Distributional Change, Pro-poor Growth and Convergence: An Application to Non-income Dimensions

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Abstract

This paper proposes new tools of analysis to determine whether growth was pro-poor. Starting from the concept of distributional change, it makes first a distinction between a non-anonymous and an anonymous analysis of pro-poor income growth based on the notion of the elasticity of non-anonymous (anonymous) income at time 1 with respect to the corresponding non-anonymous (anonymous) income at time 0, such an elasticity being measured via the concept of relative concentration curve. These concepts of distributional change and of elasticity of income at one period with respect of that of another period are then also related to the notions of σ- and β-convergences. The approach is then extended to compute non-anonymous and anonymous growth rates of another characteristic, such as the level of education, with respect to income. Each of the concepts previously mentioned is represented by some index related to the famous Gini index and graphical illustrations derived from the concept of Lorenz and relative concentration curves are also given. Empirical illustrations based on Indian data and focusing on state literacy levels confirm the usefulness of the approach introduced in this paper.

Key Words: β-convergence - σ-convergence **-** Gini index – India – literacy rates – pro-poor growth - relative concentration curve

J.E.L. Classification: D31 – I32 – O15

I. Introduction

There is by now a vast literature on the concept of pro-poor growth although there is no agreement on what this notion should refer to. Some argue that growth is pro-poor when it raises the incomes of the poor whereas others believe that growth can be labeled "pro-poor" only if it raises the incomes of poor proportionately more than it raises the average income in society (see, Kakwani et al., 2004, and Ravallion, 2004, for more details on these two approaches). While Dollar and Kraay (2002) found, on the basis of a large cross-country data set, that the incomes of the individuals who belong to the two poorest deciles of the income distribution rise on average at the same rate as the mean income (see, also the more recent study of Dollar et al., 2013, which tends to confirm the earlier findings of Dollar and Kraay, 2002). Van der Weide and Milanovic (2014) however, using micro-census data from US states for the period 1960-2010, concluded that high levels of inequality reduce the income growth of the poor and help the growth of the rich.

It is also important to make a distinction between studies of pro-poor growth that take an anonymous approach in the sense that they are usually based on cross-sections and do not follow individuals over time and works based on panel data that do not assume such anonymity (see, for example, Grimm, 2007, and Nissanov and Silber, 2009).

Note also that while studies of pro-poor growth generally looked at developing countries and, as a consequence, took an absolute approach to the definition of the poverty line (that is, they assumed a constant poverty line in real terms), the approach is quite different when looking at poverty in developed countries (e.g., see Deutsch and Silber, 2011) because there the poverty line is generally defined in relative terms (that is, the poverty line is assumed to be equal to some percentage of the median or mean standardized income).

Essama-Nssah and Lambert (2009) suggested a common analytical framework to analyze pro-poor growth which allowed them to decompose their proposed measures of pro-poorness across income sources or consumption expenditure components. Such a breakdown has important policy implications because, for example, it may help identifying income sources which may be anti-poor.

While the previously mentioned studies focused mainly on the income dimensions of poverty, Grosse et al. (2007) extended the analysis to non-income dimensions, deriving growth incidence curves and related pro-poor growth measures to non-income indicators in the domains of education, health and nutrition. More recently Bérenger and Bresson (2012), using the concept of sequential stochastic dominance, suggested a new way of testing the "pro-poor" nature of growth for poverty measures, based on both income and other characteristics such as education.

The present study follows in a certain way the approach taken by Grosse et al. (2007) but it proposes new tools of analysis to determine whether growth was pro-poor. Starting from the concept of distributional change, it makes first a distinction between a non-anonymous and an anonymous analysis of pro-poor income growth based on the notion of the elasticity of non-anonymous (anonymous) income at time 1 with respect to the corresponding non-anonymous (anonymous) income at time 0, such an elasticity being measured via the concept of relative concentration curve (see, Kakwani, 1980 for more details). These concepts of distributional change and of elasticity of income at one period with respect of that of another period are then also related to the notions of σ- and β-convergences. This approach is then extended to compute non-anonymous and anonymous growth rates of another characteristic, such as the level of education, with respect to income. Each of the concepts previously mentioned is represented by some index related to the famous Gini index and graphical illustrations derived from the concept of Lorenz and relative concentration curves are also given. Empirical illustrations based on Indian states data on literacy rates confirm the usefulness of the approach introduced in this paper.

2. Generalizations of the Gini Index and the measurement of distributional change, convergence and pro-poor growth: the case of unconditional convergence

Among the numerous algorithms which have been proposed to compute the famous Gini index (see, Yitzhaki, 1998), some (see, Pyatt, 1976, and Silber, 1989) have expressed this index in matricial form. Silber (1989) has thus proposed to express the Gini index of income inequality I_G as

$$I_G = e'Gs \qquad (1)$$

where e' is a 1 by n row vector whose elements are the individual population shares $(\frac{1}{n})$, s is a n by 1 column vector of the individual income shares s_i and G is a n by n square matrix whose typical element g_{ij} is equal to 0 if $i = j$, to -1 if $j > i$ and to +1 if $i > j$, n being the number of individuals. Note that the income shares s_i have to be ranked by decreasing individual income.

2.1. A non-anonymous analysis of pro-poor income growth

Silber (1994) has then shown that such an approach could be extended to the measurement of distributional change. He proposed two measures of distributional change, one J_{GP}, a population weighted measure, and another one, J_{GI}, which is income-weighted and is defined as follows. Let us call s_0 the vector of the income shares s_{0i} at time 0 ($i = 1$ to n, where n is the number of individuals) and let us assume that these shares are ranked by decreasing values. Similarly let us call s_1 the vector of the individual income shares s_{1i} at time 1, the shares being ranked by their values at time 0. In other words if s_{0i} is the share of an individual who has rank i at time 0 (remember that rank 1 is given to the richest individual), then s_{1i} is the income share at time 1 of the individual who had rank i at time 0.

Let us now define a row vector $\sigma_0{}'$ of the individual shares s_{oi}, these shares being now ranked by decreasing values of the ratios (s_{1i}/s_{0i}). Similarly call w_1 the column vector of the individual shares s_{1i}, these shares being also ranked by decreasing values of the ratios (s_{1i}/s_{0i}).The ratio (s_{1i}/s_{0i}) may also be expressed as

$$(s_{1i}/s_{0i}) = \frac{(y_{1i}/n\overline{y_1})}{(y_{0i}/n\overline{y_0})} = \frac{(y_{1i}/y_{0i})}{(\overline{y_1}/\overline{y_0})} = \frac{1+(\Delta y_i/y_{0i})}{1+(\Delta \bar{y}/\overline{y_0})} \quad (2)$$

where y_{ti} is the income of individual i at time t, $\overline{y_t}$ the average income at time t, $\Delta y_i = (y_{1i} - y_{0i})$ and $\Delta \bar{y} = (\overline{y_1} - \overline{y_0})$.

Therefore the product $\sigma_0{}'Gw_1$ measures in a certain way the inequality of the individual (non-anonymous) growth rates $(\Delta y_i/y_{0i})$ and is the measure of distributional change J_{GI} proposed by Silber (1994).

The index $\sigma_0{}'Gw_1$ may be given the following graphical representation. On the horizontal axis we plot the cumulative values of the shares σ_{0i}, the shares being ranked by increasing values of the ratios (s_{1i}/s_{0i}). Similarly on the vertical axis we plot the cumulative values of the shares w_{1i}, the shares being again ranked by increasing values of the ratios (s_{1i}/s_{0i}). It can then be shown that the index $\sigma_0{}'Gw_1$ is equal to twice the area lying between the curve obtained and the diagonal.

2.2. Measuring the "elasticity" of the individual non-anonymous incomes at time 1 with respect to the values of the non-anonymous incomes at time 0.

Let us now continue our analysis of the non-anonymous distributional change and assume that the shares s_{0i} are ranked by decreasing values, which gives us the row vector $s_0{}'$. Similarly let us also rank the individual shares s_{1i} by decreasing values of the income shares they had at time 0, that is, by decreasing values of the shares s_{0i} . This gives us the column vector s_1, as it was defined previously. The product $s_0{}'Gs_1$ is then somehow a measure of the "elasticity" of the individual non-anonymous incomes at time 1 with respect to the values of the non-anonymous incomes at time 0. If this product is positive, it will show that, as a whole, the higher the income share at time 0, the higher the corresponding individual non-anonymous income share at time 1; if it is negative it will show that the lower the income shares at time 0, the higher the individual non-anonymous income shares at time 1.

Here again we obtain a graphical representation by plotting on the horizontal axis the cumulative values of the shares s_{0i} and on the vertical axis the cumulative values of the shares s_{1i}, both sets of shares being ranked this time by increasing values of the shares s_{0i}. The curve obtained may cross the diagonal once or several times. It can be shown that if any area below the diagonal is given a positive sign and any area above the diagonal a negative sign, then the sum of these signed areas is equal to half the value of the index $s_0'Gs_1$.

It should also be clear that the index expressed as $s_0'Gs_1$ measures in a certain way the degree of what is called β - convergence. More precisely when $s_0'Gs_1$, which varies between -1 and +1, is negative, it indicates that the income growth rates of the poor were higher than those of the rich, so that there was β - convergence. In other words if $s_0'Gs_1$ is negative, we can say that non-anonymous growth was pro-poor. On the contrary when $s_0'Gs_1$ is positive, it shows that the income growth rates were higher for the rich than for the poor so that there was income divergence and the non-anonymous growth was somehow pro-rich.

2.3. An anonymous analysis of pro-poor income growth

Until now the whole analysis has been non-anonymous in the sense that we always compared the income shares of an individual at time 1 with his/her income share at time 0. We can however implement an anonymous analysis (which we have evidently to implement if we do not have panel data but only two different cross sections, with the same number *n* of individuals[1]) in the sense that we would always compare the income of an individual who had rank i at time 1 with the income of the individual who had rank i at time 0, these individuals being generally different.

Let us now first call, as before, s_0 the vector of the shares s_{0i}, the latter being ranked by decreasing values (at time 0). Similarly let us call η_1 the vector of the shares s_{1i} , the latter being ranked by decreasing values also (at time 1). The typical share of the vector η_1 will be denoted as η_{1i}. Let us now call τ_0 (with typical share τ_{0i}) the vector of the shares s_{0i} where the latter are ranked by decreasing ratios (η_{1i}/s_{0i}) and φ_1 the vector of the shares η_{1i} where the latter are also ranked by decreasing ratios (η_{1i}/s_{0i}). The product $\tau_0{}'G\varphi_1$ measures then in a certain way the inequality in the anonymous growth rates, that is, of the growth rates obtained when we compare the i^{th} shares at time 0 and 1, this comparison being done for each i.

It should be stressed here that the higher the value of the index $\tau_0{}'G\varphi_1$, the greater the inequality in the anonymous growth rates (growth rates of the various centiles).

A graphical representation may be derived in a way similar to that which was given previously to the product $s_0{}'Gs_1$ in the non-anonymous case.

2.4. Measuring the elasticity of the anonymous income shares at time 1 with respect to the anonymous income shares at time 0:

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[1] If, as is generally the case, the number of observations in both cross-sections is different, it is always possible to draw a random sample of the same size *n*, from each cross-section.

Finally if we compute the product $s_0{}'G\eta_1$ we compute somehow the elasticity of the anonymous income shares at time 1 with respect to the anonymous income shares at time 0.

Note that in a certain way the index $s_0'G\eta_1$ corresponds to the concept of σ-convergence because it is easy to observe that if $s_0'G\eta_1$ is negative, the growth rates of the poor (which may be different individuals at time 1 and 0) were generally higher than those of the rich, so that inequality decreased while if $s_0'G\eta_1$ is positive, growth was pro-rich so that inequality increased. Remember that in the present case the poor are not necessarily the same in periods 0 and 1, and similarly for the rich.

3. Generalizations of the Gini Index and the measurement of distributional change, convergence and pro-poor growth: the case of conditional convergence

Assume now that in addition to knowing the incomes of the individuals at times 0 and 1, we also know their educational levels at times 0 and 1.

3.1. Computing distributional change, convergence and pro-poor growth for educational levels:

We can apply the same definitions of the four indices that have been given in section 2, and compute again four different Gini-related indices. To simplify the notations, we will use the same names for the vectors as those used previously in the case of income, but apply them now to educational levels.

In the non-anonymous case, we will first compute the degree of inequality in the individual growth rates in educational levels, what was called $\sigma_0{}'Gw_1$ in section 2, when we analyzed the income distributional change.

Then we can compute somehow an index measuring the degree of β-convergence, that is, of individual non anonymous pro-"poor education" growth, what was called $s_0{}'Gs_1$ in section 2 in the case of income.

In the anonymous case we can similarly first compute the degree of inequality in the anonymous growth rates in educational levels of the various centiles, what was called $\tau_0{}'G\varphi_1$ in section 2 when we analyzed income distributional change.

Finally we can compute an index measuring somehow the degree of σ-convergence, that is, an index of anonymous pro-"poor education" growth. This corresponds to the index $s_0'G\eta_1$ that we computed in section 2 in the case of income growth.

3.2. Computing non-anonymous and anonymous growth rates of individual educational levels with respect to individual incomes

Let us call μ_0 the vector of educational shares at time 0 ranked by decreasing income at time 0 and μ_1 the vector of educational shares at time 1 ranked by decreasing income at time 0. The product $\mu_0'G\mu_1$ is then an index which measures the relationship between the individual growth rates in educational levels and the corresponding individual incomes. If the index $\mu_0'G\mu_1$ is positive, it means that the non-anonymous growth rates in educational levels were as a whole higher for higher incomes while if it is negative it implies that the growth rates in educational levels were generally higher for the individuals with a low income.
Finally let us call, as before, μ_0 the vector of educational shares at time 0 ranked by decreasing income at time 0 and θ_1 the vector of educational shares at time 1 ranked by decreasing income at time 1. The product $\mu_0'G\theta_1$ is then an index which measures the relationship between the growth rates in educational levels of the various centiles and the corresponding incomes of these centiles.

3.3. Deriving indices measuring somehow the degree of conditional (on income) (non- anonymous) $\beta-$ convergence and (anonymous) σ-convergence.

Starting with the non-anonymous case, we defined previously an index $s_0'Gs_1$ measuring somehow the degree of (non-anonymous) $\beta-$ convergence in educational levels, and an index $\mu_0'G\mu_1$ measuring the relationship between the individual growth rates in educational levels and the corresponding individual incomes.
The difference between the former and the latter index may then be considered as a measure of the conditional (on income) $\beta-$ convergence. Given that, as was stressed previously, a negative index is a sign of pro-poorness, we can conclude that if this difference is negative (positive), the growth rates in educational levels were generally

higher for individuals having low (high) values of the other determinants (income excluded) of these growth rates in individual educational levels.

Finally in the anonymous case, we defined an index $s_0{}'G\eta_1$ measuring somehow the degree of σ-convergence in educational levels and an index $\mu_0{}'G\theta_1$ measuring the relationship between the growth rates in educational levels of the various centiles and the corresponding incomes of these centiles. The difference between the former and the latter index may then be considered as a measure of the conditional (on income) σ−convergence. If this difference is negative (positive), we can conclude that the growth rates in the educational levels of the various centiles were generally higher, the lower (higher) the level of the other (than income) determinants of educational levels.

Simple illustrations of all the indices and relative concentration curves derived in the previous sections are presented in Appendix A.

4. Empirical Applications

Despite strong economic growth in the last two decades, illiteracy, especially among women, remains a major concern for India. Only about 65 percent of the women (age 7 and above) were literate in 2011, the corresponding percentage for men being 82%. Women literacy rates are of significance since they are instrumental in improving key quality of life indicators such as lowering infant mortality rates, raising life expectancy, boosting school enrollment, and increasing women's representation in legislatures. Recent efforts at improving literacy rates include the passage by the Indian Government in 2010 of the Right to Education Act which promises free and compulsory elementary education to all children between 6 and 14 years.

4.1 Data sources

The National Family Health Survey (NFHS) is a large-scale, multi-round survey conducted in a representative sample of households throughout India. Three rounds of the survey have been conducted since 1992-1993. We use data from the last two rounds, namely NFHS-2 for 1998/99 and NFHS-3 for 2005/06.[2]

The survey primarily collects information on health indicators such as fertility, infant and child mortality, reproductive health, nutrition. Additionally data on indicators of standard of living such as type of flooring and roofing, the source of drinking water, availability of electricity and ownership of consumer durables such as fan, radio, sewing machine, refrigerator, bicycle, motorcycle and car is also collected. Since NFHS is not a panel data set at the individual level, we use data at the state level. We compile data on the number of women between the age of 15-49 who are literate in each state and calculate states' share of literate women.[3] Literate women are those who have either completed at least grade six or passed a simple literacy test conducted as part of the survey. Table 1 lists states' share of literate women in the population. In both years, the state of Maharashtra had the highest share of literate women.

4.2. Indices for the Non-Anonymous Case

In the non-anonymous case we always compare the literacy share of a state i in 1998/99 with the same state's literacy share in 2005/06, regardless of its ranking in 2005/06. Recall that s_0 is the vector of literacy shares (s_{0i}), in 1998/98 and s_1 is the vector of literacy shares (s_{1i}) in 2005/06 where both are ranked by decreasing values of s_{0i}.

Inequality in the growth rates of the states

In Figure 1.1, we plot a curve showing cumulative literacy shares (σ_{0i}) in 1998/99 and cumulative literacy shares (w_{1i}) in 2005/06 by ranking states by increasing values of the ratio (s_{1i}/s_{0i}). This ranking is in fact identical to ranking states by increasing growth rates of literate women. The curve in Figure 1.1 shows the

[2] The latest round for 2014/15 is out in the field. The NFHS-2 sample covers 99 percent of India's population living in all 26 states. It has a representative sample of about 91,000 ever-married women, aged 15-49 while NFHS-3 has data on more than 230,000 women aged 15-49 and men aged 15-54 from all states in India

[3] We do not assign population weights to states while calculating the state's share of literate women; we plan in the future to use such weights and re-estimate the indices to check their robustness. Note that in this analysis we use the weighted number of observations (weighted by sample design) in each state as reported in the NFHS data files.

inequality of growth rates in women's literacy across states. The index $\sigma_0{}'Gw_1$ is calculated by after ranking each vector by decreasing values of the ratio (s_{1i}/s_{0i}). The index is equal to twice the area lying between the curve in Figure 1.1 and the diagonal and varies between 0 and 1. The estimated value of the index is equal to 0.27 (Table 2) and suggests low (non-anonymous) inequality levels among states' literacy growth rates.

Elasticity of the literacy rates in 2005/06 with respect to the literacy rates in 1998/99:

In Figure 2.1, we plot another curve showing cumulative literacy shares (s_{0i}) in 1998/99 and cumulative literacy shares (s_{1i}) in 2005/06 but in this case we rank the states by increasing values of s_{0i}. The curve obtained crosses the diagonal once. If any area below the diagonal is given a positive sign and any area above the diagonal a negative sign, then the sum of these signed areas is equal to half the value of the index $s_0'Gs_1$, where the shares in the two vectors are arranged by decreasing values of s_{0i}. The index varies between -1 and +1 and its estimated value is equal to -0.09 (Table 2). The index $s_0{}'Gs_1$ is somehow a measure of the "elasticity" of the non-anonymous state literacy in 2005/06 with respect to the values in 1998/99. Though the value of the index is close to 0, a negative sign suggests that the lower the literacy shares were in 1998/99, the higher the shares were in 2005/06. Thus there is evidence of slight (unconditional) β - convergence implying that the non-anonymous growth in literacy among states was pro-poor education.

Measuring the degree of "income pro-poorness" of the growth rates in literacy levels

In addition to data on literacy shares, to estimate this measure, we compile data on per capita state income(y_{0i}) in 1998/99. μ_0 and μ_1 are two vectors of literacy shares (s_{0i}) and (s_{1i}) ranked by per capita state income (y_{0i}) in 1998/99. In Figure 3.1, we plot a curve relating μ_0 and μ_1 by ranking the states by increasing values of y_{0i}. By assigning any area below the diagonal a positive sign and any area above the diagonal a negative sign, we find that the sum of these signed areas is equal to half the value of the index $\mu_0{}'G\mu_1$, where both vectors are ranked by decreasing values of y_{0i}. The index measures the relationship between state growth rates in literacy levels and corresponding state incomes. The estimated value of the index is 0.06 indicating that the growth in state literacy levels was not pro-poor income.

Note that the indices $s_0'Gs_1$ and $\mu_0'G\mu_1$ are both using the same data on states' share of literate women. The difference between the two is that in the former, the literacy shares are ranked by decreasing values of the literacy shares in 1998/99,(s_{0i}) and in the latter, they are ranked by decreasing values of income (y_{0i}). Hence the former measures the degree of non-anonymous $\beta-$ convergence in literacy levels and the latter measures the relationship between the growth rates in state literacy and state income levels. If we take the difference between the two indices $s_0'Gs_1$ and $\mu_0'G\mu_1$, it is equal to -0.15. This difference is somehow a measure of the conditional (on income) $\beta-$ convergence. Since a negative index indicates "pro-poorness" (either literacy "pro-poorness" or "income pro-poorness") we can conclude that, given the states' income, the growth rates in literacy rates were generally higher for those states which had low values of the other determinants of literacy.

4.3 Indices for the Anonymous Case

We now consider the anonymous case where we always compare the literacy share of a state which had rank i in 2005/06 with the literacy share of the state which had rank i in 1998/99, these states being generally, but not necessarily, different. Assume that s_0 is the vector of the shares s_{0i}, ranked by decreasing values of s_{0i} while η_1 is the vector of the shares s_{1i} , ranked by decreasing values of s_{1i}.

Inequality in literacy growth rates

Recall that τ_0 and φ_1 are vectors of the shares s_{0i} and η_{1i} respectively, where both shares s_{0i} and $\eta_{1i}$are ranked by the ratio(η_{1i}/s_{0i}). In Figure 1.2, we plot a curve relating τ_0 and φ_1 by ranking both by increasing values of the ratio. The curve depicts the inequality of the growth rates in women's literacy when we compare the i^{th} state in 1998/99 and 2005/06, this comparison being done for each $i = 1, \dots 29$. The index $\tau_0'G\varphi_1$, where both vectors are ranked by decreasing values of the ratio,(η_{1i}/s_{0i}), is equal to twice the area lying between the curve in Figure 1.2 and the diagonal. The index value varies between 0 and 1 and in this case it is equal to 0.08 (Table 2). Such a relatively low value of this index indicates that there were no big differences in the *anonymous* growth rates of the various states (as mentioned previously anonymity refers here to the fact that we compare the literacy rate of the state which had rank *i* in 2005/06 with the literacy rate of the state which had this same rank *i* in 1998/99, these states being usually different).

Computing the "anonymous" elasticity of the literacy rates in 2005/06 with respect to the literacy rates in 1998/99:

In Figure 2.2, we plot a curve which shows cumulative literacy shares (s_{0i}) in 1998/99 and literacy shares (η_{1i}) in 2005/06, each ranked by respective increasing values. As seen in Figure 2.2, the curve almost overlaps the diagonal. If any area below the diagonal is given a positive sign and any area above the diagonal a negative sign, then the sum of these signed areas is equal to half the value of the index $s_0' G \eta_1$, where each vector is ranked by respective decreasing values. As expected from the curve in Figure 2.2, the estimated value of the index is equal to 0 (Table 2). The index $s_0' G \eta_1$ measures "elasticity" of the anonymous state literacy shares in 2005/06 with respect to the values in 1998/99. The index varies between -1 and 1 and measures somehow the degree of σ - convergence because had this index been negative (positive), we would have concluded that the anonymous growth rates in literacy had been generally higher when the literacy ranking was low (high), so that the between states inequality in literacy rates would have decreased (increased). Since we found that this elasticity was close to zero we can conclude that between 1998/99 and 2005/06 there was no sign of convergence in anonymous literacy rates, that is, growth in literacy rates was not pro-poor.

Measuring the degree of "income pro-poorness" of the anonymous growth rates in literacy levels

Recall that μ_0 is a vector of literacy shares (s_{0i}) ranked by decreasing values of per capita state income in 1998/99, (y_{0i}), while θ_1 is a vector of literacy shares (s_{1i}) ranked by decreasing values of per capita state income in 2005/06, (y_{1i}). In Figure

3.2, we plot a curve relating μ_0 and θ_1 by ranking each vector by increasing values. If any area below the diagonal is assigned a positive sign and any area above the diagonal a negative sign, it can be shown that the sum of these signed areas is equal to half the value of the index $\mu_0{}'G\theta_1$. The index measures the relationship between anonymous growth rates in state literacy levels and state incomes. The estimated value of the index is 0.07 indicating that the anonymous growth in state literacy levels was not pro-poor income.

As in the non-anonymous case, we compute the difference between $s_0'G\eta_1$ and $\mu_0{}'G\theta_1$ and find that it is equal to -0.06. Since $s_0'G\eta_1$ measures somehow the degree of $\sigma-$convergence in literacy levels, that is, while $\mu_0{}'G\theta_1$ measures the relationship between the anonymous growth rates in state literacy and state income levels, the difference gives a measure of the conditional (on income) $\sigma-$convergence. Since such a difference was found to be negative and given that a negative index indicates that the growth in literacy rates was pro-poor, we can conclude that the other (than income) determinants of anonymous state literacy, favored somehow more those "anonymous" states for which the value of these determinants was low.

4.3 Summary of Empirical Results

Analyzing the regional distribution of literate women in India, we found that there were no big differences in (both the non-anonymous and anonymous) growth rates in literacy among the states. There was evidence of weak (unconditional) β - convergence implying that the non-anonymous growth in literacy among states was slightly pro-poor education; growth rates in literacy levels were slightly higher for states with lower literacy levels. However there was no sign of σ- convergence, meaning anonymous growth in literacy was not pro-poor education.

Furthermore when we combined data on state literacy with per capita state income levels, we found that both the non-anonymous and the anonymous growth in state literacy levels were not pro-poor income; growth rates in literacy levels were slightly higher for states with higher incomes. In the non-anonymous case, we found evidence suggesting conditional (on income) $\beta-$ convergence; given the states' income, the growth rates in literacy levels were generally higher for those states which had low values of the other determinants of literacy. In the anonymous case, we found evidence suggesting conditional (on income) $\sigma-$convergence; determinants (other than income) of anonymous state literacy, favored somehow more those “anonymous” states for which the value of these determinants was low.

Table 1: State-wise share of literate women

States	1998-1999	2005-2006
Andhra Pradesh	0.03	0.04
Arunachal Pradesh	0.01	0.01
Assam	0.04	0.03
Bihar	0.04	0.01
Chattisgarh	0.01	0.02
Delhi	0.04	0.04
Goa	0.02	0.05
Gujarat	0.05	0.03
Haryana	0.05	0.02
Himachal Pradesh	0.05	0.04
Jammu & Kashmir	0.02	0.02
Jharkhand	0.01	0.01
Karnataka	0.05	0.04
Kerala	0.06	0.06
Madhya Pradesh	0.05	0.03
Maharashtra	0.07	0.09
Manipur	0.02	0.05
Meghalaya	0.01	0.02
Mizoram	0.02	0.03
Nagaland	0.01	0.05
Orissa	0.04	0.03
Punjab	0.04	0.04
Rajasthan	0.04	0.01
Sikkim	0.01	0.02
Tamil Nadu	0.06	0.06
Tripura	0.02	0.02
Uttaranchal	0.01	0.03
Uttar Pradesh	0.07	0.05
West Bengal	0.05	0.05

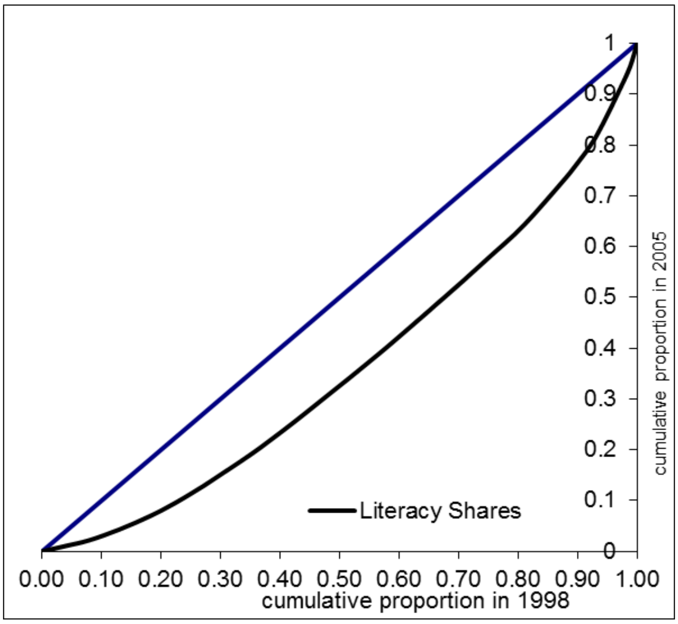
Table 2: Estimates of Proposed Indices

Index of Literacy Shares	Formula	Range	Non-Anonymous Estimates	Anonymous Estimates
Inequality in literacy growth rates	$\sigma_0{}'Gw_1$	0 to 1	0.27	0.08
Elasticity of the literacy rate in 2005/06 with respect to the literacy rate in 1998/99	$s_0'Gs_1$	-1 to 1	-0.09	0.00
Income pro-poorness of the elasticity of the literacy rate in 2005/06 with respect to the literacy rate in 1998/99	$\mu_0{}'G\mu_1$	-1 to 1	0.06	0.07

Figure 1: Inequality in state literacy growth rates

1.1 The non-anonymous case

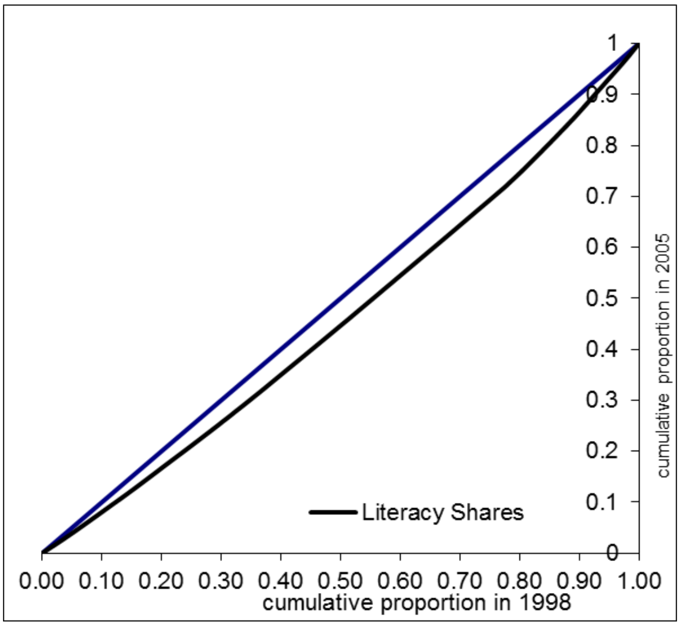
Graphical representation of the index $\sigma_0' G w_1$



Ranking states by increasing values of the ratio (s_{1i}/s_{0i})

1.2 The anonymous case

Graphical representation of the index $\tau_0{}' G \varphi_1$

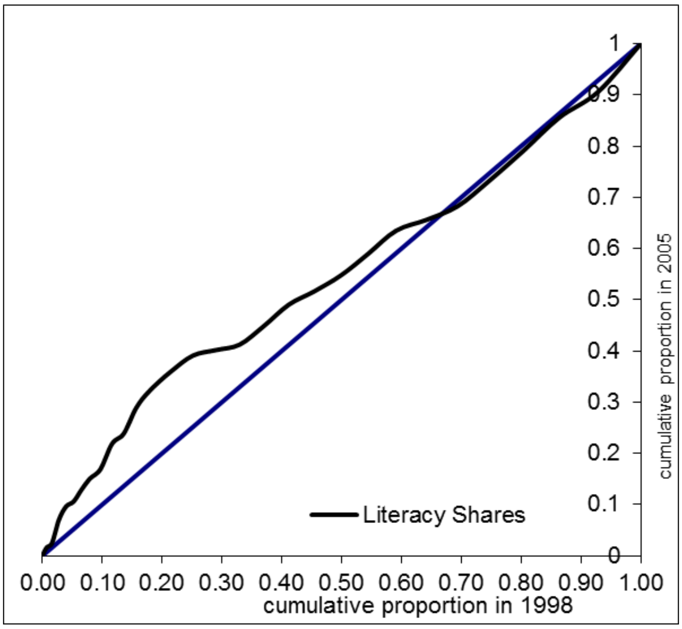


Ranking states by increasing ratio (η_{1i}/s_{0i})

Figure 2: Elasticity of the literacy rate in 2005/06 with respect to the literacy rate in 1998/99

2.1 The non-anonymous case

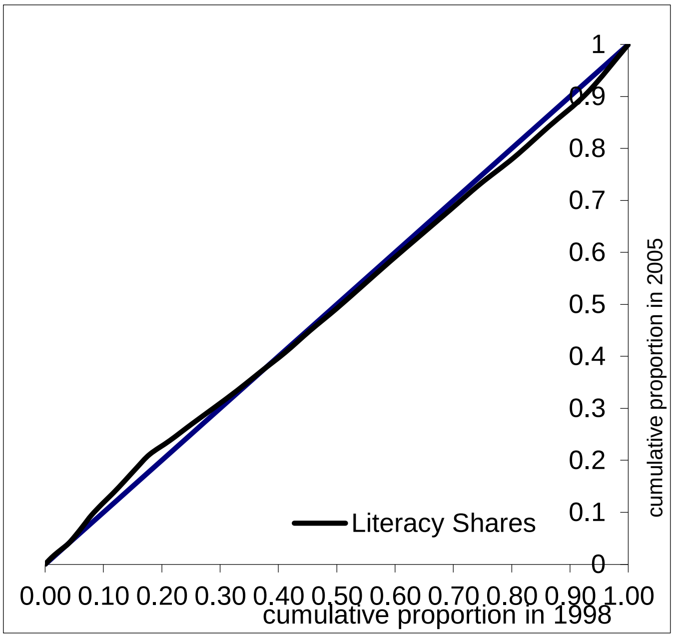
Graphical representation of the index $s_0' G s_1$



Ranking states by increasing values (s_{0i})

2.2 The anonymous case

Graphical representation of the index $s_0' G \eta_1$

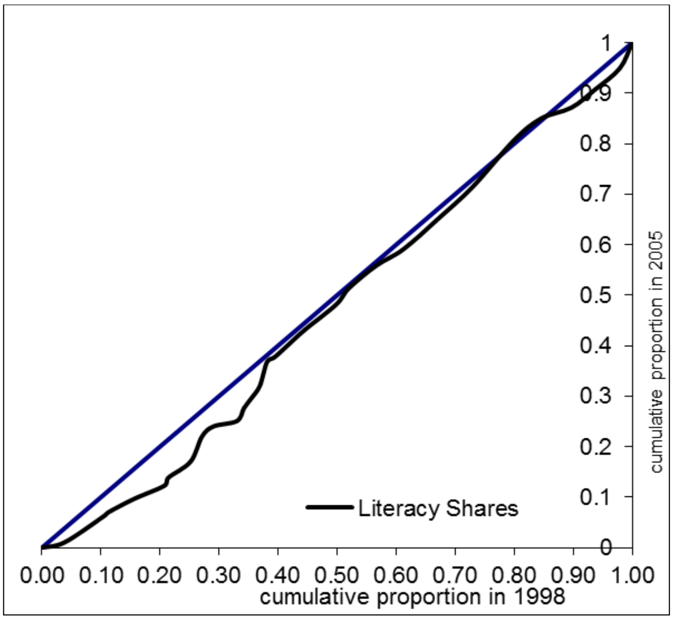


Ranking states by increasing values (s_{0i}) and $\left(\eta_{1i}\right)$

Figure 3: Income pro-poorness of the elasticity of the literacy rate in 2005/06 with respect to the literacy rate in 1998/99

3.1 The non-anonymous case

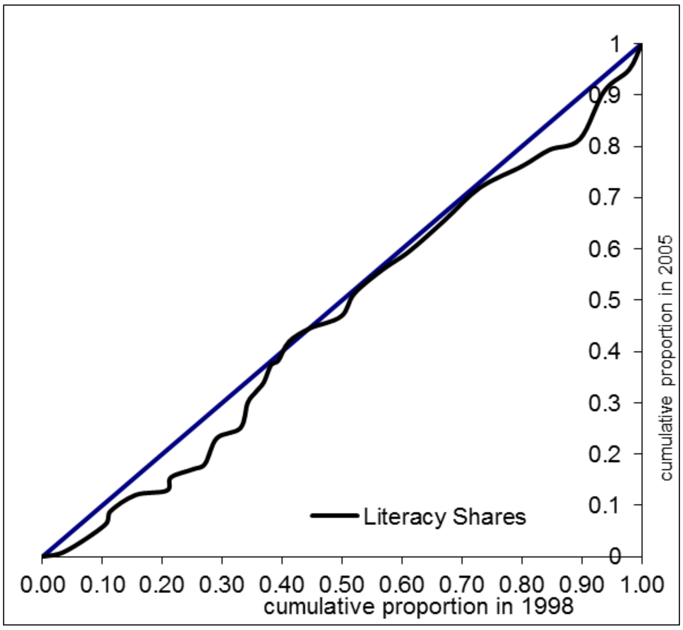
Graphical representation of the index $\mu_0{}'G\mu_1$



Ranking states by increasing income values (y_{0i})

3.2 The anonymous case

Graphical representation of the index $\mu_0{}'G\theta_1$



Ranking states by increasing values (μ_{0i}) and (θ_{1i}).

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Appendix A: A simple numerical illustration

<u>Measuring the inequality of the individual (non-anonymous) growth rates</u>:

Table 1 below gives basic data for a simple numerical illustration while Table 2 presents the values of the vectors σ_0 and w_1 . It is easy to derive that on the basis of the data shown there $\sigma_0' G w_1 = 0.45802$. Note that the vector σ_0' is expressed as $\sigma_0' = [.02\ .32\ .06\ .20\ .40]$ while the transpose $w_1{}'$ of the vector w_1 is written as $w_1' = [.400\ .267\ .033\ .100\ .200]$

Table 1: Original data

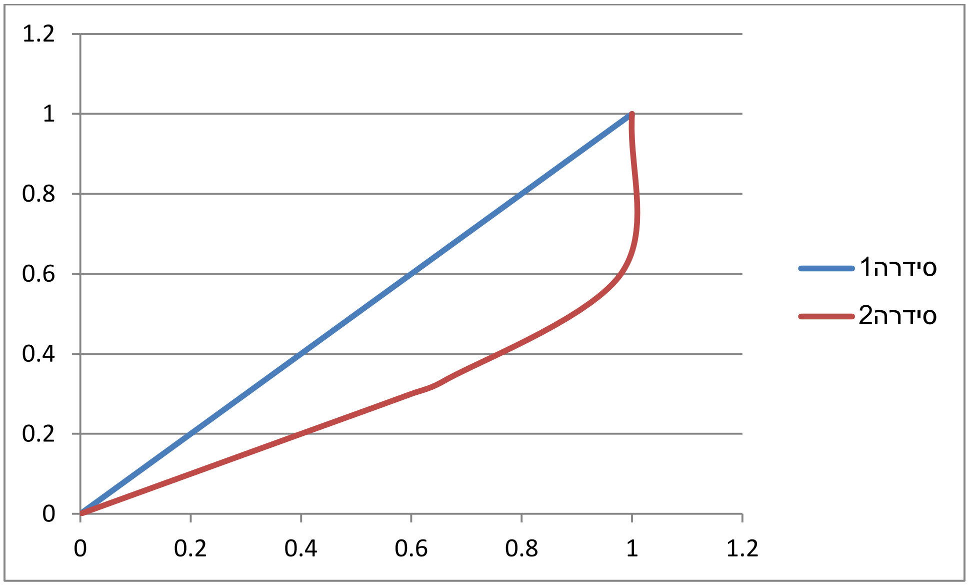
Name of individual	**Income at time 0**	**Income share at time 0 (vector s_0)**	**Income at time 1**	**Income share at time 1 (vector s_1)**	**Ratios (s_{1i}/s_{0i})**	**Ranking of the individuals (by decreasing values) of the ratios (s_{1i}/s_{0i})**
A	100	0.40	60	0.200	0.5	4
B	80	0.32	80	0.267	0.83	2
C	50	0.20	30	0.100	0.5	5
D	15	0.06	10	0.033	0.55	3
E	5	0.02	120	0.400	20	1
TOTAL	250	1.00	300	1.000		

Table 2: Computing $\sigma_0{}' G w_1$

Name of individual	**Vector $\sigma_0{}'$**	**Vector w_1**	**Ratios (w_{1i}/σ_{0i})**	**Ranking of the individuals (by decreasing values) of the ratios (s_{1i}/s_{0i})**
A	0.02	0.400	20	1
B	0.32	0.267	0.83	2
C	0.06	0.033	0.55	3
D	0.40	0.200	0.5	4
E	0.20	0.100	0.5	5
TOTAL	1.00	1.00		

The graphical representation of this index $\sigma_0{}' G w_1$ is given below in Figure 1.

Figure 1: Graphical representation of the index $\sigma_0' G w_1$



<u>Measuring of the "elasticity" of the individual non-anonymous incomes at time 1 with respect to the values of the non-anonymous incomes at time 0</u>:

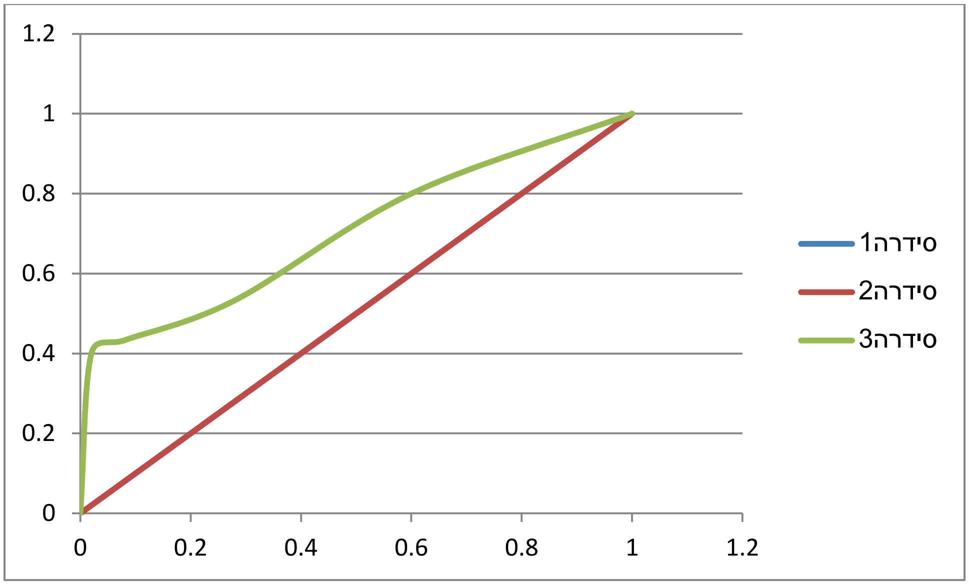
Using the simple numerical illustration of Table 1, it is easy to derive that $s_0' G s_1 = -0.398$, so that as a whole, individuals having a high income share at time 0 tend to have a low income share at time 1, and conversely. The details are given in Table 3 below.

Table 3: Computing $s_0' G s_1$

Name of individual	**Income at time 0**	**Income share at time 0 (vector s_0) ranked by decreasing income at time 0.**	**Income at time 1**	**Income share at time 1 (vector s_1) ranked by decreasing income at time 0**	**Ranking of the individuals for both vectors s_0 and s_1**
A	100	0.40	60	0.200	1
B	80	0.32	80	0.267	2
C	50	0.20	30	0.100	3
D	15	0.06	10	0.033	4
E	5	0.02	120	0.400	5

Here again we obtain a graphical representation by plotting on the horizontal axis the cumulative values of the shares s_{0i} and on the vertical axis the cumulative values of the shares s_{1i}, both sets of shares being ranked this time by increasing values of the shares s_{0i}. A graphical representation based on our numerical example is given Figure 2.

Figure 2: Graphical representation of the index $s_0' G s_1$

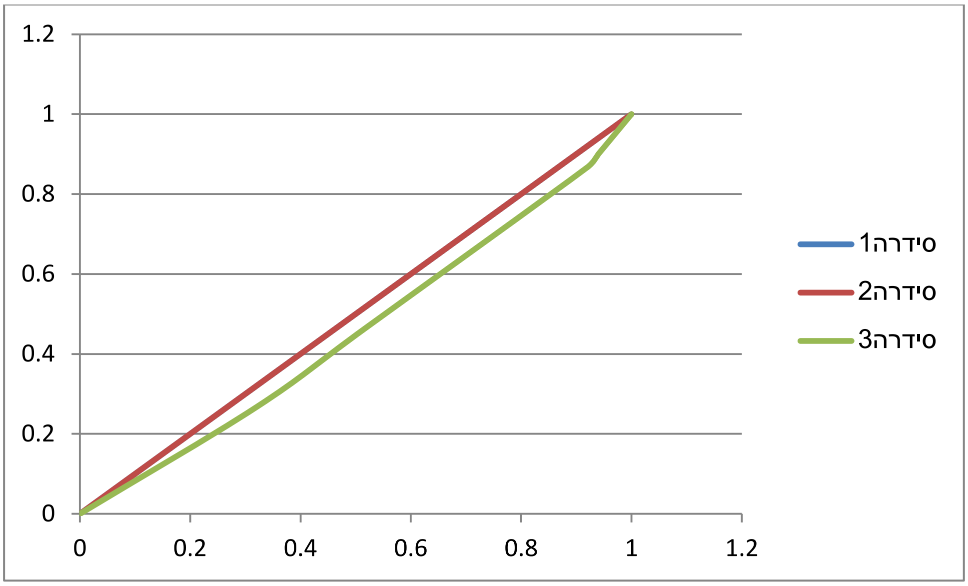


Note that the row vector $s_0{}'$ is expressed as $s_0{}' = [.04\ .32\ .2\ .06\ .02]$
while the transpose $s_1{}'$ of the vector s_1 is written as $s_1' = [.200\ .267\ .100\ .033\ .400]$.

<u>Measuring the inequality of the anonymous income growth rates</u>

On the basis of the numerical example of Table 1 we then find that $\tau_0{}' G \varphi_1 = 0.085$. The details are given in Table 4 below.
Note that the row vector $\tau_0{}'$ is expressed as $\tau_0{}' = [.06\ .02\ .40\ .20\ .32]$,
while the transpose $\varphi_1{}'$ of the vector φ_1 is written as $\varphi_1' = .100\ .033\ .400\ .233\ .267]$.
A graphical representation, based on the data of Table 4 and derived in a way similar to that in which Figures 1 and 2 were derived, is given in Figure 3.

Figure 3: Graphical representation of the index $\tau_0{}'G\varphi_1$



<u>Measuring the elasticity of the anonymous income shares at time 1 with respect to the anonymous income shares at time 0</u>

Using the numerical example of Table 4, we then find that $s_0' G\,\eta_1 = -0.04226$ where $s_0' = [0.40\ 0.32\ 0.20\ 0.06\ 0.02]$ and the transpose $\eta_1{}'$ of η_1 is expressed as $\eta_1{}' = [0.400\ 0.267\ 0.200\ 0.100\ 0.033]$.
Note that since the index $s_0' G\,\eta_1$ is negative, there was σ- convergence. This can be easily verified since the Gini index at time 0 may be shown, using (1), to be equal to 0.12 while the Gini index at time 1 is equal to 0.0724.

Figure 4: Graphical representation of the index $s_0' G\,\eta_1$

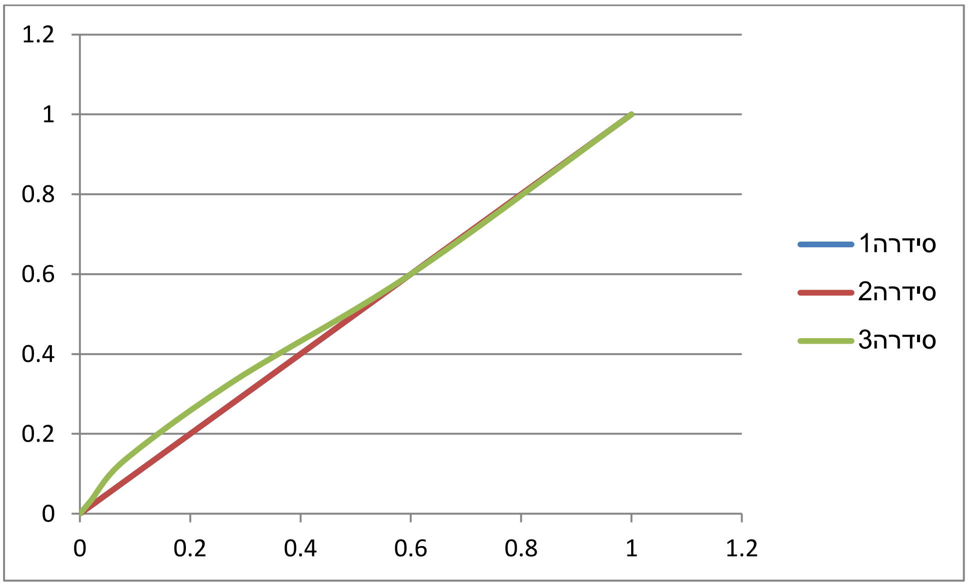


Table 4: Computing $\tau_0{}' G \varphi_1$ and $s_0' G \eta_1$

Income at time 0	**Income shares at time 0 ranked by decreasing values at time 0 (vector s_0)**	**Income at time 1**	**Income shares at time 1 ranked by decreasing values at time 0 (vector s_1)**	**Income shares at time 1 (vector η_1) ranked by decreasing values at time 1**	**Ratios** (η_{1i}/s_{0i})	**Ranking (by decreasing values) of the ratios** (η_{1i}/s_{0i})	**Vector** τ_0	**Vector** φ_1	**Ranking of vectors** τ_0 and φ_1
100	0.40	60	0.200	0.400	1	3	0.06	0.100	1
80	0.32	80	0.267	0.267	0.834	5	0.02	0.033	2
50	0.20	30	0.100	0.200	1	3	0.40	0.400	3
15	0.06	10	0.033	0.100	1.666	1	0.20	0.200	4
5	0.02	120	0.400	0.033	1.666	1	0.32	0.267	5

Computing distributional change, convergence and pro-poor growth for educational levels

The educational levels and the incomes of individuals A, B, C, D and E at times 0 and 1 are given in Table 5 below.

Table 5: Income and educational levels of the various individuals

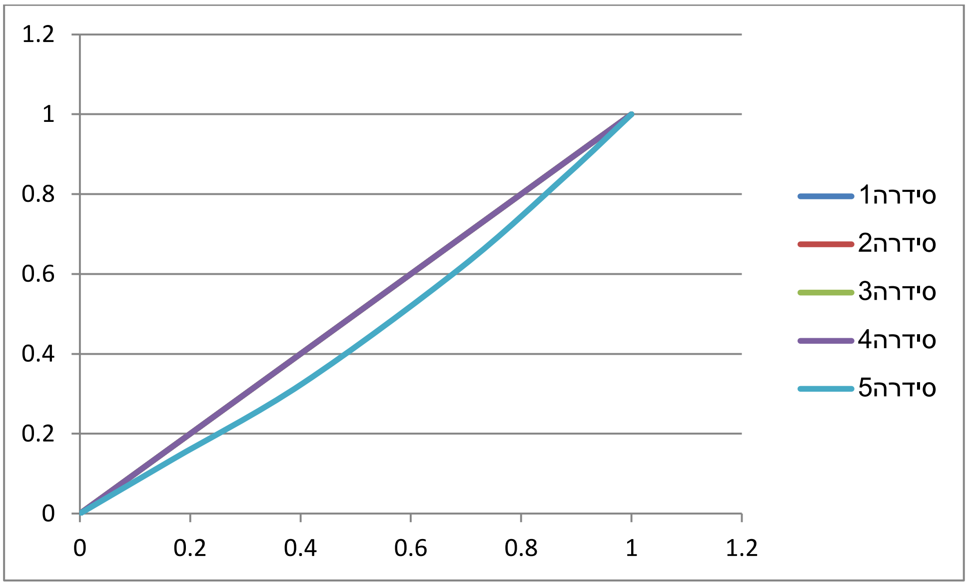
Name of individual	**Income at time 0**	**Educational level at time 0 (vector e_0)**	**Shares of educational levels at time 0**	**Income at time 1**	**Educational level at time 1 (vector e_1)**	**Shares of educational levels at time 1**
A	100	12	0.286	60	15	0.288
B	80	7	0.167	80	7	0.135
C	50	10	0.238	30	10	0.192
D	15	5	0.119	10	8	0.154
E	5	8	0.190	120	12	0.231

The inequality in the individual non anonymous growth rates in educational levels (what was called $\sigma_0' G w_1$ when we analyzed the income distributional change) is equal to 0.100 (see the data that are at the basis of this computation in Table 6).

Table 6: Computing $\sigma_0' G w_1$ for educational levels

Name of individual	**Vector σ_0'**	**Vector w_1**	**Ratios (w_{1i}/σ_{0i})**	**Ranking of the individuals (by decreasing values) of the ratios (s_{1i}/s_{0i})**
D	0.119	0.154	1.292	1
E	0.190	0.231	1.212	2
A	0.286	0.288	1.010	3
C	0.238	0.192	0.808	4
B	0.167	0.135	0.808	5
TOTAL	1.00	1.00		

Figure 5: Graphical representation of the index $\sigma_0' G w_1$ for educational levels

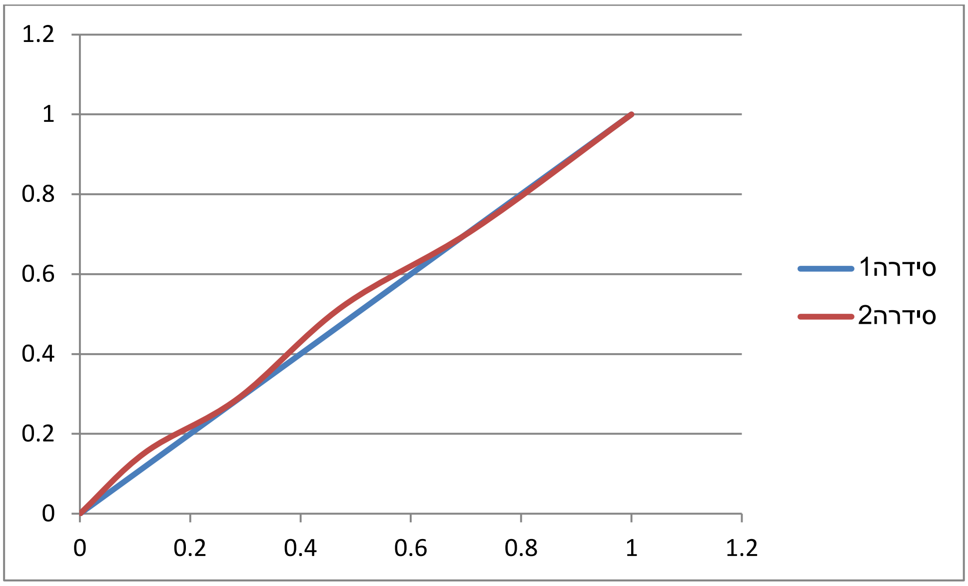


The index measuring the degree of individual non anonymous pro-"poor education" growth (what was called $s_0' G s_1$) is equal to -0.028, so that individual educational growth was slightly pro-"poor education" (see the details of the computation in Table 7).

Table 7: Computing $s_0' G s_1$ for educational levels

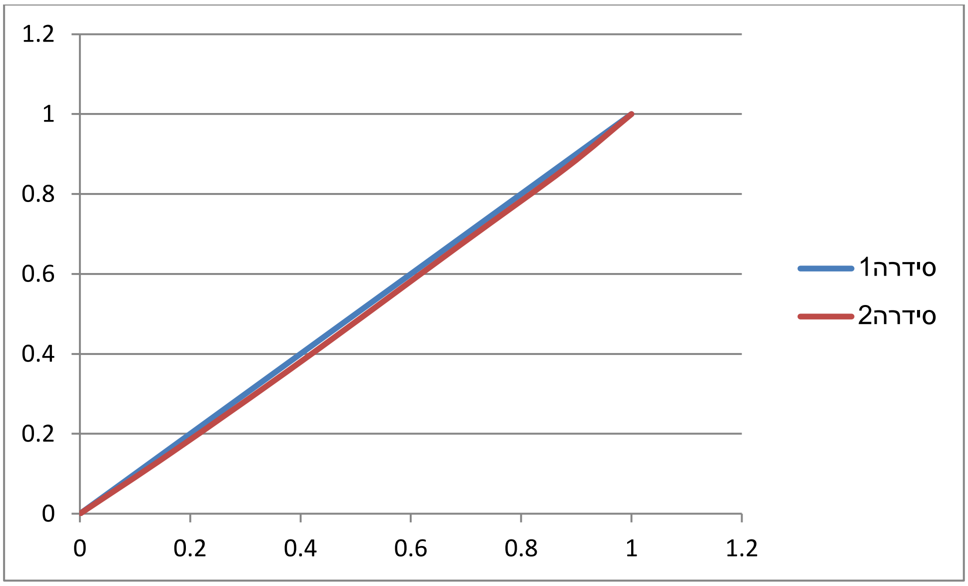
Name of individual	Shares of educational level at time 0 ranked by decreasing values at time 0 (vector s_0)	Share of educational level at time 1 ranked by decreasing values at time 0 (vector s_1)	Ranking of the individuals for both vectors s_0 and s_1
A	0.286	0.288	1
C	0.238	0.192	2
E	0.190	0.231	3
B	0.167	0.135	4
D	0.119	0.154	5

Figure 6: Graphical representation of the index $s_0' G s_1$ for educational levels



In the anonymous case if we compute the degree of inequality in the growth rates in educational levels of the various centiles (quintiles in our case), what was called previously $\tau_0{}' G \varphi_1$ when we analyzed the income distributional change) we find out that this index of inequality is equal to 0.029 (see the data that are at the basis of this computation in Table 8). The graphical representation is given in Figure 7.

Figure 7: Graphical representation of the index $\tau_0{}' G \varphi_1$ for educational levels



Finally when we compute the index measuring somehow the degree of σ-convergence, that is an index of anonymous pro-”poor education” growth (which corresponds to the index $s_0'G\eta_1$ in the case of income growth), we find out (see, Table 8) that such an index is equal to -0.006 so that anonymous growth was very slightly pro-“poor education”. The graphical representation is given in Figure 8.

Figure 8: Graphical representation of the index $s_0'G\eta_1$ for educational levels

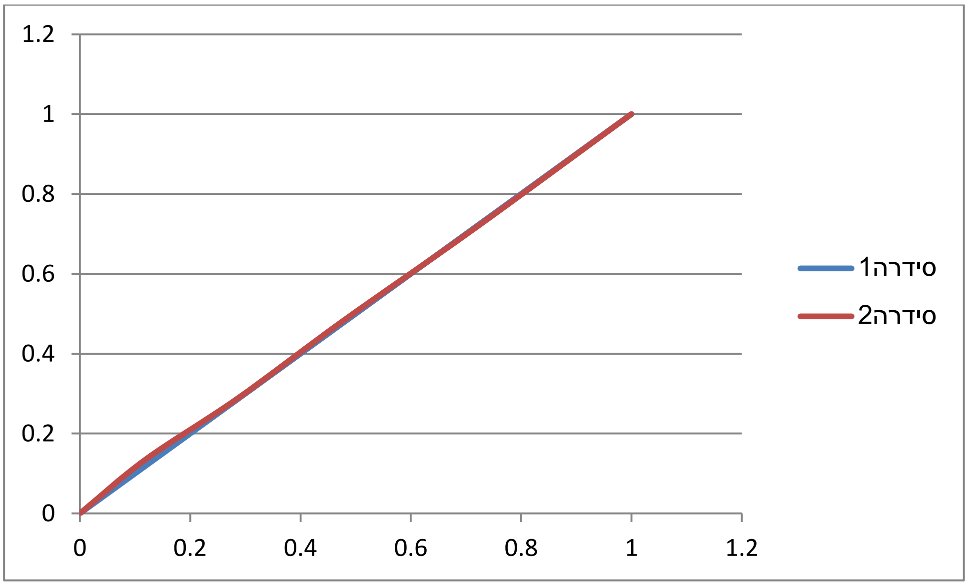


Table 8: Computing $\tau_0{}'G\varphi_1$ and $s_0'G\eta_1$ for educational levels

Educational shares at time 0 ranked by decreasing values at time 0 (vector s_0)	**Educational shares at time 1 ranked by decreasing values at time 0 (vector s_1)**	**Educational shares at time 1 (vector η_1) ranked by decreasing values at time 1**	**Ratios** (η_{1i}/s_{0i})	**Ranking (by decreasing values) of the ratios** (η_{1i}/s_{0i})	**Vector τ_0: educational shares at time 0 ranked by values) of the ratios (η_{1i}/s_{0i})**	**Vector φ_1: educational shares at time 1 ranked by values) of the ratios (η_{1i}/s_{0i})**	**Ranking of vectors τ_0 and φ_1**
0.286	0.288	0.288	1.010	3	0.119	0.135	1
0.238	0.192	0.231	0.969	4	0.190	0.192	2
0.190	0.231	0.192	1.010	2	0.286	0.288	3
0.167	0.135	0.154	0.923	5	0.238	0.231	4
0.119	0.154	0.135	1.131	1	0.167	0.154	5

Computing non-anonymous and anonymous growth rates of individual educational levels with respect to individual incomes

Table 9 summarizes the data that will be the basis for the computations in the present subsection.

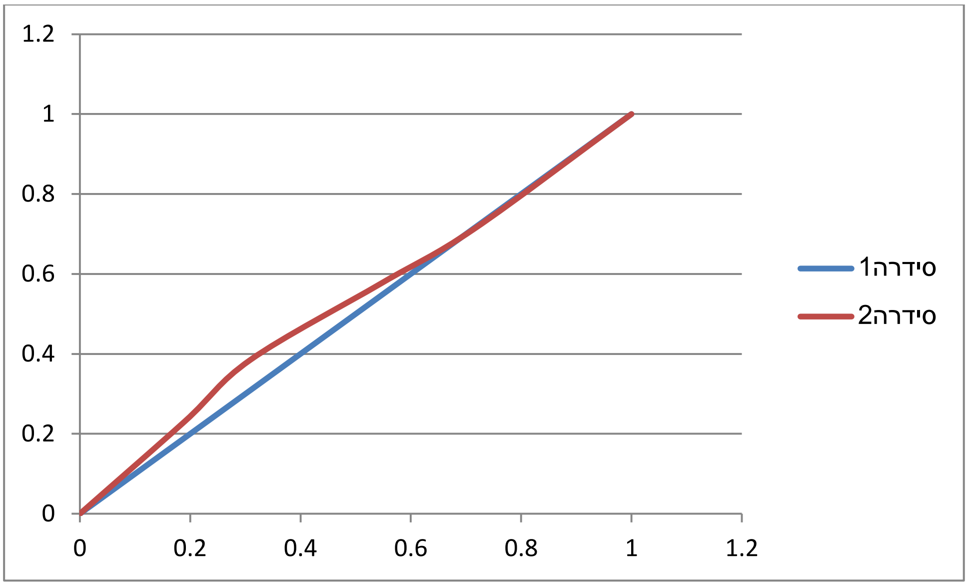
Table 9: Shares of educational levels by decreasing income: non anonymous approach.

Name of individual	Income at time 0 by decreasing levels	Shares of educational levels at time 0	Income at time 1	Shares of educational levels at time 1
A	100	0.286	60	0.288
B	80	0.167	80	0.135
C	50	0.238	30	0.192
D	15	0.119	10	0.154
E	5	0.190	120	0.231

If we call μ_0 the vector of educational shares at time 0 ranked by decreasing income at time 0, The corresponding row vector $\mu_0{}'$ will then be expressed as
$\mu_0' = .286\ 0.167\ 0.238\ 0.119\ 0.190$
Similarly let us call μ_1 the vector of educational shares at time 1 ranked by decreasing income at time 0. The corresponding row vector $\mu_1{}'$ will then be expressed as
$\mu_1' = 0.288\ 0.135\ 0.192\ 0.154\ 0.231$

The product $\mu_0{}'G\mu_1$ is then an index which measures the relationship between the individual growth rates in educational levels and the corresponding individual incomes. It is easy to find out that $\mu_0{}'G\mu_1 = -0.051$. In other words the growth in individual educational levels is slightly pro-“poor income” since the index $\mu_0{}'G\mu_1$ is slightly negative. The graphical representation is given in Figure 9.

Figure 9: Graphical representation of the index $\mu_0{}'G\mu_1$ for educational levels



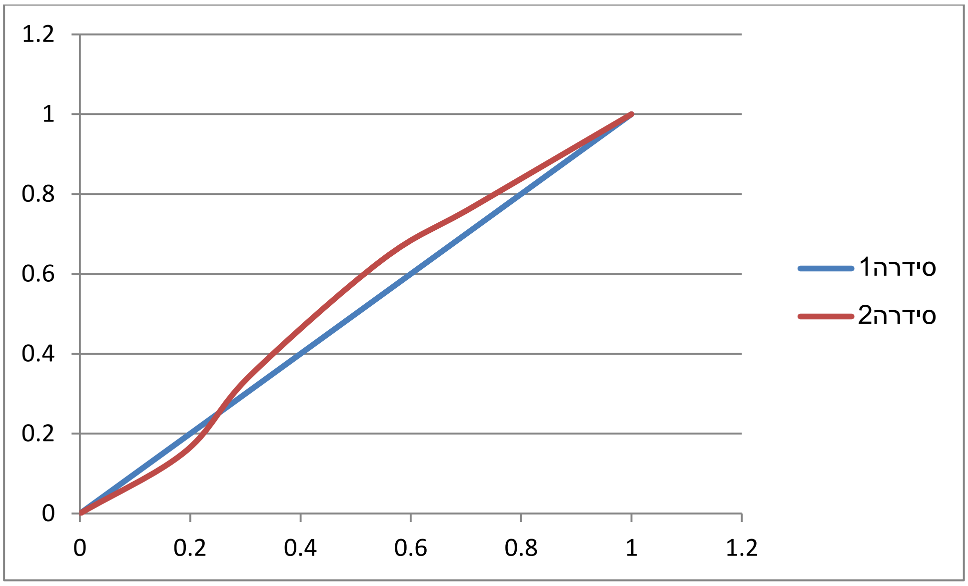
Finally let μ_0 be the vector of educational shares at time 0 ranked by decreasing income at time 0 and θ_1 the vector of educational shares at time 1 ranked by decreasing income at time 1. The corresponding row vector $\theta_1{}'$ will then be expressed as θ_1' =0.231 0.135 0.288 0.192 0.154(see, Table 10).

The product $\mu_0{}'G\theta_1$ is then an index which measures the relationship between the growth rates in educational levels of the various centiles (in our numerical example, quintiles) and the corresponding incomes of these centile (quintiles). It is easy to find out that $\mu_0{}'G\theta_1 = -0.062$. In other words the anonymous growth in individual educational levels of the various centiles (quintiles) is slightly pro-"poor income" since it the index $\mu_0{}'G\theta_1$ is slightly negative. The graphical representation is given in Figure 10.

Table 10: Shares of educational levels by decreasing income: the anonymous approach.

Quintile	**Income at time 0 by decreasing levels**	**Shares of educational levels at time 0**	**Income at time 1**	**Shares of educational levels at time 1**
1	100	0.286	120	0.231
2	80	0.167	80	0.135
3	50	0.238	60	0.288
4	15	0.119	30	0.192
5	5	0.190	10	0.154

Figure 10: Graphical representation of the product $\mu_0' G \theta_1$



Appendix B: An alternative algorithm to compute the Gini index

Since using the G-matrix directly is impossible with many observations because this square matrix would be too big, the following algorithm could be used.

Assume the index we compute is $x'Gy$ where x' is a row vector of shares ranked in a specific way and whose typical element is x_i with $i = 1$ to n, n being the number of observations, and y is a column vector of shares ranked by some criterion and whose typical share is y_i. Then, given the definition of the *G*-matrix, it is easy to derive that the product $x'Gy$ my be also computed as

$$x'Gy = \sum_{i=1}^{n} x_i \left[\left(\sum_{j=1}^{i-1} y_j \right) - \left(\sum_{j=i+1}^{n} y_j \right) \right]$$