 \right) \right]^\alpha + \frac{1}{n} \sum_{i = 1}^{n} \left[ \max \left( V(x_{i1}, x_{i2}) - V(r_1, r_2), 0 \right) \right]^\beta
\]

and

\[
P_{\text{poles, abs}} = \frac{1}{n_{\text{poor}}} \sum_{i_{\text{poor}} = 1}^{n_{\text{poor}}} \left[ \max \left( V(z_{i1}, z_{i2}) - V(x_{i1}, x_{i2}) \right) \right]^\alpha + \frac{1}{n_{\text{rich}}} \sum_{i_{\text{rich}} = 1}^{n_{\text{rich}}} \left[ \max \left( V(x_{i1}, x_{i2}) - V(r_1, r_2) \right) \right]^\beta
\]

with an analogous interpretation as the relative indices (4) and (5).

**Multidimensional Well-Being Polarization Asymmetry**

The building blocks of our multidimensional well-being polarization index allows a characterization of the polarization asymmetry between the poverty part and the affluence part. This asymmetry might be described by a multidimensional well-being polarization gap ratio between the poverty and affluence gaps for the different relative mean concepts by

\[
P_{\text{all, ratio}} = \left\{ \frac{1}{n} \sum_{i = 1}^{n} \left[ \max \left( V(z_i, z_j) - V(x_{i1}, x_{i2}) / V(z_i, z_j) \right) \right]^\alpha \right\} / \left\{ \frac{1}{n} \sum_{i = 1}^{n} \left[ \max \left( V(x_{i1}, x_{i2}) - V(r_1, r_2) / V(x_{i1}, x_{i2}) \right) \right]^\beta \right\}
\]

and

\[
P_{\text{poles, ratio}} = \left\{ \frac{1}{n_{\text{poor}}} \sum_{i_{\text{poor}} = 1}^{n_{\text{poor}}} \left[ \max \left( V(z_{i1}, z_{i2}) - V(x_{i1}, x_{i2}) / V(z_{i1}, z_{i2}) \right) \right]^\alpha \right\} / \left\{ \frac{1}{n_{\text{rich}}} \sum_{i_{\text{rich}} = 1}^{n_{\text{rich}}} \left[ \max \left( V(x_{i1}, x_{i2}) - V(r_1, r_2) / V(x_{i1}, x_{i2}) \right) \right]^\beta \right\}
\]

and in a similar way for the absolute gaps of Equations 6 and 7 as

\[
P_{\text{all, ratio, abs}} = \left\{ \frac{1}{n} \sum_{i = 1}^{n} \left[ \max \left( V(z_i, z_j) - V(x_{i1}, x_{i2}), 0 \right) \right]^\alpha \right\} / \left\{ \frac{1}{n} \sum_{i = 1}^{n} \left[ \max \left( V(x_{i1}, x_{i2}) - V(r_1, r_2), 0 \right) \right]^\beta \right\}
\]

and

\[
P_{\text{poles, ratio, abs}} = \left\{ \frac{1}{n_{\text{poor}}} \sum_{i_{\text{poor}} = 1}^{n_{\text{poor}}} \left[ \max \left( V(z_{i1}, z_{i2}) - V(x_{i1}, x_{i2}) \right) \right]^\alpha \right\} / \left\{ \frac{1}{n_{\text{rich}}} \sum_{i_{\text{rich}} = 1}^{n_{\text{rich}}} \left[ \max \left( V(x_{i1}, x_{i2}) - V(r_1, r_2) \right) \right]^\beta \right\}
\]

Larger multidimensional well-being polarization gap ratios than a value of one describe a greater asymmetry between the pole gaps.

**4 Minimum Multidimensional Polarization Gap (2DGAP)**

Multidimensional polarization by the compensation approach, as so far discussed, is captured by a multidimensional well-being function which is at the heart of our new polarization indices described above. The virtue of measuring multidimensional well-being and any well-being gap by a CES-type or other type well-being function is that it respects and quantifies the interdependence of multiple well-being attributes by a one-value well-being index. However, such an aggregation of dimensions into a single well-being value is questionable if it is still
measuring “multidimensional” poverty, affluence or polarization, since the single attributes are no longer transparent. Transparency for the single attributes in the multidimensional approach is however desirable in a manner which allows a targeted policy aimed at the specific attributes of poverty, affluence or polarization.

The main motivation for developing the multidimensional polarization 2DGAP is thus to “disentangle” the single attributes of a well-being gap to obtain a unique multidimensional intensity measure with transparent single attributes. This approach extends the minimum poverty 2DGAP concept proposed in Merz and Rathjen 2014b to the multidimensional polarization case.

In the polarization case, both poles of a distribution with their respective poverty threshold and affluence threshold contours are of interest. Figure 3 (top) shows the CES well-being graph and describes the two-dimensional poverty and affluence case: \( V(z) = V(z_i, z_j) \) is the well-being contour at the poverty threshold \( z=(z_i, z_j) \), \( V_i = V(x_i, x_j) \) is the individual well-being contour at \( x_i=(x_i, x_j) \). The difference \( V_c - V_i \) is the multidimensional poverty well-being gap. In an analogous way \( V'_c - V'_i \) defines the multidimensional affluence well-being gap for a rich person with \( V'_c = V(x'_i, x'_j) \) and \( V'_r = V(r_i, r_j) \) as the affluence threshold.

The mapping of the multidimensional well-being on its (two) single dimensional input space now allows for another attractive integrated approach for describing multidimensional polarization intensity while disentangling the single attribute contributions. It consists of a unique distance between an individual situation and the poverty or, respectively, the affluence threshold.

As an illustration, consider the two-dimensional case from the compensation approach and its attribute space as in Figure 3 (at the bottom) and regard first the poverty situation at \( x=(x_i, x_j) \) for an individual. With respect to both dimensions there is a fan of distances from that point \( x=(x_i, x_j) \) to the IMD isopoverty threshold. Indeed, each distance yields the same well-being difference \( V_c - V_i \). However, each distance requires that different single attribute input mixtures need to be overcome in order to escape multidimensional poverty.

The shortest path between \( x=(x_i, x_j) \) and the corresponding point \( p=(p_i, p_j) \) at the isopoverty threshold contour is shown in Figure 3. It requires a minimum input mixture in the sense of an optimized compensation intensity, a minimum combined input “length”, in order to escape multidimensional poverty. A natural measure for that length is the Euclidian distance of the single attributes \( c = \sqrt{a^2 + b^2} \) with their components \( a \) and \( b \) as the single poverty attribute gap intensities.

The same approach will be used for a shortest path from a multidimensional affluent individual situation to the isoaffluence line. It is the minimum combined input length necessary to lose multidimensional affluence.

Following the proposed minimum 2DGAP for the poverty case in Merz and Rathjen 2014b, we call this shortest distance \( c \) the minimum multidimensional poverty/affluence 2DGAP (for two polarization dimensions).
Figure 3: Multidimensional Polarization: Well-Being Gap and Minimum 2DGAP

The solution for the shortest (or closest) length is characterized by the orthogonal path from a poor individual \( x = (x_1, x_2) \) to the tangent at \( p = (p_1, p_2) \) for the poverty case of the CES-type isothreshold line by

\[
 c = \left[ a^2 + b^2 \right]^{0.5} = \left[ (p_1 - x_1)^2 + (p_2 - x_2)^2 \right]^{0.5} = \left[ (p_1 - x_1)^2 + f(p_1 | V) - x_2)^2 \right]^{0.5} = \min
\]

where \( f(p_1 | V) \) is the isothreshold contour with regard to the ordinate values \( x_2 \) (say, time) of the CES multidimensional well-being function.
The solution $p_1$ of the minimizing problem then allows calculation of $c$ by Equation 12 for a given $x = (x_1, x_2)$. Since the proposed CES well-being function is well behaved, there is always a unique solution for the minimum 2DGAP (distance $c$).

Because of the quadratic distances in Equation 12, the solution procedure for the affluence situation is similar to the poverty situation. The solution $p_1$, respectively $p^r = (p_1^r, p_2^r)$ in the affluent case, of the nonlinear Equation 12 (see also Figure 3) then allows calculating $c$ again by Equation 12 for a single poor $x = (x_1, x_2)$ or a single affluent $x^r = (x_1^r, x_2^r)$ individual.

**Single Poverty/Affluence Attribute Gaps**

Once $p = (p_1, p_2 = f(p_1 | V_\zeta))$ is found, the single poverty and affluence attribute gaps can be calculated by

$$a = p_1 - x_1 \text{ unde } b = f(p_1) - x_2$$

**Relative Minimum 2DGAP**

For the poverty case the 2DGAP might be defined relative to the maximum 2DGAP distance $c_{\text{max}}$, which is the distance from the origin $(0,0)$ to the respective orthogonal slope of the IMD isopoverty threshold:

$$c_{\text{rel}} = c / c_{\text{max}} \text{ where } c_{\text{max}} = \left[ \frac{\rho}{\gamma} \cdot w_1 p_1 - \rho \right] / w_2$$

with its corresponding relative single poverty attribute gap intensities

$$a_{\text{rel}} = \left[ a / p_1 \right] a_{\text{max}} \text{ and } b_{\text{rel}} = \left[ b / f(p_1, V_\zeta) \right] b_{\text{max}}$$

However, for the affluence case, as already discussed, there is no comparable genuine maximum distance since any affluence well-being or 2DGAP gap faces the problem of an open top interval.

A possible relative minimum affluence 2DGAP relating to the isoaffluence line precludes direct comparisons to poverty pole measures. Relating it to the overall median for both pole minimum 2DGAPs would have a comparable reference but would not deliver transparent single attribute components.

**Aggregation and Mean Minimum Polarization 2DGAP**

To retain the polarization single attribute contributions in the multidimensional approach, we propose a straightforward aggregation by the sum of the respective minimum 2DGAP pole means, the mean minimum polarization 2DGAP:

$$C = \frac{1}{n_{\text{poor}}} \sum_{i \in \text{poor}} c_i + \frac{1}{n_{\text{rich}}} \sum_{i \in \text{rich}} c_i$$
with its single aggregated components\(^4\)

\[
A = \frac{1}{n_{\text{poor}}} \sum_{i=\text{poor}}^{n} a_i + \frac{1}{n_{\text{rich}}} \sum_{i=\text{rich}}^{n} a_i, \quad B = \frac{1}{n_{\text{poor}}} \sum_{i=\text{poor}}^{n} b_i + \frac{1}{n_{\text{rich}}} \sum_{i=\text{rich}}^{n} b_i.
\]

**The Benefit: Transparency of Single Attributes of Multidimensional Polarization**

The minimum 2DGAP distance \(c\) itself measures the shortest multidimensional gap as the shortest length of all dimensional gap intensities in combined attribute units but without direct interpretation in terms of, say, money or time units. However, and this is the benefit of our proposed approach, both sides (attributes) of the right-angled triangle \((a\text{ and }b)\) are measurable and interpretable in the single dimension, say, income in money units (euros) and time in time units (minutes). This information and transparency then allows targeted single-dimension anti-polarization policies while respecting its multidimensional interdependence. We briefly discuss such polarization policies in our concluding section.

**The n-Dimensional Case**

The minimum multidimensional polarization gap (2DGAP) can be extended to the n-dimensional case, called minimum NDGAP, by a multivariate minimum search, where the slopes of the NDGAP linear distance are subject to the orthogonality of the n-dimensional tangents to their respective isothreshold contours.

**Input Distance Function Approach and Minimum 2DGAP**

Another input-related gap measurement is the input distance function (IDF) approach (Malmquist 1953, Deutsch and Silber 2005, Anderson et al. 2008). It is a measure of efficiency where all input factors would have to be changed proportionally by a certain value to achieve a total (technical and allocative) economically efficient output.\(^5\) This measurement requires that the slope (marginal rate of substitution) of the output level isoquants is identical at any crossing point with a given input vector at a given fixed allocative input price ratio. Then the IDF is an appropriate scaling ratio along the input vector, an arrow from the input space origin with a fixed input factor ratio, to efficiently achieve a certain output level.\(^6\)

With respect to multidimensional poverty (Deutsch and Silber 2005), the IDF approach can be interpreted and adapted as a standard of living gap measure in the input (resource) space. The farther the resource vector is below from a well-being (utility) frontier, the lower is the standard of living. In particular, the IDF approach describes the proportional increase of the multidimensional poverty attributes (time and income, say) to reach the efficient allocation at

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\(^4\) The aggregation of the single poverty attributes \(a\) and \(b\) and of the 2DGAP \(c\) over all individuals might not result in the joint aggregate condition \(c = (a^2 + b^2)^{0.5} \) with two degrees of freedom one remaining component \((a, b\text{ or }c)\) is computable from the other aggregates. In our application, alternative computations of the respective remaining components have shown close accordance with the orthogonal condition.

\(^5\) The distance value is one if the input factors result exactly in a certain output level (total economically efficient situation). Depending on the definition of the input arrow lengths relation, a distance value different to one requires more or less input factor quantities (in a fixed relation) to be efficient.

\(^6\) Imagine an individual two-dimensional input situation \((x = (x_1, x_2))\) below a multidimensional well-being threshold with its respective individual optimal allocation at \(x\) and regard the input space of a linear homogeneous output function (e.g. Cobb Douglas well-being function). Then a distance function value of 1.5 (times the length of \(x\)) requires 50\% more of both inputs \(x_1\) and \(x_2\) to achieve the poverty threshold isoquant in an efficient way.
the multidimensional poverty isoquant. Thereby, and this is the important difference to our minimum 2DGAP intensity, the individual input price ratio (individually evaluated time and income) from a budget constraint with regard to the optimal allocation at the individual situation remains constant also at the crossing point of the input vector and the poverty isoquant.

Take as an example the wage rate as the price for income (working hours). In case of a higher wage offer for a working poor, a constant price ratio in the IDF approach would require foregoing a better paid working situation to escape poverty.

A poverty gap measured by our minimum 2DGAP approach, however, provides a minimum gap from the individual situation to the multidimensional threshold isoquant while respecting the attribute compensation. The minimum 2DGAP thereby allows a changing input factor ratio and a changing individual price ratio on the path to the multidimensional poverty threshold isoquant. In contrast to the IDF approach, this would allow an individual, for example, to take advantage of better paid jobs to escape poverty.

In brief, whereas with the IDF gap the individual situation (slope of the individual budget line of the optimal allocation ratio of time and income) remains constant, the 2DGAP allows a changing individual situation and allocation to achieve the poverty respectively affluence frontier led by the minimum compensation as evaluated by, in our study, the German population.

5 Multidimensional Time and Income Polarization in Germany – Justification of Attributes and Polarization Threshold Line Concept

The empirical application part of our study is about multidimensional polarization in Germany. Some empirical polarization findings for Germany are already available (see Goebel et al. 2010, Grabka and Frick 2008); they are however primarily based on unidimensional income polarization. Merz 2006 and Merz and Zwick 2005 analyse unidimensional income polarization of the self-employed as free-lancers or entrepreneurs in particular. Scheicher 2009 provides empirical results based on his multidimensional approach with working hours and income as polarization attributes. Education and income are the polarization attributes of the Gigliarano and Mosler 2009 multidimensional analysis according to their group-specific approach.

Though the available German results show, roughly speaking, some increasing polarization, mainly in the decade 2001-09, the empirical results with our new polarization measures, polarization attributes, and type and periods of data are obviously new.

In the empirical part of this study, Section 5 justifies time and income as multidimensional polarization attributes and their expected interdependence, and discusses the polarization threshold lines concept. Section 6 describes the data base and the empirical polarization threshold lines. Section 7 provides polarization results based on other measures found in the literature as well as the results of our new (CES) well-being gap and minimum 2DGAP interdependent multidimensional polarization indices. Section 8 is about the degree of polarization for various socio-demographic groups.

A general remark to the empirical part: since our study is an extension of our multidimensional time and income poverty study (Merz and Rathjen 2014a, b), further in-depth empirical justifications and information is available there concerning the variables under investigation, time and income, and all further empirical definitions regarding poverty. These will not be elaborated on in this study.
5.1 Time and Income as Multidimensional Polarization Attributes

To understand poverty in a broader sense, empirical multidimensional poverty studies incorporate various poverty attributes. An example is the European Union Laeken social inclusion/exclusion indicator set with educational disadvantages, health inequalities, unemployment and worklessness as poverty dimensions (Atkinson 2003). Whereas these and a number of other attributes are broadly accepted and discussed in the context of economic and social policies combating poverty, a simple mirror image of affluence is obviously misleading in many aspects.

Why use income as a polarization dimension?

Income is the traditional and most-widely accepted poverty attribute and typically the focus of much policy-making. The affluent are commonly defined as controlling a large amount of material resources, in particular income and wealth. Thus, income is a natural candidate as a polarization dimension for both poles.

Why use time as a polarization dimension?

We argue that, in addition to income as a fundamental material resource, time, though immaterial, is a similar fundamental resource and should be incorporated as a second attribute to better understand societal polarization. Time is important for individual well-being simply by allowing or prohibiting desired everyday activities for the poor and the affluent alike. The importance of the time dimension for poverty studies is stressed meanwhile by other studies (see the discussion in Merz and Rathjen 2014a, Goodin et al. 2008, Burchardt 2008, Harvey and Mukhopadhyay 2007, Bittman 1999, or Vickery 1977).

Time as an enabling factor is also a condition for social participation, or social inclusion/exclusion, which is an important aspect in the extended poverty discussion (Sen 1999, 1995). We are also convinced that social participation has a similar importance for the affluent if they are to lead an integrated social life.

Instead of a broad leisure time concept we propose genuine personal leisure time as being essential to the multidimensional approach. Time poverty occurs when genuine personal leisure time – which is defined as the time left after all paid and unpaid, market and non-market obligations have been met – is below a given threshold level and does not allow or limits social participation in society (see Merz and Rathjen 2014a for further discussion). Correspondingly, time affluence occurs when genuine personal leisure time is above a given threshold level.

Why interdependent time and income polarization?

Time availability restricts both market and non-market activities. Thus the more time is spent for income-related activities; the less is available for leisure, and vice versa. This trade-off is well-known and is central to optimal allocation theory in microeconomics and Becker’s 1965 household production function approach. The microeconomic approaches should illustrate the general competing time-income interdependence, however, the individual time-income optimal allocation admittedly is not in the focus of our analysis.

As discussed above, the trade-off will be quantified in our study by a CES well-being function with time and income as weighted input factors. Instead of arbitrarily chosen trade-off weights and situations with different compensation degrees, we let the German population (through survey data) identify the degree of interdependence and substitution between income and genuine personal leisure time.
5.2 Income, Time, and Multidimensional Poverty and Affluence Threshold Line Concepts

Single poverty threshold lines $z_i$ and single affluence lines $r_i$ ($i = 1, 2$) identify the poor and the affluent respectively, the respective multidimensional well-being thresholds and finally the set of individuals in our polarization analysis. Yet, empirical analysis requires concrete values.

Income: Poverty and Affluence Threshold Line Concept

Income poverty studies commonly use monthly household net equivalence income with equivalence scales like the OECD scale\(^7\). Conventional income-based poverty studies in the European Union identify a person as income poor if her net equivalence income is below 60% of the median income of all households (Bundesregierung 2005, XV). As a result, the 60% median line of the monthly household net equivalence income is adopted in this study as the income poverty line. For the sake of comparison, all subsequent income information for 1991/92 is adjusted to 2001/02 price levels.

Whereas there is common agreement about the income poverty line, there is a longstanding and still open discussion about a respective affluence line. The German government explicitly focused for the first time on affluence in addition to poverty in their first “Poverty and Affluence Report” (Bundesregierung 2002), which was followed by three further government reports (Bundesregierung 2004, 2011, 2013). During that period, top incomes gained increasing attention not only in Germany (Atkinson and Piketty 2007, Dell 2007 with German income tax microdata from 1891-1998, Merz, Hirschel and Zwick 2007 with German income tax microdata from 1992-2003). Several affluence lines were proposed in this literature, including an affluence line as a multiple of an income fraction, such as 200% or 150% of mean median income, or as a top income percentile.

As a pragmatic approach, we are choosing 150% as the cut-off for the median monthly household net equivalence income affluence threshold line (this figure is supported, for example, by the polarization studies of Goebel et al. 2010 or Grabka and Frick 2008 from the German Economic Institute).

Time: Poverty and Affluence Threshold Line Concept

Compared to income, the discussion about time poverty or even time affluence is still in its infancy. Bittman 1999 mentions a 50% time poverty line. To be consistent to our income poverty and affluence line, we chose 60% of the median genuine personal leisure time as defining an individual which is time poor, and 150% of the median as the time affluence threshold line. Admittedly, such threshold lines are certainly debatable.

6 Multidimensional Time and Income Polarization in Germany – Data and Empirical Threshold Lines

6.1 Data: GSOEP 2002 and GTUS 1991/92 and 2001/02

The German Socio-Economic Panel (GSOEP)

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\(^{7}\) With a weight 1 for a household head, a weight of 0.5 for additional household members aged 15 years or older, and a weight of 0.3 for all others.
The German Socio-Economic Panel (GSOEP) provides representative individual longitudinal data for all persons older than 16 years living in German households. The representative panel study started in 1984 and provides annual subjective as well as objective information about individual living conditions in Germany (for details, see Wagner, Frick and Schupp 2007). In particular, the GSOEP asks about satisfaction with regard to different topics, such as income, as well as general questions about life satisfaction. The 11-point scale regarding general satisfaction information is used for our CES well-being estimation and refers to the recent happiness/satisfaction literature (Clark et al. 2008, Frey and Stutzer 2005).

Since appropriate well-being data are only available in the German Socio-Economic Panel, we use the GSOEP 2002 for the CES well-being estimation. Although in principle we could use the GSOEP for further analyses, we prefer to use time use diary data from the German Time Use Surveys (GTUS) from 1991/92 and 2001/02 (with no appropriate well-being information) since the time use diaries provide more in-depth information.

The German Time Use Surveys (GTUS) 1991/92 and 2001/02

The German Federal Statistical Office conducted two large representative time use surveys, the German Time Use Surveys 1991/92 and 2001/02 (Ehling, Holz and Kahle 2001, Ehling 2003). In these surveys all respondents in a household older than 11 years noted their everyday routines in diaries in their own words for two working days and a Saturday or Sunday. Personal and household questionnaires also provided socio-economic background information. The final available data for our analysis comprised 6,774 households with 15,366 persons and 30,732 diaries for 1991/92, and 5,144 households with 11,908 persons and 35,685 diaries for 2001/02.

6.2 Empirical Time, Income and Well-Being Multidimensional Poverty and Affluence Threshold Lines, Germany 1991/92 and 2001/02

The time and income single poverty and affluence threshold lines are summarized in Table 1. All income data are adjusted for price inflation.

The median income, and as a result the income poverty and affluence thresholds, increased by 19.2% to 1322.59 euros in the ten-year period between 1991/92 and 2001/02.

The median time, and again the time poverty and affluence thresholds, increased somewhat less by 17.0% to 310 minutes per day within that ten-year period.

Concerning the empirical well-being and isopolarization thresholds an estimation of the CES well-being function is necessary. With the GSOEP-reported general life satisfaction data on an 11-point scale, an estimation of individual well-being requires rather a type of ordered response modelling. The Kmenta 1967 Taylor series approach, however, allows a simple OLS estimation of the log-transformed non-linear CES well-being function (see Merz and Rathjen 2014a) and results in the CES well-being function

\[
V = f(I, L) = 3.550 \cdot (0.519 \cdot I^{0.297} + 0.481 \cdot L^{0.297})^{0.108}
\]

8 As mentioned, income is defined as monthly net equivalence income. Time is personal genuine leisure time, which is detailed in the individual time use diaries and includes one of the main categories “Contact, Conversations, Sociality” or “Media Use, Free-time Activities” in GTUS 1991/92 and the categories “Social Life and Entertainment”, “Participation in Athletic Activities e.g. Outdoor Activities”, “Hobbies and Games” and “Mass Media” in GTUS 2001/02.

9 GSOEP 2002 question 11 in the personal questionnaire.
with $I = x_1$ for income and $L = x_2$ for genuine personal leisure time. Significantly estimated coefficients together with the fulfillment of further consistency rules quantify the relevance of substitution/compensation between time and income. The population-based evaluation\(^\text{10}\) of the substitution/compensation between genuine time and income yields a substitution elasticity of $\sigma = 1.422$, which shows an easier substitution than in the Cobb-Douglas type ($\sigma = 1$) situation\(^\text{11}\). For comparison reasons, the 1991/92 well-being function is specified by the same estimated parameters as in 2001/02.

Table 1: Income, Time and Well-Being Multidimensional Poverty and Affluence Lines, Germany 1991/92 and 2001/02

<table>
<thead>
<tr>
<th></th>
<th>1991/92</th>
<th>2001/02</th>
</tr>
</thead>
<tbody>
<tr>
<td>Median Net Equivalence Income</td>
<td>1109.64</td>
<td>1322.58</td>
</tr>
<tr>
<td>(in euros per month and 2002 prices)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Median Personal Leisure Time</td>
<td>265</td>
<td>310</td>
</tr>
<tr>
<td>(in minutes per day)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Income Poverty Line ($=60%$ median net equivalence income)</td>
<td>665.78</td>
<td>793.55</td>
</tr>
<tr>
<td>Time Poverty Line ($=60%$ median personal leisure time)</td>
<td>159</td>
<td>186</td>
</tr>
<tr>
<td>Well-Being Poor $V^{\text{poor}} = f(I^{\text{poor}}, L^{\text{poor}})$</td>
<td>6.704</td>
<td>6.827</td>
</tr>
<tr>
<td>Income Affluence Line ($=150%$ of median)</td>
<td>1664.46</td>
<td>1983.97</td>
</tr>
<tr>
<td>Time Affluence Line ($=150%$ of median)</td>
<td>397.50</td>
<td>465.00</td>
</tr>
<tr>
<td>Well-Being Rich $V^{\text{rich}} = f(I^{\text{rich}}, L^{\text{rich}})$</td>
<td>7.402</td>
<td>7.538</td>
</tr>
</tbody>
</table>

Source: own calculations with data from GTUS 1991/92 and 2001/02. The time and income poverty lines and affluence lines by GTUS data are calculated for the total population for the median income, and the median genuine personal leisure time for the available population older than 11 years.

The evaluated well-being poverty threshold line at the intersection of the single time and income thresholds in Equation 19 has a well-being level of 6.704 in 1991/92 and 6.827 in 2001/02. By contrast, the evaluated well-being affluence threshold line at the intersection of the single time and income thresholds yields a well-being level of 7.402 in 1991/92 and 7.538 in 2001/02.

Thus, the CES results suggest a slight increase in overall well-being over the ten-year period. The estimated input coefficients, the weight $w$ for income and $(1-w)$ for personal leisure, indicate a relative importance of income. However, the evaluated time contribution is not that far away from a balanced 50% situation, and reflects the importance of time.

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\(^{10}\) Our CES well-being function estimates are based on the working population because the economically active population actually experiences both work and leisure and therefore its judgment of the trade-off between the two dimensions is probably more appropriate.

\(^{11}\) Perfect substitution: ($\rho = -1, \sigma = \infty$), Cobb-Douglas case with ($\rho = 0, \sigma = 1$), no substitution at all (complementary input factors, $\rho = \infty, \sigma = 0$).
Multidimensional Time and Income Polarization in Germany – Headcount Ratios, Well-Being Gap and Minimum 2DGAP Results

7.1 Overall Multidimensional Polarization Results: Headcount Ratios and Well-Being Gaps

Our analysis concentrates on the economically active population. With regard to the working poor, we shed light on a situation where despite governmental efforts poverty still exists. Similarly, we regard the economically active affluent without taking any further wealth situation into account. Thus, we will focus for both distributional poles on those who are economically active with more than five daily working hours.12

We identify the poor and the affluent by their headcount ratios subdivided in six multidimensional regimes for both available years (Figure 4). The regimes identify unidimensional time and income polarization as well as interdependent multidimensional time and income polarization with their compensation regimes. Table 2 provides unidimensional and multidimensional polarization well-being results of our respective indices as well as of indices from the literature. With a focus on our new well-being indices, we only briefly summarize some results of the literature indices. Together with their formulas, they are discussed in more detail in Merz and Scherg 2013.

Unidimensional Polarization Results

Table 2 and Figure 4 summarize the unidimensional income and time results. The combined time polarization situation is less selective than the income polarization situation; the still divergent poverty and affluence developments only result in a significantly increased median specific polarization (Wang and Tsui). All other polarization measures (Foster and Wolfson; Esteban, Gradin and Ray; and Scheicher) are insignificant with respect to time. However with respect to income, all these measures (except Scheicher) describe a significant income polarization decrease.

Multidimensional Well-Being Results

The discussed compensation approach (weak focus axiom) allows substitution between time and income and respects the interdependence of the polarization dimensions. New results of our well-being multidimensional polarization measures and components over that decade in Germany are presented (IMD results in Figure 4 and Table 2). We regard $P_{\text{median}}$ (Equation 3) which is related to the median $P_{\text{poles}}$ (Equation 4), which in turn is related to the poverty and affluence lines, and $P_{\text{poles, ratio}}$ (Equation 9) which measures the asymmetry of both pole gaps. We divide the discussion with respect to the poverty and the affluence poles, and then discuss them combined.

Interdependent Multidimensional Poverty: The headcount ratios of the multidimensional poor (Figure 4 top, regimes P1, P2, P3) declined slightly from 12.6% in 1991/92 to 12.2% in 2001/02. Though the change is not significant, the absolute level of the working poor in both years is still remarkable. Regime P3 is of particular importance as even an income above the poverty threshold is assigned not to compensate time poverty for 9.3% and 8.7%, respectively, of the economically active population. Regime P3 is also the prominent poverty regime in the multidimensional perspective. A detailed discussion of time and income

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12 A graphical inspection of the time and income situations is found in our discussion paper Merz and Scherg 2013.
multidimensional poverty development (compensation approach, weak focus axiom) with the same data is provided by Merz and Rathjen 2014a.

**Figure 4: Multidimensional Polarization Headcount Ratios in Different Poverty and Affluence Regimes for Germany 1991/92 and 2001/02**

**Poverty**

![Image](https://example.com/image1.png)

![Image](https://example.com/image2.png)

**Affluence**

![Image](https://example.com/image3.png)

![Image](https://example.com/image4.png)

IMDP (IMDA) line is the multidimensional time and income isopovety/isoaffluence threshold based on the GTUS 2002 CES estimates.

Source: own calculations with GTUS 1991/92 and 2001/02, economically active population

**Interdependent Multidimensional Affluence**: there is a significant reduction in multidimensional affluence by *headcount ratios* from 1991/92 to 2001/02 from 8.1% to 5.5% (Table 2 and Figure 4 bottom, regimes R1, R2, R3). The prominent regime in both years is regime R4. These individuals (18.26% respectively 20.22%) are assigned not to be multidimensional affluent though income rich but time poor; their time deficit is assigned not to be compensated by high income. In contrast, regime R3 individuals (only 7.50% respectively 5.06%) are assigned to be compensated their time deficit by their high income; they are multidimensional affluent.

Furthermore, whereas this group and regime decreases from 7.5% to 5.06% in 2001/02, regime R4 developed in the opposite direction, from 18.26% to 20.22%. Thus the headcount ratio of the income but not time affluent increased in Germany over that decade. Notably, there is only less than a half percent of the economically active population which is affluent in both attributes and both years (regime R1).

**Interdependent Multidimensional Polarization**: Now regard the combined poverty and affluence situation. The IMD polarization *headcount ratio* decreases significantly from a remarkable 20.7% in 1991/92 to 17.6% in 2001/02. Of specific interest and empirical
importance are regimes with time deficits (Table 3): for poverty, where the time deficit is assigned not to be compensated even by above poverty threshold income (regime P3); for affluence, where the time deficit is assigned to be compensated by above affluence threshold income (regime R3). These regimes show the highest IMD polarization headcount ratios for both years and emphasize the importance of genuine personal leisure time for the German population.
IMD polarization well-being intensity decreases significantly over that decade if the common polarization threshold is the median \( P_{\text{median}} \). However, if the isopoverty and isoaffluence thresholds are farther from the median, then the multidimensional polarization decrease is only of minor significance \( P_{\text{poles}} \). This indicates particular differences between the lower and upper middle classes relatively close to the median.

Thus, based on the compensation evaluation of the German population with a CES well-being function, time and income polarization only declined in the 1990s when both distributional poles were measured with respect to the median time and income values.

If differences to median pole thresholds are regarded, which pole is driving the polarization measure? An answer is given by our multidimensional well-being polarization gap ratio \( P_{\text{poles, ratio}} \) (Equation 9), which measures the pole asymmetry. The mean well-being poverty gap is more than twice as large as the mean well-being affluence gap, and, the pole asymmetry grew. The dominance of the poverty well-being gap might be explained as follows. First, less than 1% is considered affluent in both dimensions (possibly due to underreporting) and are expected to be not far away from the IMD affluence line, which results in small well-being differences. Second, and a related reason, instead of being affluent in both dimensions it is expected instead that high income together with high working hours restricts genuine personal leisure time. Relatively high regime R4 headcount ratios (Figure 4) support this argument.

To summarize the overall situation: the unidimensional consideration of income and time suggests a decrease in income polarization and a slight increase in time polarization in Germany over the decade. Concerning the single poles, time as well income poverty increases whereas income affluence decreases. Therefore, there is no definite evidence of an increasing bipolarity of income and time distribution if the unidimensional measures are regarded separately.

The proposed interdependent multidimensional polarization well-being approach however provides such a combined analysis. The results up to now show, only with respect to the median but not to the specific poverty and affluence lines, a significant overall decrease in interdependent multidimensional time and income polarization.

### 7.2 Multidimensional Polarization by the Minimum Multidimensional Polarization Gap (2DGAP)

The multidimensional polarization gap results so far discussed embrace the interdependence of time and income under the CES well-being shield. However, and in particular for targeted policy analysis, one might argue that the single polarization attributes suffer from a lack of transparency when only a one-value indicator is given. Our proposed multidimensional polarization 2DGAP measure, developed in Section 4 above, in fact provides such a single time and income polarization contribution while respecting the assigned attribute compensations.

Table 3 shows polarization results as mean minimum multidimensional polarization gaps (2DGAP) (Equation 17) with the disentangled income and genuine personal leisure time components (Equation 18) for 1991/92 and 2001/02 overall and divided for the respective poverty and affluence regimes. In addition, Table 3 presents regime specific mean well-being gaps as the underlying one-value well-being indicator, and further regime specific polarization headcount ratios.
2DGAP Overall ($C$) (see the last line of Table 3): the mean multidimensional polarization $2DGAP$ ($C$, Equation 17) increased significantly by 18% starting at 141.30 in 1991/92. Thus, respecting the compensation between genuine personal leisure time and income as evaluated by the German population, polarization increased in the 1990s, a remarkable result.

Of particular interest are its mean components for income and time ($A$ and $B$ of Equation 18) under compensation:

2DGAP Income component ($A$): The summarized mean minimum income 2DGAP component increases from €29.27 to €38.58 (2DGAP a). Though the mean income gap is small, the relative monetary increase of 32% is remarkable and highly significant. There is an asymmetry with a stronger poverty than affluence intensity; the poverty income component (€17.72) is greater than the affluence income component (€11.55). This might be an indication for an antipoverty policy instead of a policy to tax higher income in an effort to successfully decrease polarization.

2DGAP Time component ($B$): The summarized mean minimum time 2DGAP component of the poverty and affluence gaps (2DGAP b) increased significantly from 136 minutes per day to 150 minutes of genuine personal leisure time by 18%. The asymmetry between the poor and the affluent mean minimum time gap component is remarkable. The affluence time gap in 1991/92 is twice as large as the poor time gap in 1991/92. It is reduced to a 1.6 multiple in 2001/02. A steeper mean minimum 2DGAP c for the affluent with a higher time/income relation indicates the importance of the compensated time deficits (regime R3, Figure 4). By some contrast, the less steep mean minimum 2DGAP c for the poor with a lower time/income relation indicates the importance of the time situation below the income threshold.

Overall, though genuine personal leisure time is an important contribution to well-being polarization, the significant overall 2DGAP polarization growth is mainly due to the significant growth of the income 2DGAP component.

2DGAP polarization regimes: The minimum polarization gaps (2DGAP) measure the poverty and affluence intensities. The strongest polarization intensity is given for the intersection of time and income poverty together and for each respective pole separately (Table 3, regimes P1, R1 and IMD poor and IMD rich). Those individuals face poverty or affluence beyond any compensation. This holds for the combined 2DGAP $C$ as well as for the single income (2DGAP $A$) and time (2DGAP $B$) components.

The strongest gap growth rate is seen in the R2 regime for the affluent, based however on a relatively low headcount ratio. Individuals there are multidimensional affluent though income is below the affluence income threshold; less income there is assigned to be compensated by time affluence. Further results concerning the single income and time 2DGAP components can be found in Table 3.

The 2DGAP results, different for various regimes, show different combinations of different time/income compensation and stress the differences to escape poverty or to leave affluence by the shortest way.
### Table 3: Multidimensional Polarization: Mean Minimum Multidimensional Polarization Gap (2DGAP) of Interdependent Multidimensional Time and Income, Regime Specific Headcount Ratios and Mean Well-Being Gaps 1991/92 and 2001/02, Germany*

<table>
<thead>
<tr>
<th>Headcount Ratio</th>
<th>Well-Being Gap</th>
<th>2DGAP: Mean Minimum (in €)</th>
<th>2DGAP: Mean Minimum Income (in €)</th>
<th>2DGAP: Mean Minimum Time (in minutes per day)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Poor</td>
<td>1.22</td>
<td>2.46</td>
<td>107</td>
<td>0.2593</td>
</tr>
<tr>
<td>P2</td>
<td>1.04</td>
<td>1.02</td>
<td>98</td>
<td>0.0932</td>
</tr>
<tr>
<td>P3</td>
<td>9.27</td>
<td>8.69</td>
<td>94</td>
<td>0.2186</td>
</tr>
<tr>
<td>IMD</td>
<td>Poor</td>
<td>12.9</td>
<td>12.17</td>
<td>97</td>
</tr>
<tr>
<td>Rich</td>
<td>R1</td>
<td>0.49</td>
<td>0.37</td>
<td>76</td>
</tr>
<tr>
<td>R2</td>
<td>0.12</td>
<td>0.04</td>
<td>33</td>
<td>*</td>
</tr>
<tr>
<td>R3</td>
<td>7.50</td>
<td>5.06</td>
<td>67</td>
<td>***</td>
</tr>
<tr>
<td>IMD</td>
<td>Rich</td>
<td>8.11</td>
<td>5.47</td>
<td>67</td>
</tr>
<tr>
<td>Rich &amp; Poor</td>
<td>P1+R1</td>
<td>2.77</td>
<td>2.83</td>
<td>102</td>
</tr>
<tr>
<td>P2+R2</td>
<td>1.16</td>
<td>1.06</td>
<td>91</td>
<td>0.1223</td>
</tr>
<tr>
<td>P3+R3</td>
<td>16.77</td>
<td>13.75</td>
<td>82</td>
<td>***</td>
</tr>
<tr>
<td>IMD P</td>
<td>Poor</td>
<td>20.70</td>
<td>17.64</td>
<td>85</td>
</tr>
</tbody>
</table>

* Poor describes multidimensional poverty, Rich multidimensional affluence and Rich & Poor multidimensional polarization in respective regimes; Headcount Ratio as \( P_{all} \) of Equation 4 with \( \alpha=\beta=0 \); Mean well-being gap as \( P_{polar} \) of Equation 5 with \( \alpha=\beta=1 \)

1 P1/R1: regime of income and time poor/rich individuals. 2 P2: regime of income poor individuals but not time poor. 3 P3: regime of time poor individuals but not income poor; R2: regime of time rich individuals but not income rich; R3: regime of income rich individuals but not time rich. 4 Two-sample difference in means test with variance inhomogeneity and unequal variances; *** = significant at the 1% level; ** = significant at the 5% level; * = significant at the 10% level. 5 Poverty: CES well-being at 60% of income respective time median (CES well-being in 1991/92 = 6.704, CES well-being in 2001/02 = 6.827); IMD: interdependent multidimensional compensation approach; Affluence: CES well-being at 150% of income respective time median (CES well-being in 1991/92 = 7.402, CES well-being in 2001/02 = 7.538).

Source: own calculations with GTUS 1991/92 and 2001/02, economically active population
**Polarization centers:** The mean minimum multidimensional polarization gaps (2DGAP) can also describe polarization centers. In particular, a polarization center is the starting point \((x_{c1}, x_{c2})\) of the mean minimum 2DGAP (Table 4). \(^{13}\)

**Table 4: Multidimensional Polarization Centers 1991/92 and 2001/02, Germany**

<table>
<thead>
<tr>
<th>2DGAP components</th>
<th>1991/92</th>
<th>2001/02</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>poor</td>
<td>rich</td>
</tr>
<tr>
<td>Income ((x_{c1})) (€)</td>
<td>656.72</td>
<td>2818.95</td>
</tr>
<tr>
<td>Time ((x_{c2})) (minutes)</td>
<td>109</td>
<td>224</td>
</tr>
</tbody>
</table>

Source: own calculations with GTUS 1991/92 and 2001/02, economically active population

At the polarization center the relation \(B/A\) of the 2DGAP poverty intensity components illustrates the compensation there. For 2001/02 poverty, the slope at the IMD isopoverty line where \(c\) is crossing (= marginal rate of substitution) is \(c'_c = -A/B = -0.42\). Thus the assigned amount of time to compensate one euro income locally is about 0.42 minutes; i.e. it is less than a 1:1 compensation; 0.42 minutes are enough to compensate one euro, which highlights the particular importance of time. For 2001/02 affluence, the comparable slope is 0.12; i.e. 0.12 minutes are enough to compensate one euro, which highlights the even stronger time importance for the affluent.

Figure 5 illustrates the positions of the mean minimum multidimensional polarization gaps (2DGAP) and their polarization centers for 1991/92 (black) and 2001/02 (blue). Three results illustrate the numerical findings: first, the mean gaps are relatively small, thus the poverty and affluence positions are relatively near their respective thresholds. Second, there is a particular move of the mean affluence gap to higher income over the decade. Third, relatively steep rising mean gaps pinpoint the importance of the time component.

**Figure 5: Multidimensional Polarization: Mean Minimum Multidimensional Polarization Pole Gaps (2DGA\(P_c\)) and Polarization Centers 1991/92 and 2001/02 (blue)**

The Kernel densities of the poverty and affluence gap distributions for 1991/92 and 2001/02 shown in Figure 6 illustrate the different pole distributions of the minimum 2DGAP \(c\) measure. The affluent pole distributions are more right-skewed than the poor pole distributions and characterize situations farther away from their polarization threshold.

\(^{13}\) It is calculated by an iterative process via 2DGAP mean components and the orthogonal requirement.
Figure 6: Kernel Densities of Minimum Multidimensional Polarization Pole Gaps (2DGAP c) 1991/92 and 2001/02, Germany

Source: Own calculations with GTUS 1991/92 and 2001/02, economically active population

Multidimensional Polarization Well-Being and Minimum 2DGAP Results Compared

A last remark is about differences between well-being and minimum 2DGAP results. The mean minimum polarization gaps (2DGAP) indicate a significant increase in multidimensional time and income polarization, whereas the well-being polarization gaps indicate some decreasing polarization, which is however significant only for the median reference and of minor importance for the pole threshold specific measure ($P_{poles}$).

One explanation is the following: Any well-being difference is described by two respective contours as isoquants in the 2D attribute space. There, a fan of multitude gaps describes the differences from one individual situation to the isopoverty or the isoaffluence line. Yet, there is only a one-value well-being gap, which covers the whole area between the individual isoquant and the respective isothreshold. However, the indefinite number of possible distances (gaps) between a single situation and the isopoverty or the isoaffluence line results in a fuzzy overall well-being situation. The minimum 2DGAP approach, by contrast, builds on a unique well-defined multidimensional shortest distance from the individual situation to the respective threshold with interpretable polarization components. So, the “fuzziness” of the well-being gap might be the reason for different well-being gap and minimum 2DGAP results.

To summarize the 2DGAP multidimensional polarization results: With highly significant results we face a strong case for an increased interdependent multidimensional time and income polarization intensity for Germany between 1991/92 and 2001/02.

8 Multidimensional Time and Income Polarization in Germany – Results for Socio-Demographic Groups

It is to be expected that different individual resources and limitations will result in a different polarization situation for different socio-economic and socio-demographic groups. Finally, for those groups which experience attention in the public discussion, Table 5a presents polarization headcount ratios, mean IMDP well-being gaps (compensation approach, weak focus) and mean minimum multidimensional 2DGAPs with their income and time components respecting compensation. The 2001/02 information also provides indices which describe the development since 1991/92 in Germany (detailed results are given in the Appendix, Tables 5b,c including information for the single poles in our downloadable discussion paper Merz and Scherg 2013).
There are many interesting individual results given the compensation evaluation by the German society. To be brief, we will focus on selected results, in particular with regard to our new multidimensional 2DGAP polarization measure and its components, which describe the polarization intensity concerning income (in euros) and genuine personal leisure time (in minutes).

**Gender:** Females are affected more often than males by poverty or affluence referring to unidimensional income and time polarization. In contrast males face a deeper multidimensional polarization gap (2DGAP c) overall and with respect to income and time. And, the 2DGAP polarization increased the most (2DGAP a,b,c) between 1991/92 to 2001/02 for males.

**Age:** Individuals who are over 65 years old and who are still working more than 5 daily hours are the group with the highest unidimensional polarization headcount ratios in the poles as well as under the IMD polarization regimes. The older the individuals are, the deeper are the polarization gaps (2DGAP c), too. The importance of age for both distributional poles is remarkable and underlines a particular erosion of the middle class for the elderly.

**Education:** Individuals with an A level (Abitur) – in contrast to all other educational levels – show the most intense polarization. The higher the education level, the higher is the headcount ratio of the affluent (Merz and Scherg 2013, Appendix Table 4b). Secondary schooling is connected with the fastest polarization growth, which in contrast to A-level education is driven by a prominent poverty pole importance.

**Occupation:** The self-employed are remarkably more often affected by income (52.88%), time (59.44%) and IMD polarization (33.95%) than any other occupational group. Subdividing the self-employed into the liberal professions (architects, lawyers, physicians etc.) and entrepreneurs reveals that the high percentages should be traced back to the high percentages of entrepreneurs with regard to time and IMD polarization, however not for the reverse income polarization. Furthermore, polarization intensity measured by multidimensional gaps show the highest spread for the self-employed (2DGAP c) and in particular for genuine personal leisure time (2DGAP b). Finally, multidimensional polarization grew fastest for the self-employed, followed by blue-collar workers.

For the self-employed, one notable result is that since two-thirds of the self-employed in the two poles under IMD polarization are found at the poverty pole, the deprived situation is of particular importance for the self-employed despite the relatively dominant affluence gap contribution (see Merz and Scherg 2013, Appendix Tables 5b,c). This underpins and is in line with the self-employed multidimensional time and income poverty results of Merz and Rathjen 2011.

**Working Hours:** The highest polarization headcount ratios and the largest IMD polarization intensity (2DGAP c) are found for those with the most weekly working hours.

**Household/Family Structure:** Whereas the IMD polarization headcount ratio for couples with two or more children is the highest among the family groups, the polarization is strongest for single parents with children (2DGAP c). As to be expected, single parents with children also show the relatively highest time gap. Further analysis show that this is mainly due to the poverty pole. They faced a strong polarization increase by 33% (2DGAP c) over that decade. The increase is even stronger for single parents with more children (51%) and pinpoints growing difficulties for single parents.

**Region:** Though unidimensional income and time poverty headcount ratios are higher in the old German states, this is not the case for the multidimensional situation. The relative number of individuals in the distributional poles is higher in the new German states (23.65% vs. 20.16%) and relative to both the old and new states.
Table 5a: Multidimensional Polarization in Socio-Economic Groups of Interdependent Multidimensional Time and Income Polarization 2001/02, Germany

<table>
<thead>
<tr>
<th>Gender</th>
<th>Occupation</th>
<th>Working Hours</th>
<th>HH-Size</th>
<th>Region</th>
</tr>
</thead>
<tbody>
<tr>
<td>Male</td>
<td>52.88</td>
<td>21.70</td>
<td>30.13</td>
<td>32.98</td>
</tr>
<tr>
<td>Female</td>
<td>59.84</td>
<td>24.74</td>
<td>56.07</td>
<td>19.38</td>
</tr>
<tr>
<td>Age 12-17</td>
<td>17.05</td>
<td>27.98</td>
<td>27.98</td>
<td>32.98</td>
</tr>
<tr>
<td>Age 18-24</td>
<td>20.32</td>
<td>24.92</td>
<td>24.92</td>
<td>32.98</td>
</tr>
<tr>
<td>Age 25-44</td>
<td>28.79</td>
<td>33.50</td>
<td>33.50</td>
<td>32.98</td>
</tr>
<tr>
<td>Age 45-65</td>
<td>36.16</td>
<td>22.90</td>
<td>22.90</td>
<td>32.98</td>
</tr>
<tr>
<td>Age &gt;65</td>
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</table>
16.26% in the old German states) and the polarization intensity overall and with regard to income and time is greater in the new than in the old German states.

To summarize: As expected, different socio-demographic groups show different uni- and multidimensional polarization and different polarization growth depending on gender, age, education, family structure and geographic region. Notably, multidimensional polarization of time and income of the self-employed as well single parents deserves specific attention.

Our quantification of multidimensional time and income polarization for various socio-demographic groups Germany is important to detect groups of specific concern. It is expected that many further factors will be needed to explain and to formulate targeted policies. This discussion has to be postponed until further research has been done.

9 Concluding Remarks

This study contributes to the conceptual refinement of multidimensional polarization by using new methodological approaches which provide new empirical results. In particular, we propose a CES well-being function to capture interdependence/compensation/substitution between polarization attributes. This is the basis for new well-being polarization measures and for the new minimum multidimensional polarization (2DGAP) approach. In particular, the 2DGAP approach disentangles the single polarization attributes and ensures at the same time that compensation between the polarization attributes is accounted for. This is important for the development of targeted economic and social policies.

The empirical application uses genuine personal leisure time in addition to income as polarization attributes in order to supplement the traditional use of income in polarization indices with social participation aspects. This is done by using the German time use diary data for the period 1991/92 to 2001/02. Beyond unidimensional and multidimensional polarization results of measures found in the literature, we develop new findings about polarization while accounting for compensation, which is not given an arbitrary value but instead one which is estimated and evaluated by the German population.

There are three prominent empirical results for Germany: First, genuine personal leisure time, in addition to income, appears to be an important and significant polarization attribute. Second, compensation between these two attributes, evaluated by the German population, is found to be of economic and statistical significance. Third, as shown by the new minimum 2DGAP approach, multidimensional polarization significantly increased in Germany over the decade 1991/92-2001/02.

In particular, the IMD polarization headcount ratio decreases significantly from a remarkable 20.7% in 1991/92 to 17.6% in 2001/02. Of specific interest and empirical importance are regimes with time deficits (Table 3): for poverty, where the time deficit is assigned not to be compensated even by above poverty threshold income (regime P3); for affluence, where the time deficit is assigned to be compensated by above affluence threshold income (regime R3). These regimes show the highest IMD polarization headcount ratios for both years and emphasize the importance of genuine personal leisure time for the German population.

IMD polarization well-being intensity decreases significantly over that decade if the common polarization threshold is the median. However, if the isopoverty and isoaffluence thresholds are farther from the median, then the multidimensional polarization well-being intensity decrease is only of minor significance. This indicates particular differences between the lower and upper middle classes relatively close to the median. The new minimum 2DGAP polarization measure however shows a significant increase of multidimensional
time and income polarization intensity by 18%. The disentangled components increased significantly for income (32%) and less distinct but also significantly for genuine personal leisure time (18%). With regard to polarization regimes, the strongest multidimensional polarization 2DGAP intensity is given for the intersection of time and income for the poverty as well as for the affluence pole.

The starting point of the overall mean minimum 2DGAP intensity can be interpreted as a **polarization center**. For the poverty pole this center is in the intersection of time as well income poverty. For the affluence pole this center is in the regime where the time deficit is assigned to be not compensated by above affluence threshold income. The polarization centers thus show time deficits in both poles and highlights the strong importance of genuine personal leisure time for the multidimensional polarization in Germany that time period.

We argue that some differences in well-being and minimum 2DGAP intensity results are due to the “fuzziness” of the well-being gap. In summary, with highly significant minimum 2DGAP results we face a strong case for an increased interdependent multidimensional time and income polarization intensity for Germany between 1991/92 and 2001/02.

Further results for socio-demographic groups show greater polarization for the self-employed and increasingly for single parents than for differences based on gender, age or geographic region.

These and all other findings in this study stress the relevance of genuine personal leisure time with its social participation aspect as an important polarization dimension both for the poor and for the affluent. Economic and social policy will probably deal differently with the poverty and affluence pole when a reduction in polarization is sought. However, more detailed information about both the poor and the affluent is needed to understand polarization trends; and this is what our contribution provides to a certain extent.

Obviously, data both contributes to and restricts the explanatory power of any analysis. In our case it is questionable whether available survey data would describe in sufficient detail the situation of the affluent and in particular the high income affluent. Compulsory income tax data, which arguably provides the most meaningful information about high income individuals, showed an increased unidimensional income polarization in Germany during the same period of analysis (Merz 2006). This is an indication that the increase of the multidimensional time and income polarization gap described in our study would probably have been even greater if more detailed data had been available.
Appendix: Polarization Measures in the Literature

Unidimensional Polarization Measures

Foster and Wolfson (mimeo 1992, released 2010)

(A1) Income spread: $S(q) = \left| \frac{F^{-1}(q) - m}{m} \right|

(A1b) Bipolarity: $B(p) = \left| \frac{\int_q^{0.5} F^{-1}(p) - m}{m} dp \right|

(A1c) $P_{FW} = 2 \left| \frac{\int_q^{0.5} F^{-1}(p) - m}{m} dp \right|

where $F$ is the cumulative distribution function of income, $q$ is the population fraction, and $m$ is the median income. This polarization index is closely related to the Gini coefficient.

(A1d) $P_{FW}$=$C_{FW}$

Wang and Tsui 2000

(A2) $P_{WT} = \frac{1}{n} \sum_{i=1}^{n} \left| \frac{y_i - m^\alpha}{m} \right|

where $\alpha \in [0,1]$, $m$ is the median income, $y_i$ is the income of individual $i$, and $n$ is the number of observations.

Scheicher 2010

(A3) $P_{univ}^s = \frac{1}{n_{poor}} \sum_{i \in poor} \left| \frac{z - y_i}{z} \right|^\alpha + \frac{1}{n_{rich}} \sum_{i \in rich} \left| \frac{y_i - r}{y_i} \right|^\beta

where $z$ is the poverty line, $r$ is the affluence line, $m$ is the median, and $y_i$ describes individual income.

Polarization with separate groups by Esteban and Ray 1994

(A4) $P_{ER} = K \sum_{i=1}^{g} \sum_{j \neq i} \pi_i^{x_{ij}} \pi_j \left| \mu_i - \mu_j \right|

where $\pi$ is the population fraction of group $i$, and $\mu$ is the mean income of group $i$. Thus if all individuals have the same income, the index will be zero so that the minimum of the index is achieved at minimal inequality. The maximum of the measure is achieved when half of the
population has the minimum income and half of the population has the maximum income. Then the measure will be one normalized by a constant $K$.

Esteban, Gradín and Ray 2007

\begin{equation}
P_{ERG}^{\alpha} = \sum_{i=1}^{g} \sum_{j=1}^{g} \pi_i^{1+\alpha} \pi_j |\mu_i - \mu_j| - \beta \epsilon(G - G_{\text{grouped}})
\end{equation}

$\beta$ is a weight for measurement error. An advantage of this extended polarization measure is that it is not necessary to classify the groups on the basis of arbitrary income constraints. Only the number of groups has to be chosen, and the group assignment is instead done by a classification algorithm.

Duclos, Esteban and Ray 2004

\begin{equation}
P_{\alpha}^{DER}(F) = \int f(y)^{\alpha} a(y)dF(y)
\end{equation}

where $F$ is the distribution function, $f$ the associated density, $\mu$ is the income mean, $\alpha \in [0,1]$.

Multidimensional Polarization Measures

Gigliarano and Mosler 2009

\begin{align}
P_1^{GM} &= \phi \left( \frac{B(X)}{W(X) + c} \right) \cdot S(X) \\
P_2^{GM} &= \psi \left( B(X) - W(X) \right) \cdot S(X) \\
P_3^{GM} &= \tau \left( \frac{B(X)}{B(X) + W(X) + c} \right) \cdot S(X)
\end{align}

where $B(X)$ is the inequality between the groups, and $W(X)$ is the inequality within the groups. $\varphi$, $\psi$, $\tau$ are strictly increasing functions of a specific inequality measure. $S(X)$ is the relative group size, which is measured by an inverse concentration measure like the Herfindahl index. $X$ is a matrix which contains all individuals in the columns and their multiple attributes in the rows. Polarization is measured either via 7a, 7b or 7c. An increasing inequality or an increasing group size raises polarization, an increasing inequality within the groups lowers polarization.

Scheicher 2010

\begin{equation}
d\left( y_i, \left[ z_j, r_j \right] \right) = \begin{cases} 
\min \left( |y_i - z_j|, |y_i - r_j| \right) & \text{if } y_i \notin \left[ z_j, r_j \right] \\
0 & \text{if } y_i \in \left[ z_j, r_j \right]
\end{cases}
\end{equation}

where $z_j$ is the poverty line, $r_j$ is the affluence line of attribute $j$, $y_{ij}$ stands for the value of the $j^{th}$ attribute of individual $i$. For aggregation, the single attribute specific distances are summed up over all attributes:
Finally, the mean of all aggregated distances of the poor and the rich individuals given the respective middle class thresholds builds the Scheicher 2010 multidimensional polarization index:

\[
p_{\text{mult}}^{s} = \frac{1}{n} \sum_{i} d\left(\{y_i, z, r\}\right)
\]

References


