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Abstract

In this paper, a quarterly growth accounting data set is built for the Canadian business sector, and the [Diewert and Yu \(2012\)](#) estimates of annual productivity growth are revised and updated to reflect changes in the new national economic accounts and national balance sheets. These data enable one to study the contribution of productivity and the terms of trade to growth of real income and real output. Since the early 2000s, the contribution of the terms of trade became significant in real income growth whereas that of total factor productivity (TFP) growth was stagnant. Improvement in the terms of trade arises from a faster decline in the real import price relative to the real export price, in particular in manufacturing.

The quarterly growth accounting data are also compared with existing quarterly and annual estimates produced by Statistics Canada. In addition, we find that quarterly data present a somewhat different picture from the annual data regarding the effects of changes in TFP and in the terms of trade.

Keyword: growth accounting, productivity, terms of trade

1 Introduction

Two important determinants of real domestic income in an open economy are productivity and the terms of trade. As noted by [Krugman \(1994\)](#): “Productivity isn’t everything, but in the long run it is almost everything. A country’s ability to improve its standard of living over time depends almost entirely on its ability to raise its output per worker.” At the same time, an improvement in

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the terms of trade, that is an increase in the price of exports relative to the price of imports, will tend to increase domestic income and wealth.

In recent years, resource-rich small open economies, such as Canada, have been faced with large swings in their terms of trade. In addition to having implications for real domestic income, these swings have led to a significant reallocation of economic resources. However, large movements in the relative productivity of different sectors of an economy, or in the relative productivity of domestic versus foreign production could likewise lead to significant structural change. Whether they be driven by productivity differentials or shifts in the terms of trade, economic welfare depends on how well and how quickly an economy can adjust to such changes in economic circumstances (Duguay, 2006).

One challenge for policymakers interested in having a real-time assessment of the extent and implications of structural adjustment is data availability. In Canada, annual data sets of total factor productivity (TFP) have been produced by Statistics Canada (using a bottom-up industry-based approach) and by Diewert and Yu (2009, 2012) (using a top-down approach with National Accounts data). However, two weaknesses of annual data are that it is only available with a considerable lag and the low frequency of the data limits its usefulness for examining issues where meaningful economic variation and responses may occur within a year. For example, the delay in the publication of the annual data complicates real-time estimation and understanding of the drivers of potential output and the output gap in Canada, important economic measures for the conduct of monetary policy. Likewise, movements in the terms of trade can have implications for income, wealth, investment, employment, labour productivity, and possibly TFP, within a year.

In this paper, a quarterly time series of Canadian TFP is constructed using the top-down growth-accounting methodology of Diewert and Yu (2009, 2012). The availability of such a series will facilitate economic research and timely analysis and monitoring of structural economic developments. The advantage of the Diewert-Yu methodology is that, because it uses national accounts data which includes exports and imports, it provides a unified framework for measuring TFP and quantifying the contribution of the terms-of-trade to real income growth. By contrast, this is not currently possible using an industry-based bottom-up approach for Canada as exports and imports are not currently separated from other outputs and inputs in the Canadian input-output tables.

In other dimensions, the top-down approach has disadvantages relative to the bottom-up approach. As implemented in this paper, the top down methodology uses measures of labour and capital inputs which are subject to aggregation errors, potentially leading to biased estimates of TFP.¹ For example, aggregation errors may arise in the measurement of capital inputs—from hetero-

¹See Diewert and Yu (2012) for the sources of aggregation error in the top-down approach. It is unclear whether,

ogeneity of assets in depreciation rates and returns. Another challenge is that it doesn't provide the necessary decomposition to examine the role of the reallocation of inputs in aggregate productivity growth.

The outline of the paper is as follows. The next section performs the growth accounting exercise on Canadian data at both annual and quarterly frequencies. The annual analysis updates Diewert and Yu to implement the revised Canadian System of National Accounts (CSNA12) and revised national balance sheets data (NBSA12) that were introduced by Statistics Canada initially in 2012. The quarterly analysis is a new contribution. The robustness of the estimated quarterly TFP series is assessed according to its sensitivity to data used and assumptions made in the growth accounting exercise. The properties of the constructed quarterly TFP series are compared to those of the annual Diewert-Yu TFP series as well as to TFP as constructed by Statistics Canada. In this section, statistics are also compared between quarterly and annual data, suggesting the usefulness of quarterly growth accounting. The third section of the paper decomposes annual growth of real gross domestic income (GDI) into contributions from changes in TFP, the terms of trade, and real labour and capital inputs. This decomposition provides insights into important differences in the sources of real income growth in different episodes of Canadian economic history.

2 Growth accounting in the business sector

This section provides an overview of the methodology and data used to construct Canadian TFP. Both annual and quarterly TFP estimates are constructed. The annual constructions differ from those provided by Diewert and Yu, primarily owing to major revisions to the Canadian National Accounts data that were made subsequent to their analysis. The first subsection reviews the growth accounting framework and provides details on the level of disaggregation of the analysis. The second subsection reviews the revisions to the Canadian National Accounts data (CSNA12) and National Balance Sheet data (NBSA12), presents the new annual estimates based on CSNA12 and compares these estimates to those provided by Diewert and Yu as well as those of Statistics Canada. The third subsection reviews the quarterly data sources and presents the quarterly estimates of TFP. Differences between quarterly and annual TFP constructions are discussed.

2.1 Methodology

This section provides a brief review of the growth accounting methodology and describes the level of disaggregation of the subsequent analysis. To understand the relationship between real

overall, the bias is upward or downward.

income, TFP, input quantities, and relative prices, it is convenient to introduce some notation: y^t is the output vector of $m = 1, \dots, M$ products, y_m^t , with p^t the corresponding vector of p_m^t , the ratio of M output prices to a measure of the aggregate price level, such as the consumption price; x^t is the the input vector of $n = 1, \dots, N$ inputs, x_n^t with w^t be the corresponding vector of w_n^t , the ratio of N input prices to the same aggregate price level; and $g^t(p, x)$ is real income generated in the business sector. In this notation, the superscript denotes the time period.

The growth of real income can be decomposed into contributions from three components: TFP growth $\gamma^t(p, x)$, the rate of change or real output prices $\alpha^s(p^{t-1}, p^t, x)$, and the growth of real inputs $\beta^s(p, x^{t-1}, x^t)$. Total factor productivity (TFP) growth captures the change in real output between two periods assuming that in both periods the same amount of inputs (capital and labour) are used for production:

$$\gamma^t(p, x) = g^t(p, x)/g^{t-1}(p, x). \quad (1)$$

Such a change in output while keeping inputs constant is often interpreted as a change in technology. The growth of real income due to changes in prices measures the change in real income produced by the market sector that arises from the change in output prices between period $t - 1$ and period t , with the output produced using the technology in period s and some reference input level x :

$$\alpha^s(p^{t-1}, p^t, x) = g^s(p^t, x)/g^s(p^{t-1}, x). \quad (2)$$

Analogous to this concept, is the growth of real income due to changes in input quantities:

$$\beta^s(p, x^{t-1}, x^t) = g^s(p, x^t)/g^s(p, x^{t-1}). \quad (3)$$

Using these definitions, it can be shown that the growth rate of real income can be decomposed as follows:

$$g^t(p^t, x^t)/g^{t-1}(p^{t-1}, x^{t-1}) \approx \gamma^t \alpha^t \beta^t \quad (4)$$

where, as in [Diewert and Yu \(2012\)](#), under the assumption that the underlying real revenue function has a translog form, this expression holds exactly.

Following [Diewert and Yu \(2012\)](#), assuming the real revenue function has the translog form,

the measures of the three components of growth in real revenue can be constructed as follows.

$$\ln \alpha^t = \sum_{m=1}^M \frac{1}{2} \left[\frac{p_m^{t-1} y_m^{t-1}}{p^{t-1} \cdot y^{t-1}} + \frac{p_m^t y_m^t}{p^t \cdot y^t} \right] \ln \left(\frac{p_m^t}{p_m^{t-1}} \right); \quad (5)$$

$$\ln \beta^t = \sum_{n=1}^N \frac{1}{2} \left[\frac{w_n^{t-1} x_n^{t-1}}{w^{t-1} \cdot x^{t-1}} + \frac{w_n^t x_n^t}{w^t \cdot x^t} \right] \ln \left(\frac{x_n^t}{x_n^{t-1}} \right); \quad (6)$$

$$\gamma^t = \frac{g^t(p^t, x^t)/g^{t-1}(p^{t-1}, x^{t-1})}{\alpha^t \beta^t}. \quad (7)$$

where, α^t is the Törnqvist index of output price, and β^t is the Törnqvist index of input quantity. In this formulation, the aggregate real output price contribution can be decomposed into the product of separate price contributions:

$$\alpha^t = \alpha_1^t \alpha_2^t \cdots \alpha_M^t, \quad (8)$$

$$\ln \alpha_m^t = \frac{1}{2} \left[\frac{p_m^{t-1} y_m^{t-1}}{p^{t-1} \cdot y^{t-1}} + \frac{p_m^t y_m^t}{p^t \cdot y^t} \right] \ln \left(\frac{p_m^t}{p_m^{t-1}} \right). \quad (9)$$

The contribution from changes in quantities of all inputs has a similar exact decomposition:

$$\beta^t = \beta_1^t \beta_2^t \cdots \beta_N^t, \quad (10)$$

$$\ln \beta_n^t = \frac{1}{2} \left[\frac{w_n^{t-1} x_n^{t-1}}{w^{t-1} \cdot x^{t-1}} + \frac{w_n^t x_n^t}{w^t \cdot x^t} \right] \ln \left(\frac{x_n^t}{x_n^{t-1}} \right). \quad (11)$$

Additional details of the methodology are summarized in Appendix A.²

In the data constructions that follow, business sector output is disaggregated into $M = 21$ components: household final consumption expenditure, sales of businesses to non-businesses, government investment, business investments, changes in business inventories, exports and imports. Business investment includes investment in each of residential structures, non-residential structures, machinery and equipment, and intellectual property products (IPP). Exports and imports are both disaggregated into 13 products and services: farm, fishing and intermediate food products; energy products; metal ores and non-metallic minerals; metal and non-metallic mineral products; basic and industrial chemical, plastic and rubber products; forestry products and building and packaging materials; industrial machinery, equipment and parts; electronic and electrical equipment and

²Diewert and Yu (2012) construct annual measures of the growth of TFP in the business sector for the period of 1961-2011. As noted above, their main assumption that the real revenue function has the translog form, which makes the chained Törnqvist index an exact measure of growth. To measure output and inputs, Diewert and Yu use the data from the national economic accounts and national balance sheets. Output is the value added in the business sector that is measured using the final demand. Inputs include the quality-adjusted labour input and capital services. The Diewert-Yu estimates of historical annual TFP growth are substantially higher than the estimates by Statistics Canada, with the difference between the two estimates largely owing to differences in the period of 1961 to mid-1990s.

parts; motor vehicles and parts; aircraft and other transportation equipment and parts; consumer goods; and, services.³

The labour input is aggregated from hours worked and compensation of 36 types of workers (including 3 levels of education achievement, 3 age groups, 2 gender groups, and employees versus self-employed). The same data are used for the labour input series as in [Diewert and Yu \(2012\)](#) up to 2010. The data are extended to 2012 using series of labour inputs constructed from the Labour Force Survey (LFS) public use microdata files.

The capital service input is aggregated from stocks of 19 types of assets, weighted by the user cost of capital. Capital stocks are constructed using the perpetual inventory method. The constructed capital consists of 9 types of machinery and equipment, 3 types of IPP, 4 types of non-residential structures, and 2 types of land. To minimize the potential for aggregation error due to, for example, heterogeneity of assets, asset categories should be disaggregated to the maximum possible extent.⁴ [Table A](#) reports the average depreciation rates of 16 types of assets. Some of these rates are similar to those estimated by Statistics Canada. Of special interest, the depreciation rate of R&D is on average 29.5 per cent. This measured rate of depreciation also displays a large downward trend, from 40 per cent in 1981 to 23 per cent in 2012. The measured average depreciation rate implies that half of investment in R&D is depreciated after two years, and 90 per cent is depreciated after six years of investment. The depreciation rates of R&D stocks are larger than estimates in other papers. For instance, of the estimated depreciation rates of R&D for selected sectors in [Huang and Diewert \(2011\)](#), the largest estimate is 29 per cent for manufacturing, and the estimates are smaller for other sectors.

2.2 Annual growth accounting with CSNA12 and NBSA12

While annual estimates of productivity are already available from [Diewert and Yu \(2012\)](#), Statistics Canada has since made major revisions to national economic data for Canada. This

³Special transactions and Other balance of payments adjustments are excluded from measuring the export and import aggregates.

⁴In theory, flows and stocks of physical assets can be constructed using the national balance sheets (for stocks) and the national accounts (for quantities and prices of investment). However, at the time the analysis of this paper was conducted, as a result of data limitations in CSNA12, such constructions were possible only at an aggregate level with 6 types of assets (Residential structures, Non-residential structures, Machinery and equipment, Inventories, Land, and IPP). Due to the limited availability of capital stock data at the disaggregated level in NBSA12, in this study, the same data as in Diewert-Yu are used for stocks of fixed non-residential capital (Cansim 031-0003). While CSNA12 does have investment data at the disaggregated level, the categories of investments in machinery and equipment in CSNA12 do not align with those in the the table of capital flows and stocks, leaving series from the latter table the only choice for constructing real capital stocks and depreciation rates. The exception is that investment in IPP and its components are from CSNA12. An additional challenge, with data series of flows and stocks of fixed non-residential capital, is that investment values differ from values of the same investment in CSNA12. For example, although investment in IPP was added to Cansim 031-0003, the reported data differs from the “same” investment in CSNA12.

Table A: Annual capital depreciation rates

Machinery and equipment		Non-residential structures	
Furniture	0.23	Industrial buildings	0.06
Agricultural machinery	0.33	Commercial buildings	0.05
Industrial machinery	0.20	Institutional buildings	0.03
Automobiles	0.40	Engineering	0.09
Trucks	0.34	Intellectual property products	
Other transportation	0.19	Software	0.44
Other machinery and equipment	0.14	Mineral exploration and evaluation	0.15
Computers	0.53	Research and development	0.30
Communication equipment	0.24		

section briefly reviews these revisions and provides revised estimates of TFP using the new data.

Revisions to national accounts: In October 2012, Statistics Canada published a major revision to the Canadian National Accounts and to the National Balance Sheets, respectively referred to as CSNA12 and NBSA12. Major changes include:

- **Research and development was reclassified as an investment.** Prior to CSNA12, business expenditures on research and development were treated as an intermediate expenditure, and expenditures on research and development by government were treated as the final consumption expenditure of government. In CSNA12, research and development expenditures are treated as an addition to the fixed capital formation. This change led to an upward revision to real GDP.
- **A new type of asset, Intellectual property products (IPP), was introduced.** IPP consists of research and development, mineral exploration and evaluation, and software. Prior to CSNA12, mineral exploration and evaluation was a component of non-residential structures, and software was treated as a type of machinery and equipment.
- **A new sector, non-profit institutions serving households (NPISH), was created.**
- **Revisions were made to exports and imports of goods and services.** New price indexes for exports and imports (based on price surveys) are used in CSNA12, reflecting more accurately changes in export and import prices. In addition, exports and imports are re-classified with more detailed categories in accordance with the North American Product Classification System (NAPCS) Canada 2007.

- **A new method is applied to evaluate the stocks of non-financial assets.** One important change in NBSA12 is that the machinery and equipment in corporate sector is now valued at the replacement cost, instead of the historical cost (price at the time of purchase). This change led to a downward revision to the value of fixed assets in corporate sector.

Annual growth accounting on revised data: Constructed indexes of real GDI, real GDP, real labour and capital services inputs, and TFP are shown in Figure 1.⁵ A few observations are noteworthy in this figure. First, although real GDP and real GDI tracked each other quite closely for the first 20 years of the sample, a wedge between the two opened up in the 2000s. As will be discussed later, this wedge reflects improvements in the terms of trade that provided additional positive contributions to real GDI. Second, TFP has stagnated since the early 2000s. While there is some evidence of cyclicity in the constructed TFP series, this cannot explain the weakness in TFP through the mid-2000s. Third, this stagnation in TFP implies that between 2002 and 2012, the growth of inputs—capital services and labour—were the driving forces behind real GDP growth. Fourth, capital services is the least cyclical of the contributors to real GDP growth. For instance, TFP and labour input accounted for more than 90 percent of the 4.6 per cent decline of real GDP in the business sector observed in the 2009 recession. Likewise, the recovery from the recession was mainly driven by TFP and labour input, with TFP expanding in 2011 for the first time since 2005.

The re-classification in CSNA12 of R&D as a final investment expenditure increased real GDP and also capital services. The share of investment in R&D in GDP is about 1.5 per cent per year, over the period of 1981 to 2012, but it has in general been increasing, particular since 2000.⁶ To assess the implications of R&D for the estimates of TFP, alternative series were constructed excluding R&D from the growth accounting. Both TFP series are shown in Figure 2, and normalized to be one in 1981. On average, including R&D as a final investment expenditure results in estimates of TFP growth that are 0.03 percentage point slower.

The estimates of TFP growth based on CSNA12 and NBSA12 are slightly lower than those by Diewert and Yu, but the evolution over time of the two index levels is quite similar (Figure 3). Over the period 1981 to 2011, TFP grows 0.51 per cent per year on average in the revised estimates, weaker than the 0.68 per cent growth rate of the Diewert and Yu series. Since labour inputs in the two estimates coincide, weaker average TFP growth in the new estimates can be traced to faster growth in capital services in the pre-1992 period. While inclusion of investment in R&D leads to slightly faster growth in real GDP in the revised estimates, this increase is not strong enough to offset the implications for TFP of the stronger growth in capital services.

⁵All indexes are normalized to one in 1981.

⁶However in 2010-2012, investment in R&D stayed around 2.2 per cent of GDP.

Given that the revised data have only relatively modest implications for the estimates of TFP following the Diewert-Yu methodology, the differences between the new constructions in this paper and the Statistics Canada productivity series are similar to those between the Diewert-Yu estimates and the Statistics Canada productivity series. Statistics Canada’s annual multifactor productivity program constructs measures of the the annual growth rate of productivity in the business sector as part of the KLEMS data set.⁷ As with the Diewert-Yu estimates, the top-down TFP estimates constructed in this paper grow at a much faster rate than the bottom-up TFP measure in the KLEMS data (Figure 4).⁸ Annual TFP growth in the KLEMS data set is only 0.06 per cent over the period of 1982 to 2011. While the average growth rates are considerably different, the cyclical behaviour of the two estimates is similar and in the recent decade the two TFP estimates are fairly close to each other.

The theoretical assumptions in methodology used by [Diewert and Yu \(2012\)](#) are largely the same as those used by Statistics Canada (see [Baldwin et al., 2007](#)). Both estimates of TFP assume perfect competition and constant returns to scale of the aggregate production function. The main difference lies in that [Diewert and Yu \(2012\)](#) use a real revenue function of the translog form, while Statistics Canada does not assume any particular form of production function. The translog revenue function is the second-order approximation of a flexible production (or revenue) function. It makes exact the Törnqvist index and the decomposition of growth in real revenue. For an arbitrary production function with constant returns to scale, the Törnqvist index is an approximation. Theoretically, these differences between the translog form and the general form of production function are unlikely to cause any significant difference in TFP estimates.

Differences in estimated TFP growth between our estimates and KLEMS arise primarily from the measurement of capital services and labour inputs. Statistics Canada uses the input-output table to estimate the outputs and inputs in the business sector. In this bottom-up approach outputs and inputs in many industries are measured and aggregated to a top level. [Diewert and Yu \(2012\)](#) instead take a top-down approach, by using the national accounts and national balance sheets for most components. In principle, these two approaches should lead to similar estimates of TFP growth. In practice, the bottom-up approach of Statistics Canada uses many more disaggregated input and output types, suggesting that there may be potential aggregation errors in the top-down approach applied at a less-detailed disaggregation. The differences in data inputs between two approaches lead to substantially faster growth in capital services in Diewert-Yu estimates for the entire sample period. In addition, labour inputs grew slower in Diewert-Yu in 1980s.

⁷KLEMS data are from CANSIM Table 383-0021.

⁸A detailed comparison of the KLEMS and Diewert-Yu measures is provided in Appendix 4 of [Diewert and Yu \(2009\)](#).

2.3 Quarterly growth accounting: data and results

This section overviews the construction of quarterly growth-accounting data for Canadian business sector, and leaves the details in Appendix B. In estimating quarterly TFP, the same “top-down” methodology is applied to the quarterly series of output and inputs from the new national accounts CSNA12 and new national balance sheets NBSA12 as was applied to construct the annual estimates. Because the data in NBSA12 are available only starting from 1990q1, the quarterly growth accounting exercise also starts from this period. One important issue that does not exist with annual data, but which must be addressed in constructing the quarterly estimates is that of seasonality. The preferred approach would be to use raw seasonally unadjusted data at all stages to construct seasonally unadjusted indexes. These seasonally unadjusted data could then be seasonally adjusted if such a format of data were desired. Unfortunately, data limitations prevented straightforward application of the same approach as for the annual exercise to quarterly seasonally unadjusted data. The process followed is described below, and robustness analysis to alternatives is discussed.

Discussion of Data: Measures of seasonally unadjusted chained-dollar quantities are preferred, but in general only seasonally adjusted chained-dollar quantities are available. To overcome this limitation of data, whether a price seasonality presents is checked. When necessary, the price seasonality is added to the implicit price indexes (deflators) of seasonally adjusted chained-dollar values, giving new implicit price indexes which are used to obtain seasonally unadjusted chained-dollar quantities. For some variables, the implicit price indexes of seasonally unadjusted constant-dollar series are identical or very close to the implicit price indexes of the seasonally adjusted measures, suggesting that there is no seasonality in those price indexes.⁹ For these variables, the real quantity used in the growth accounting exercise is the seasonally adjusted chained-dollar quantity. For others, price indexes of seasonally-adjusted and unadjusted constant-dollar series differ, suggesting the existence of price seasonality. When this is the case, the price seasonality is obtained as a ratio of implicit price indexes that are calculated, respectively, using seasonally unadjusted and adjusted current values and constant-dollar quantities. This price seasonality ratio is then applied to the implicit price indexes calculated using seasonally adjusted chained-dollar quantities, giving a measure of implicit price indexes which is used to obtain seasonally unadjusted

⁹Implicit prices of the chained-dollar series can differ from those of the constant-dollar series because of changes in the composition of the variable over time. One such an example is the investment in machinery and equipment, with the price index of the seasonally adjusted chained-dollar series being very different from that of the seasonally adjusted constant-dollar series. This arises because the significantly declined prices of computers and increased use of computers.

chained dollar quantities.¹⁰

In the case of output measures, seasonally unadjusted current and chained-dollar values of all components except for exports and imports can be obtained from CSNA12. The exception is the exports and imports of 12 types of goods, for which seasonally unadjusted constant-dollar values are unavailable. Examination of the price indexes of merchandise trade indicates that there is no significant price seasonality for these exports and imports except for the prices of imports of energy products. The price indexes of the seasonally adjusted chained-dollar series for the exports and imports of goods are therefore used as proxies for the price indexes of the seasonally unadjusted series for those with small differences between price indexes of adjusted and unadjusted series. For other exports and imports, the price seasonality of the aggregate export and import prices is applied respectively to the implicit price indexes of the seasonally adjusted chained-dollar exports and imports at the 2-digit NAPCS level.¹¹

The quarterly labour inputs, including hours worked and compensation for 36 types of workers (3 levels of education achievement, 3 age groups, 2 gender groups, and employees versus self-employed), are constructed from the Labour Force Survey (LFS) public use microdata files. Hours worked are measured as actual hours worked on all jobs.¹² LFS also contains hourly earnings for usual hours worked on the main job, available starting from 1997Q1. For each of these 36 worker types, total quarterly compensation for all hours worked is estimated assuming the compensation rates for usual hours and actual hours are the same, since the LFS only reports compensation for usual hours worked. For compensation series, quarterly data before 1997Q1 is obtained by interpolating the annual data, and applying the aggregate wage seasonality obtained from the Survey of Employment, Payrolls and Hours (SEPH). Thus, for periods before 1997Q1, the seasonal changes in compensation are the same for all types of workers. Finally, to ensure consistency, the 72 quarterly series of hours worked and compensation are benchmarked to the data used in the annual growth accounting exercise.

The quarterly capital stocks, used to measure capital services, are constructed using initial

¹⁰Let $P^{adj,const}$ be the deflator of the seasonally adjusted constant-dollar series, calculated as the current value divided by the constant-dollar quantity, $P^{adj,const} = \frac{V^{adj}}{Q^{adj,const}}$. The deflator of the seasonally unadjusted constant-dollar series, $P^{unadj,const}$, is defined and calculated in the same way. The price seasonality is defined as $\xi = \frac{P^{unadj,const}}{P^{adj,const}}$, which is used to calculate the deflator for seasonally unadjusted chained-dollar quantities as $P^{unadj,chain} = \xi \cdot P^{adj,chain}$. The seasonally unadjusted chained-dollar quantity is then calculated as $Q^{unadj,chain} = \frac{V^{unadj}}{P^{unadj,chain}}$.

¹¹This may create a bias in the volatility for some exports and imports because the degree of seasonal variation for some exports and imports are smaller than that for the aggregate export and import. For instance, the seasonality for the imported energy products is significant, a large portion of seasonality of the aggregate import may arise from that of the imported energy products. At the time of our estimates, no other ways appear to exist for obtaining the price seasonality for individual exports and imports.

¹²The LFS includes information on usual and actual weekly hours worked for both main jobs and all jobs.

capital stocks from the annual data, quarterly depreciation rates based on annual rates, and investment series from CSNA12. In addition, owing to data limitations, quarterly capital stocks are disaggregated into fewer asset types than in the annual growth accounting exercise. Since the quarterly capital stock data in NBSA12 is only available for 3 types of assets (non-residential structures, machinery and equipment, and IPP), real quarterly capital stock data are instead constructed using the perpetual inventory method based on quarterly investments and annual data of capital flows and stocks. Annual capital stocks are used to construct depreciation rates and the initial-period capital stocks, which together with the quarterly investments from CSNA12 are used to recursively calculate quarterly real capital stocks. However, as the types of capital stocks in annual data cannot be matched to the types of investment in quarterly data, primarily for machinery and equipment, it was necessary to reduce the number of assets to only include 4 types of machinery and equipment, 2 types of non-residential structures, and 3 types of intellectual property products (IPP). Machinery and equipment includes Computers, Telecommunications equipment, Industrial machinery, and Transportation and furniture. Grouping machinery and equipment in this way permits the matching of asset types between annual and quarterly data by current-value investments.¹³

Quarterly growth accounting results: The general trends in the quarterly series for output, TFP, and labour and capital services inputs are as in the annual series (Figure 5).¹⁴ First, although quarterly TFP expanded on average over the period of 1990Q1 to 2013Q1 (0.07 per cent per quarter), as in the annual data, TFP contracted on average between the early 2000s and 2013Q1. Some positive TFP growth was observed in the early part of the recovery from the recession, although growth subsequently stalled. Second, the business sector experienced capital deepening since late 2003, at about the same time that TFP started contracting noticeably. This suggests that the business sector may have experienced structural transformation over this period.

Some of the basic correlation properties of TFP and other important macroeconomic measures of inputs and relative prices are summarized in Table B. First, TFP growth is contemporaneously positively correlated with growth of real GDP in the business sector and non-residential investment growth. Second, the growth rates of both hours worked and the labour input (quality adjusted hours worked) are negatively correlated with contemporaneous TFP growth, but positively correlated with TFP growth lagged by one quarter. These negative correlations are consistent with the findings

¹³Extensive comparison between quarterly investments (Cansim 380-0068) and annual investments (Cansim 031-0003) appears to suggest that annual investments used for constructing annual capital stocks are not as updated as the quarterly series. For the same series (with identical names), quarterly and annual data are different.

¹⁴The series shown in the charts are seasonally adjusted with the X-12-ARIMA Seasonal Adjustment Program developed by the United States Census Bureau.

in Basu et al. (2006).¹⁵ Finally, while contemporaneous (or lagged) improvements in the terms of trade are moderately positively correlated with non residential investment growth, correlations with the other variables are weaker.

Table B: Partial correlations over the business cycle

	ΔTFP	Lagged ΔTFP	ΔToT	Lagged ΔToT
Output (Δy)	0.88	-0.30	0.16	0.28
Labour input (Δl)	-0.37	0.32	0.17	0.21
Hours (Δh)	-0.34	0.32	0.17	0.23
Investment (Δi)	0.22	0.08	0.39	0.58
TFP (ΔTFP)	1.0	-0.45	0.08	0.15
Terms of trade (ΔToT)	0.08	0.02	1.0	0.36

Just as differences in the measurement of capital services by Diewert and Yu (2012) and Statistics Canada had implications for the measurement of TFP in the top-down versus bottom-up approaches, the differences in quarterly and annual measures of capital services also have important implications holding the methodology constant. The correlation between the growth rate of annual TFP calculated from the quarterly constructions and the growth rate of the directly constructed annual TFP series is 0.75, indicating important gaps between the two—differences which can be traced primarily to the the measurement of capital services (Figure 6). In addition, differences in investment in business inventories between annual and quarterly data has contributed to large differences in the years of 1997, 2001, 2005, 2009 and 2010.

Recognizing that measurement of inputs to the growth accounting exercise may be relevant, it is nonetheless noteworthy that basic correlations among the same macroeconomic variables examined above, can be significantly different in the annual and quarterly data. To illustrate this difference, Table C reports the correlation coefficients with the annual data for the period of 1990 to 2012, which can be compared with the quarterly correlations in Table B. For example, in the quarterly data, the growth rates of hours and labour input are negatively correlated with TFP growth, but in the annual data, the correlations between these same variables are positive.

¹⁵A more detailed exercise along the lines of that in Basu et al. (2006) requires separately identifying capacity utilization, and is left for future research. By not controlling for changes in capacity utilization, the quarterly TFP measures constructed in this study may confuse technological changes with changes in capacity utilization. While there is no consensus on how the effects of capacity utilization should be separated from measured TFP, a blunt adjustment for capacity utilization can be made by subtracting the change in capacity utilization rates estimated by Statistics Canada from TFP growth. Assuming an additive relation between technological change and change in capacity utilization, $\Delta \ln TFP = \Delta \ln A + \Delta \ln U$, where A is the ‘true’ technology and U is the capacity utilization rate. Subtracting capacity utilization from the measured TFP gives the utilization-adjusted TFP (or A). Figure 10 shows that measured TFP and utilization-adjusted TFP do deviate, most notably over the recent recession. A more formal study incorporating capacity utilization into growth accounting is left for future research.

Table C: Partial correlations, annual data

	ΔTFP	Lagged ΔTFP	ΔToT	Lagged ΔToT
Output (Δy)	0.88	0.22	0.31	0.14
Labour input (Δl)	0.52	0.35	0.57	-0.04
Hours (Δh)	0.50	0.31	0.57	-0.02
Investment (Δi)	0.21	0.18	0.71	-0.01
TFP (ΔTFP)	1.0	0.04	0.04	0.18
Terms of trade (ΔToT)	0.04	-0.04	1.0	-0.15

Finally, as a robustness check, the labour productivity series implicit in the quarterly growth accounting exercise can be compared to the quarterly labour productivity data published by Statistics Canada. In general, real GDP, hours worked, and average labour productivity (ALP) are similar, although the measures constructed in this paper are more volatile (Figures 7, 8, and 9).¹⁶ The real output series are fairly close—they differ in that both paid rent and the rental value of owner-occupied dwellings are excluded in the analysis of this study, whereas only the owner-occupied dwellings are excluded in the Statistics Canada estimates. The quarterly hours worked series are more different between the series as they are drawing on different source data. Hours worked in the Statistics Canada labour productivity Program take into account information in both LFS and SEPH data sources, as well as multiple-job holders, while the analysis in this study relies solely on the LFS data.¹⁷

3 Real GDI, productivity and Terms of trade

In this section, annual growth of real gross income in Canada is decomposed into contributions from changes in productivity, the terms of trade, and real labour and capital inputs.

Growth of real income can be decomposed into five factors as follows:¹⁸

$$\rho^t / \rho^{t-1} = \gamma^t \alpha_X^t \alpha_M^t \beta_L^t \beta_K^t,$$

where, ρ^t is aggregate real income, γ^t is TFP growth, α_X^t is the rate of change of the real export price, and α_M^t is the rate of change of the real import price. If $\alpha_X^t > 1$, it means that Canadian

¹⁶This analysis reflects work in progress.

¹⁷In constructing quarterly hours worked, Statistics Canada use the geometric mean of hours worked in LFS and SEPH for employees to smooth the measured hours worked, and use hours worked from LFS for the self-employed. In addition, multiple-job holders in LFS are counted twice to calculate the aggregate hours. This paper doesn't use the SEPH data as it does not include information on detailed worker types.

¹⁸To be consistent with [Diewert and Yu \(2012\)](#), growth again is the ratio of levels between period t and period $t - 1$.

export prices grow faster than the price of domestic output, hence α_X^t measures the contribution of rising export prices to the growth of real income generated by the business sector. Conversely, if $\alpha_M^t > 1$, it measures the contribution of falling real import prices to the growth of real gross output.¹⁹ The larger the value of α_M^t , the larger the decline in the real import price (again relative the price of domestic output). The export price and import price can be pooled together to form the change of the terms of trade, $\alpha_{XM}^t = \alpha_X^t \alpha_M^t$. Finally, the last two terms in the above equation represent respectively the growth of quantities in labour inputs (β_L^t) and capital services (β_K^t). The cumulative counterparts of the above decomposition equation can be obtained for each term. For example, the cumulative TFP growth is $\Gamma^t = \Gamma^{t-1} \cdot \gamma^t$ with $\Gamma^0 = 1$. Before proceeding to the results of this decomposition, it is important to reiterate that the analysis in this paper is a growth accounting exercise, not an evaluation of economic relationships or causation. Understanding economic links between terms of trade, real output, and reallocation requires an analysis that admits structural economic interpretations, such as through the use of a structural model, and is to be investigated in separate research.

3.1 Decomposing real GDI growth

Real GDI growth in Canada has been supported by TFP growth and improvements in the terms of trade, although their relative importance has changed over time. Real GDI and real output tracked each other relatively closely prior to 2000, but over the past decade real gross income in Canada has exceeded and grown faster than real output, owing to an improvement in the terms of trade (Figure 11). From 2001 to 2012, real gross income in the business sector increased 23.9 per cent, of which real gross output grew by 16.3 per cent and the terms of trade improved by 7.6 per cent.²⁰ Thus, about one-third of real income growth over the last decade is accounted for by the improvement in the terms of trade.

A decomposition of the cumulative growth in real income since 1981 is shown in Figure 12. The terms of trade contributed significantly to the real income in the early 2000s at about the same time that TFP growth started to slow. Contributions to average annual growth rates of real GDI are provided for different sub-periods in Figure 13.²¹ Over 1982-2000, the terms of trade made

¹⁹Note that imports make a negative contribution to gross domestic product. The more the imports, the lower the gross domestic product. In Eq. (25) of Appendix A, the term in the square bracket is negative because imports add to the gross domestic product as a negative value. When the real import price falls, α_M^t rises, leading to growth in the real income.

²⁰Again, growth rates are log-differences of values in 2001 and 2012, for the purpose of decomposition.

²¹In Figure 13, the Canadian recession year, 2009, is omitted because the large magnitude of the recession affects the average growth rates excessively. In 2009, the aggregate real income growth in business sector dropped 9.5 per cent, and real GDP dropped by 4.6 per cent. Half of the decline in real income was due to the deterioration of the terms of trade. The decline of TFP accounted for 41 per cent of drop in real output and 20 per cent of drop in real income.

only a small contribution to annual real income growth whereas close to one third of real income growth came from the contribution of TFP growth. By contrast, over the period of 2001 to 2008, productivity slowed considerably and the terms of trade began to contribute significantly to the annual growth of real gross income. In this period, one third of real income growth is attributed to improvement in the terms of trade, a sufficiently sizable boost that the standard of living measured by real income growth did not worsen even though TFP growth slowed. Finally, in the recovery period after the Great Recession, the terms of trade continued to be important for real income growth, and TFP started to show modest growth.

The improvement in the terms of trade, the price index of exports relative to the price index of imports, over the period 1981 to 2012 reflects the fact that the real import price has been declining at a faster rate than the export price. Over this period, both the real export price (price of exports relative to price of domestic output) and the real import price (price of imports relative to price of domestic output) have fallen (Figure 14). In particular, since 2000, the terms of trade has been improving, primarily because real import price experienced a sharp decline, falling at an average of 2.1 per cent per year, while the real export price index fell at the rate of 0.6 per cent per year.

3.2 Decomposing the terms of trade

Understanding movements in the terms of trade at a disaggregated level can help contribute to an understanding of the potential reallocation effects of changes in relative export prices. Canada exports and imports different products, and trade is unbalanced for the same product. For example, exports of energy products far exceed imports of energy products. To better understand changes in the aggregate terms of trade, the terms of trade are decomposed into contributions from four product categories, namely, non-energy commodity, energy products, manufactured goods, and services.

Let α_A^t be the change in the terms of trade for non-energy commodity, α_E^t be the change in the terms of trade for energy products, α_I^t be the change in the terms of trade for manufactured products, and α_S^t be the change in the terms of trade for services. The decomposition of contribution of the aggregate terms of trade is as follows:

$$\alpha_{XM}^t = \alpha_A^t \cdot \alpha_E^t \cdot \alpha_I^t \cdot \alpha_S^t.$$

The 12 product categories of exports and imports from CSNA12. are aggregated into four categories using Fisher indexes: non-energy commodity, energy products, manufactured products, and

services.²² Energy products include crude oil, natural gas, refined petroleum, electricity, and other energy products. Non-energy commodities include farm, fishing and intermediate food products; metal ores and non-metallic minerals; metal and non-metallic mineral products; and, forestry products, building and packaging materials. Manufactured goods include chemical, plastic and rubber products; industrial machinery, equipment and parts; electronic and electrical equipment and parts; motor vehicles and parts; aircraft and other transportation equipment; and consumer goods.

Figure 15 shows the shares of the four types of exports in total current-value exports, and Figure 16 shows the shares for imports. Export in manufactured goods on average accounts for 47 per cent of total exports, but since 2000 this share started to shrink. Due to rising global prices of oil, the share of energy export in total export started to increase in 2000. On the import side, manufactured products are the single largest imports, accounting for 62 per cent of total imports on average. The import of energy products, though staying small, has been increasing since 2000.

Manufacturing and services are the most important contributing factors to the changes in the aggregate terms of trade during 2001-2012, as shown in Figure 17. Each bar in this figure shows the contribution of the terms of trade of individual goods (in percentage changes), adding all changes in each bar gives the contribution of the aggregate terms of trade shown in Figure 13. Manufactured goods accounted for more than half (55 per cent) of the contribution of the aggregate terms of trade to real income growth over the period of 2001 to 2012 (excluding 2009), with services accounting for another 20 per cent per year.

Energy products shared a large contribution to the changes in the aggregate terms of trade over the period of 2001 to 2008 when the oil price boomed. These products were also responsible for close to half of the decline in the aggregate terms of trade in 2009. In the recovery periods (2010 to 2012), terms of trade in energy products deteriorated from the pre-recession periods.

Though the terms of trade in manufacturing and in energy products has been improving since 2000, the underlying driving forces of these improvements are different between these two types of products. Figure 18 shows the real price indexes of exports and imports for the four product categories. The import price index for manufacturing products declined at a faster pace than export price over this period, leading to a large contribution to the improvement in the aggregate terms of trade. For energy products, the oil price boom in 2000s led to an increase of both export and import prices, though the export price increased slightly more than the import price. These relative changes in energy prices led to a large contribution to the improvement in aggregate terms of trade, but this contribution stopped during and after the 2008-2009 recession.

In summary, the improvement in the terms of trade has become increasingly important in the

²²We ignore the exports and imports of Special transactions for this analysis.

growth of the real income over the last decade. This improvement arises mainly from the sharp decline in import prices of manufacturing products and services.

4 Concluding remarks

In this paper, both annual and quarterly growth accounting data for Canadian business sector have been constructed. Terms of trade, in the absence of strong productivity growth, has contributed to real income growth, helping maintain the standard of living. Looking forward, in the longer run, boosting productivity growth is important for the stable growth of output and real income, in particular if the terms of trade deteriorate.

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A Appendix A: Diewert-Yu methodology of growth accounting

The growth accounting approach to productivity growth decomposes the growth of real income in the business sector into contribution from productivity growth, growth in primary inputs, and changes in real prices. Given data for real income growth, growth in primary inputs and changes in real prices, productivity growth can be constructed. Here we briefly summarize the methodology, with additional details available in [Diewert and Yu \(2012\)](#).

Notation In most cases, the same notation as in [Diewert and Yu \(2012\)](#) is used. Let $y^t = (y_1^t, y_2^t, \dots, y_M^t)$ be the **output** vector of M products, $x^t = (x_1^t, x_2^t, \dots, x_N^t)$ be the **input** vector of N inputs, $P^t = (P_1^t, P_2^t, \dots, P_M^t)$ be the **output price** vector, and let $W^t = (W_1^t, W_2^t, \dots, W_N^t)$ be the **input price** vector.

The production technology of a firm producing y^t is summarized by f^t . Then, for example, the production function for output m is represented as $y_m^t = f^t(y_1^t, \dots, y_{m-1}^t, y_{m+1}^t, \dots, y_M^t; x^t)$. The cost function of producing all M outputs is $c^t(y_1^t, y_2^t, \dots, y_M^t; W_1^t, W_2^t, \dots, W_N^t)$ given the above production technology $f^t(\cdot)$. The aggregate revenue function is $g^t(P^t, x^t)$ if firms produce the optimal amount of outputs y^t . If the economy is in perfect competition and the production function displays constant returns to scale, then $g^t(P^t, x^t) = P^t \cdot y^t = W^t \cdot x^t$.

The real income generated in the business sector would be $g^t(p^t, x^t) = p^t \cdot y^t = w^t \cdot x^t$, where $p^t = P^t/P_c^t$ and $w^t = W^t/P_c^t$. P_c^t is the price of consumption.

Decomposing growth in real income The growth of real income can be decomposed into three components: growth of technology or total factor productivity (TFP), growth of real output prices, and growth of inputs.

TFP growth is a ratio of the real income that can be produced using the period t versus $t - 1$ technology with input volumes held constant at some reference level x and with prices held constant at some reference unit price p : $\gamma^t(p, x) = \frac{g^t(p, x)}{g^{t-1}(p, x)}$. It is natural to use the geometric mean of a Laspeyres measure and a Paasche measure of productivity growth as follows

$$\gamma^t = \left[\frac{g^t(p^{t-1}, x^{t-1})}{g^{t-1}(p^{t-1}, x^{t-1})} \cdot \frac{g^t(p^t, x^t)}{g^{t-1}(p^t, x^t)} \right]^{\frac{1}{2}}. \quad (12)$$

It should be noted that $g^t(p^{t-1}, x^{t-1})$ and $g^{t-1}(p^t, x^t)$ are theoretical terms, no data are available to measure them.

The **growth of real income** due to changes in prices is defined as $\alpha(p^{t-1}, p^t, x, s) = \frac{g^s(p^t, x)}{g^s(p^{t-1}, x)}$. It measures the change in real income produced by the market sector that arises from changes in

output prices between period $t - 1$ and period t , and the output is produced using technology in period s and using some reference input level x . This growth can be expressed as the geometric mean of a Laspeyres measure and a Paasche measure, given by

$$\alpha^t = \left[\frac{g^{t-1}(p^t, x^{t-1})}{g^{t-1}(p^{t-1}, x^{t-1})} \cdot \frac{g^t(p^t, x^t)}{g^t(p^{t-1}, x^t)} \right]^{\frac{1}{2}}. \quad (13)$$

Analogous to the growth of real income due to changes in output prices, the theoretical index of **growth of real income** due to changes in input quantities is defined as $\beta(x^{t-1}, x^t, p, s) = g^s(p, x^t)/g^s(p, x^{t-1})$. It measures the growth of real income, where output is produced using technology in reference period s and using the reference real output prices p . It can again be expressed as

$$\beta^t = \left[\frac{g^{t-1}(p^{t-1}, x^t)}{g^{t-1}(p^{t-1}, x^{t-1})} \cdot \frac{g^t(p^t, x^t)}{g^t(p^t, x^{t-1})} \right]^{\frac{1}{2}}. \quad (14)$$

The **growth of real income** is defined as $gr^t = \frac{g^t(p^t, x^t)}{g^{t-1}(p^{t-1}, x^{t-1})}$, which can be written as

$$gr_1^t = \frac{g^t(p^t, x^t)}{g^{t-1}(p^t, x^t)} \cdot \frac{g^{t-1}(p^t, x^t)}{g^{t-1}(p^{t-1}, x^t)} \cdot \frac{g^{t-1}(p^{t-1}, x^t)}{g^{t-1}(p^{t-1}, x^{t-1})}. \quad (15)$$

The growth of real income can also be written as

$$gr_2^t = \frac{g^t(p^{t-1}, x^{t-1})}{g^{t-1}(p^{t-1}, x^{t-1})} \cdot \frac{g^t(p^t, x^{t-1})}{g^t(p^{t-1}, x^{t-1})} \cdot \frac{g^t(p^t, x^t)}{g^t(p^t, x^{t-1})}. \quad (16)$$

Take the geometric mean of Eq. (15) and Eq. (16), we obtain the following

$$gr^t = \gamma^t \cdot \left[\frac{g^{t-1}(p^t, x^t)}{g^{t-1}(p^{t-1}, x^t)} \cdot \frac{g^t(p^t, x^{t-1})}{g^t(p^{t-1}, x^{t-1})} \right]^{\frac{1}{2}} \cdot \beta^t, \quad (17)$$

or

$$gr^t = \gamma^t \cdot \alpha^t \cdot \left[\frac{g^{t-1}(p^t, x^t)}{g^t(p^{t-1}, x^{t-1})} \cdot \frac{g^{t-1}(p^{t-1}, x^t)}{g^t(p^t, x^{t-1})} \right]^{\frac{1}{2}}. \quad (18)$$

Diewert and Yu claim that, in Eq. (17), the square root term in the middle is approximately equal to α^t ; and in Eq. (18), the square root term is approximately equal to β^t . The **growth of real income** is then decomposed as follows

$$gr^t \approx \gamma^t \alpha^t \beta^t, \quad (19)$$

where γ^t is TFP growth, α^t is growth in real output prices, and β^t is growth in primary inputs.

Assumptions for exact index number [Diewert and Yu \(2012\)](#) impose assumptions on functional forms and parameters for revenue so that the decomposition (19) holds exactly.

Assumption 1. *Translog revenue function*

$$\begin{aligned} \ln g^t(p^t, x^t) = & a_0^t + \sum_{m=1}^M a_m^t \ln p_m^t + \frac{1}{2} \sum_{m=1}^M \sum_{j=1}^M a_{mj} \ln p_m^t \ln p_j^t + \sum_{n=1}^N b_n^t \ln x_n^t \\ & + \frac{1}{2} \sum_{n=1}^N \sum_{j=1}^N b_{nj} \ln x_n^t \ln x_j^t + \sum_{m=1}^M \sum_{n=1}^N c_{mn} \ln p_m^t \ln x_n^t. \end{aligned} \quad (20)$$

The translog form of revenue function enables the decomposition equation to hold exactly. Translog form is a second-order approximation of a general twice differentiable revenue function.

Assumption 2. *Following restrictions on parameters hold*

- i) $\sum_{m=1}^M a_m^t = 1$ for any t ;
- ii) $\sum_{n=1}^N b_n^t = 1$ for any t ;
- iii) $a_{mj} = a_{jm}$ for all m, j ;
- iv) $b_{nj} = b_{jn}$ for all n, j ;
- v) $\sum_{j=1}^M a_{mj} = 0$ for $m = 1, \dots, M$;
- vi) $\sum_{j=1}^N b_{nj} = 0$ for $n = 1, \dots, N$;
- vii) $\sum_{n=1}^N c_{mn} = 0$ for $m = 1, \dots, M$;
- viii) $\sum_{m=1}^M c_{mn} = 0$ for $n = 1, \dots, N$.

[Diewert \(1974\)](#) shows that the revenue function is homogeneous of degree one in p^t if assumptions i), iii), v), and viii) hold. Similarly, the revenue function is homogeneous of degree one in x^t if assumptions ii), iv), vi), and vii) hold. These assumptions are necessary for the revenue function to be well defined and to have the properties of a revenue function as in the production theory.

Decomposition with translog function Under Assumptions 1 and 2, the three components of growth in real revenue are given by

$$\ln \alpha^t = \sum_{m=1}^M \frac{1}{2} \left[\frac{p_m^{t-1} y_m^{t-1}}{p^{t-1} \cdot y^{t-1}} + \frac{p_m^t y_m^t}{p^t \cdot y^t} \right] \ln \left(\frac{p_m^t}{p_m^{t-1}} \right); \quad (21)$$

$$\ln \beta^t = \sum_{n=1}^N \frac{1}{2} \left[\frac{w_n^{t-1} x_n^{t-1}}{w^{t-1} \cdot x^{t-1}} + \frac{w_n^t x_n^t}{w^t \cdot x^t} \right] \ln \left(\frac{x_n^t}{x_n^{t-1}} \right); \quad (22)$$

$$\gamma^t = \frac{gr^t}{\alpha^t \beta^t}. \quad (23)$$

Where α^t is the Törnqvist index of output price, and β^t is the Törnqvist index of input quantity.

Contribution of output prices Diewert and Yu (2012) show that the above decomposition can take into account changes in relative prices of GDP components. Suppose the m th output price p_m changes from period $t-1$ to t . The overall measure of the effects on real income from changes in the real output price p_m is given by the geometric mean of a Laspeyres measure and a Paasche measure, as follows

$$\alpha_m^t = \left[\frac{g^{t-1}(p_1^{t-1}, \dots, p_{m-1}^{t-1}, p_m^t, p_{m+1}^{t-1}, \dots, p_M^{t-1}, x^{t-1})}{g^{t-1}(p^{t-1}, x^{t-1})} \cdot \frac{g^t(p^t, x^t)}{g^t(p_1^t, \dots, p_{m-1}^t, p_m^{t-1}, p_{m+1}^t, \dots, p_M^t, x^t)} \right]^{\frac{1}{2}}. \quad (24)$$

Under Assumptions 1 and 2,

$$\ln \alpha_m^t = \frac{1}{2} \left[\frac{p_m^{t-1} y_m^{t-1}}{p^{t-1} \cdot y^{t-1}} + \frac{p_m^t y_m^t}{p^t \cdot y^t} \right] \ln \left(\frac{p_m^t}{p_m^{t-1}} \right). \quad (25)$$

This implies that the period t aggregate real output price contribution is exactly decomposed into the product of separate price contributions, as follows

$$\alpha^t = \alpha_1^t \alpha_2^t \cdots \alpha_M^t. \quad (26)$$

In the case of the **terms of trade** (export price relative to import price), let the real export price be export price deflated by domestic output price, and let the real import price be import price deflated by domestic output price. Note that at this aggregate level, there are three output components: domestic output, export, and import. The contribution of changes in real export and import prices are respectively α_{EX}^t and α_{IM}^t , both defined as in Eq (25). The contribution of

changes in the terms of trade to real income growth is then

$$\alpha_{XM}^t = \alpha_{EX}^t \cdot \alpha_{IM}^t.$$

Contribution of primary inputs In the same fashion, the contribution of changes primary inputs β^t can be further decomposed. Suppose that the n th input x_n changes. Its contribution to real income is β_n^t , measured as the geometric mean of a Laspeyres measure and a Paache measure, given by

$$\beta_n^t = \left[\frac{g^{t-1}(p^{t-1}, x_1^{t-1}, \dots, x_{n-1}^{t-1}, x_n^t, x_{n+1}^{t-1}, \dots, x_N^{t-1})}{g^{t-1}(p^{t-1}, x^{t-1})} \cdot \frac{g^t(p^t, x^t)}{g^t(p^t, x_1^t, \dots, x_{n-1}^t, x_n^{t-1}, x_{n+1}^t, \dots, x_N^t)} \right]^{\frac{1}{2}}. \quad (27)$$

With the translog form of real GDP function, β_n^t can be written as

$$\ln \beta_n^t = \frac{1}{2} \left[\frac{w_n^{t-1} x_n^{t-1}}{w^{t-1} \cdot x^{t-1}} + \frac{w_n^t x_n^t}{w^t \cdot x^t} \right] \ln \left(\frac{x_n^t}{x_n^{t-1}} \right). \quad (28)$$

The contribution from changes in quantities of all inputs has the following exact decomposition.

$$\beta^t = \beta_1^t \beta_2^t \cdots \beta_N^t. \quad (29)$$

B Appendix B: Data for growth accounting

This section describes in detail how the outputs and inputs in the Canadian business sector are measured, for both quarterly and annual growth accounting. The focus is on quarterly data series since quarterly growth accounting is new. Cansim tables used for quarterly estimates are listed in Table G. For the annual estimates, only differences from Diewert-Yu are delineated.

Here, the business sector includes sectors except the public administration sector and non-profit institutions serving households (NPISH), as well as the non-business parts of the education sector and the health care sector. NBSA12 starts from 1990Q1, so do the quarterly series.

CSNA12 is a major revision to the national accounts. In addition to revisions to many tables, several changes are relevant for measuring productivity. First, CSNA12 introduced a new economic sector, Non-profit institutions serving the household (NPISH). This sector is treated as part of the general government. Second, CSNA12 introduced Intellectual property products (IPP), a new asset type consisting of Software, Mineral exploration and evaluation, and Research and development. Before CSNA12, Software was a component of machinery and equipment; Mineral exploration and evaluation was a component of non-residential structure; and Research and development was treated as an intermediate input for businesses and as a final consumption expenditure for government. Third, exports and imports are re-categorized using the North American Product Classification System (NAPCS) Canada 2007. In addition, the measurement of prices of exports and imports is improved by using micro price surveys.

In constructing each variable, two of the three values are needed: current nominal value, real value (or quantity), and the implicit price index. In most cases, current nominal values and real quantities are available with few exceptions. Often, aggregating different series is necessary. Chained Fisher index is used when aggregating multiple components of the same type. For example, Fisher index is used to construct the aggregate consumption expenditure by removing housing services from the total expenditure. Chained Törnqvist index is used when aggregating all output (input) components into the total output (input). Nevertheless, the results are not sensitive to the choice of price indexes.

To construct the seasonally unadjusted series of quantities and prices, two issues need to be taken care of. First, some variables have no quarterly data and it is impossible to estimate them (e.g., no quarterly series exist for current value of GDP at basic price), quarterly data are obtained by interpolating annual series at the quarterly frequency when needed. Second, seasonally unadjusted chained-dollar series are not available for many variables. In CSNA12 tables, seasonally unadjusted series are often available only in constant dollar, not in chained dollar. In such a case, seasonality

is detected by checking whether there is a significant difference between seasonally unadjusted and adjusted constant-dollar values. If the underlying variable displays no seasonal variation, seasonally adjusted chained-dollar values are used. If the underlying variable displays an obvious seasonal variation, seasonally unadjusted chained-dollar values are obtained by applying a seasonality ratio to the implicit price indexes of the seasonally adjusted chained-dollar values. This seasonality ratio is obtained as the ratio of implicit price indexes calculated from seasonally unadjusted and adjusted current values and constant-dollar values.²³

B.1 Quarterly output

Output is the final demand expenditure, which is a measure of value added because it excludes the intermediate inputs. It consists of 5 categories of domestic output, 12 categories of exports, and 12 categories of imports. Domestic output includes household final consumption expenditure, sales of businesses to non-businesses, business investment, government investment, and changes in business inventories. Most output components in the business sector are from the expenditure-based GDP table in CSNA12.

Household final consumption expenditure. Final consumption expenditure is the total final household consumption spending on goods and services, excluding paid rent and the imputed rent for owner-occupied dwellings.²⁴ To obtain the imputed rent for owner-occupied dwellings in the business sector requires GDP at basic prices in business sector. No current-value quarterly GDP at basic prices is available in CSNA12. First, the implicit price of the imputed rent is constructed. The seasonality of the imputed rent from the table Detailed household final consumption expenditure, using seasonally unadjusted and adjusted constant-dollar prices.²⁵ It is then applied to the interpolated annual prices of the imputed rent, resulting in the seasonally unadjusted chained-dollar price of the imputed rent. For real values of the imputed rent in the business sector, series from the tables of GDP at basic prices by North American Industry Classification System (NAICS) is used. The quantity and price of the paid rent are from the table of detailed household final consumption expenditure. The seasonally adjusted prices in constant dollar and in chained dollar are identical

²³The differences between seasonally adjusted chained-dollar values and seasonally adjusted constant-dollar values arise from changes in the composition of the underlying outputs and inputs. For example, the constant-dollar value and chained-dollar value of investment in machinery and equipment are very different in 1980s and 1990s, due to the significantly declined prices of computer equipment and significantly increased use of computers. For some other variables chained-dollar price and constant-dollar price are virtually the same.

²⁴In the growth accounting data at Statistics Canada, only the imputed rent from owner-occupied dwellings is excluded from the output measure.

²⁵Here, the term “seasonally adjusted constant-dollar price” refers to the implicit price index (deflator) calculated from dividing seasonally adjusted current values by seasonally adjusted real values at constant dollar. Other similar terms are defined in the same way.

for paid rent, seasonally unadjusted price (in constant dollar) of paid rent is used without further adjustment.

For the aggregate household consumption expenditure, the seasonally adjusted chained-dollar implicit price and the seasonally adjusted constant-dollar implicit price diverge over time, thus requiring adjustment. The price seasonality is obtained in the table of GDP at 2007 constant prices (expenditure-based). The seasonally adjusted chained-dollar price is obtained from the expenditure-based GDP table. The seasonally unadjusted chained-dollar price is then calculated as the seasonally adjusted chained-dollar price multiplied by the price seasonality.

The final consumption expenditure net of residential rents is obtained using the Fisher price and quantity indexes. Consumption tax is then removed from the final net consumption expenditure. The effective consumption tax rate is calculated, and used to adjust the seasonally unadjusted chained dollar price, as well as the real value of net final consumption expenditure.

Net sales of businesses to non-business sector. Calculating the net sales of businesses to non-business sector (NSBN) is described in detail in [Diewert and Yu \(2012\)](#). According to their theory, the current value of NSBN equals to the current value of the general government's final consumption expenditure (at market prices), subtracting the current value of GDP at basic prices in the non-business sector. The general government's consumption expenditure equals to the sum of government consumption expenditure and the NPISH's final consumption expenditure, which are obtained from the expenditure-based GDP table.

For GDP at basic prices in the non-business sector, CSNA12 has no quarterly current-value GDP at basic prices by sector. The real value of GDP at basic prices in the non-business sector is from the table of GDP at basic prices by NAICS. The implicit price of GDP at basic prices for the non-business sector is obtained by interpolating its annual data.

The net sales of businesses to non-businesses are then calculated as the Fisher quantity index from the general government spending and the non-business GDP at basic prices.

Government investment. Government investment consists of investment by government and by NPISH. Current and real values of these investments are from the table of GDP (expenditure-based) and the table of GDP at 2007 constant-dollar price. The seasonality of investment prices is obtained from the GDP table at 2007 constant-dollar prices.

Business investment. Business investment includes that in residential structure, non-residential structure, machinery and equipment, and intellectual property products. The current values and seasonally adjusted chained-dollar prices of these variables are from the table of expenditure-based

GDP. These variables display various degrees of seasonality, so the table of GDP at 2007 constant dollar prices is used to obtain the price seasonality to obtain seasonally unadjusted chained-dollar prices.

In calculating the price seasonality for the investment in residential structure, the seasonally unadjusted constant-dollar quantity of investment in residential structure starts only from 2007Q1. For periods before 2007Q1, it is thus calculated with the Fisher quantity index using the total business investment subtracting investments in non-residential structure, in machinery and equipment, and in IPP.

Business inventories. The real values of changes in business inventories are obtained from the quarter-to-quarter changes of real stocks of inventories, instead of values from the expenditure-based GDP table. Real stocks of inventories are in turn measured using the current values of business inventories in NBSA12 and the implicit prices. But, appropriate implicit price is unavailable in CSNA12, so it is borrowed from Diewert-Yu which in turn was constructed based on prices provided to Diewert by Statistics Canada. This price series is interpolated then extended to the latest quarter using the Industrial product price index (IPPI).

Exports and imports. There are 12 types of goods and one type of services in both exports and imports, at the 2-digit NAPCS level. Seasonally unadjusted real values of exports and imports are not available in CSNA12, except for the aggregate goods and services. It turns out that most exports and imports of goods at the 2-digit NAPCS level do not exhibit significant seasonality, there are evidences showing that the seasonally adjusted prices are fairly close to unadjusted prices.²⁶ For exports and imports of goods exhibiting seasonality, the price seasonality of exports and imports for the aggregate good is applied.

For export of services, seasonally adjusted prices are fairly different from unadjusted prices, suggesting a price seasonality of exports in services.

At the aggregate level, the price of the aggregate export of goods do not exhibit strong seasonality. The seasonally adjusted and unadjusted prices of aggregate export of goods, based on 2007 constant dollar quantities, are fairly close to each other, with a deviation of 0.45 per cent on average. The largest deviation is 1.6 per cent, in 2007Q4. At the 2-digit NAPCS level, data for both seasonally adjusted and unadjusted prices are available for the period of 1997Q1 to 2013Q1. These prices are compared, it is found that the average deviation of the two price series among all 11 types of exports of goods is 0.3 per cent, suggesting that the price seasonality of exports is small.

²⁶Seasonally unadjusted prices of exports and imports (Laspeyres and Paache indexes) are available in tables of international merchandise trade.

The export of Energy products observes the largest deviation, at 1.7 per cent on average, followed by the export of Farm and food products, at 1.2 per cent on average. The mean deviation for the export of Metal and non-metallic mineral products is 1.0 per cent. The deviations for the rest of exports are all very small, significantly below 1.0 per cent. These evidences suggest that the prices of exported goods do not exhibit strong seasonality. Therefore, the following exported goods are assumed to have no price seasonality, including Farm and food products, Energy products, Metal and non-metallic mineral products, Motor vehicles and parts, Aircraft and other transportation equipment and parts, and Consumer goods. For other exports with larger differences between seasonally adjusted and unadjusted prices, the seasonality of the aggregate export of goods is applied to the chained-dollar seasonally adjusted prices.

The import of services does not display strong seasonality as the seasonally adjusted and unadjusted prices (at 2007 constant dollar) are close to each other, with the deviation being 0.54 per cent on average. In order to be consistent with the quantities and prices of export of services, price seasonality is applied to the seasonally adjusted prices, obtaining the seasonally unadjusted chained dollar import prices of services.

The price seasonality of imports of goods is largely similar to that of exports of goods. The deviation between seasonally unadjusted and adjusted prices of the aggregate import of goods is on average 0.33 per cent. For prices of imports at the 2-digit NAPCS level, the average deviation among 11 types of imported goods is 0.44 per cent. Different from export prices, the import prices of Energy products observe large seasonality, with the average deviation being around 3.3 per cent. The largest deviation is in 2009Q1 at 6.5 per cent. The mean deviation of prices is 1.6 per cent for imports of Farm and food products, and 1.2 per cent for Metal ores and non-metallic minerals. All other imported goods have very small differences between seasonally adjusted and unadjusted prices. For imports with small differences between adjusted and unadjusted prices, the seasonally adjusted chained dollar implicit prices are used. These imports include Metal and non-metallic mineral products, Basic and industrial chemical and plastic and rubber products, and Forestry products and building and packaging materials. For the rest of imports, price seasonality of import price of the aggregate imports in goods is applied to the chained dollar seasonally adjusted import prices.

Finally, import custom duties are removed from the import prices.

B.2 Quarterly labour Inputs

Labour inputs are the aggregate of quality-adjusted hours worked of 12 types of workers. In [Diewert and Yu \(2012\)](#), annual data series are available for labour compensation and hours worked

for each of 36 types of workers (including 2 gender groups, 3 age groups, 3 education groups, and paid versus self-employed workers). These 36 types of workers are aggregated to 12 types (2 sex groups, 3 education groups, and paid versus self-employed workers).

There are no readily available data to measure quarterly labour inputs. The Labour Force Survey (LFS) public user micro files are used to construct hours worked and compensation for the 36 types of workers.²⁷ These quarterly series are then benchmarked to the Diewert-Yu annual series with the Denton's method. After being benchmarked, the labour inputs display quarterly variation according to the LFS data, and adding quarterly values in each year will replicate the annual values.

Hours worked and earnings in LFS. LFS is a primary survey of the labour market, it covers employment status, hours worked, and compensation. To measure the quarterly labour inputs in the business sector, workers are selected who reports themselves working in business sector (all sectors except Education, Health care and Public administration). Individuals are also included if they are private sector employees working in Education services (NAICS 61) or Health care and social assistance (NAICS 62). Fraction of private sector workers is very small in Education services, while it is large in Health care (close to 40 per cent in more recent years). All self-employed workers are in business sector, as in LFS they are labeled as private sector workers.

The LFS has information on both usual and actual weekly hours for both main job and all jobs. Here, the actual weekly hours worked in all jobs are used. The quarterly total hours worked for each of the 36 types of workers is the quarterly employment multiplied by the average weekly hours worked, then multiplied by 13. The total quarterly compensation of each type of worker is obtained as the average hourly earnings (for main jobs) multiplied by total quarterly hours for each type of the worker.

Benchmarking. The Denton's method is used to benchmark the 72 series of hours worked and compensation to the annual data as in Cansim 383-0024.²⁸ Compensation in LFS is missing before 1997Q1 for all types of workers. For the period of 1990Q1 to 1996Q4, the annual series of compensation are interpolated where the quarterly variation accords with the quarterly wages and salaries and supplementary labour income in business sector from SEPH.²⁹ Therefore, before 1997Q1, the

²⁷Another micro data, Survey of Employment, Payrolls and Hours (SEPH), also measure quarterly employment, compensation and hours. But SEPH does not have labour inputs by education and age. In Statistics Canada's Labour Productivity program, the employment and hours of employees in business sector are obtained as the geometric mean of series from LFS and SEPH, then benchmarked to the annual data.

²⁸In the Stata software, this method is developed by [Baum and Hristakeva \(2011\)](#).

²⁹In SEPH, Cansim table 382-0021 is used for the aggregate quarterly compensation. In this Cansim table, it is not possible to separate the business sector part of labour income from Education services and Health care services,

quarterly variation of compensation is common for all types of workers.

The quarterly data for quarters after 2010Q4 cannot be benchmarked, because the last period of the annual data is 2010. All benchmarked series are extended to the latest quarter using the LFS quarterly series.

B.3 Quarterly capital services.

Capital service is the user cost of capital multiplied by real capital stocks. The challenging part is measuring real capital stocks. The standard practice is to construct capital stocks using investment and depreciation rates by the perpetual inventory method where the book value of capital stocks in the initial period is also needed. Let K_{it+1} be the capital stock of asset i at the beginning of year $t + 1$, I_{it} be the real values of investment, and δ_{it} be capital depreciation rates. The perpetual inventory method to obtain the real capital stocks is given by

$$K_{it+1} = (1 - \delta_{it})K_{it} + I_{it}.$$

Asset categories should be at a reasonably disaggregated level, to overcome the aggregation error (for example, due to the heterogeneity of assets). Presumably one can construct flows and stocks of physical assets using the national balance sheets NBSA12 (for stocks) and the national accounts CSNA12 (for quantities and prices of investment). In fact, this is possible only at the aggregate level at which there are 6 types of assets: Residential structures, Non-residential structures, Machinery and equipment, Inventories, Land, and IPP (IPP is new in NBSA12). Capital stocks at the detailed level are unavailable in NBSA12.

We construct the real capital stocks at the disaggregated level, using the investment data from CSNA12, and using the capital stocks (for initial period stocks and depreciation rates) from the annual data of capital flows and stocks. But the new (quarterly) and annual data do not match exactly. On the investment side, in CSNA12, there are 9 types of machinery and equipment: Industrial machinery and equipment; Computers and computer peripheral equipment; Communications and audio and video equipment; Other electrical and electronic machinery and equipment; Other machinery and equipment; Furniture, fixtures and prefabricated structures; Passenger cars; Trucks, buses and other motor vehicles; and Aircraft and other transportation equipment. In the annual data (of capital flows and stocks), there are also 9 types of machinery and equipment, but they do not align with those in CSNA12. For example, there is no stand-alone investment in Trucks and investment in Agricultural machinery in CSNA12. These differences make it difficult to combine

so these two sectors are excluded from the business sector (which is inconsistent with the definition of business sector used for quarterly growth accounting).

the new investment data with the annual capital stocks data (for the perpetual inventory method).

Table D lists the types of investment in machinery and equipment. Judged by names and also from concordance table provided by Statistics Canada, it appears that the types of quarterly investment and of annual investment (capital flows) do not match.

Table D: Types of machinery and equipment in quarterly and annual series of investment

Quarterly (Cansim 380-0068)	Annual (Cansim 031-0003)
Industrial machinery and equipment	Industrial machinery
Computers and computer peripheral equipment	Computers
Communications and audio and video equipment	Telecommunication equipment
Other machinery and equipment	Other machinery and equipment
Furniture, fixtures and prefabricated structures	Furniture
Passenger cars	Automobiles
Trucks, buses and other motor vehicles	Trucks
Aircraft and other transportation equipment	Other transportation
Other electrical and electronic machinery and equipment	Agricultural machinery

Moreover, even for the seemingly same types of investment, differences between new quarterly data and the annual data are large, in both current values and real values, and these differences appear to be random over time. To obtain the annual investments from new quarterly data (Cansim 380-0068), we use the annual averages of quarterly investments that are seasonally adjusted at annual rates. Table E shows differences for the same or very similar capital types between Cansim 380-0068 and Cansim 031-0003. Investment in engineering structures is different between annual and quarterly data in pretty much all years. For some other asset types, the large differences of investment in current values occur mostly in more recent years, suggesting that Cansim 031-0003 may not have the latest possibly revised values. In particular, investments in IPP is new in both Cansim 031-0003 and in CSNA12, the differences between annual and quarterly values are large since 2007. Investment in Mineral exploration and evaluation in 2011 and 2012 is much lower in Cansim 380-0068 than in Cansim 031-0003. For investment in software, the quarterly-annual differences start since 2000, quarterly values become significantly larger than annual values starting from 2009. Investment in machinery and equipment has been increasingly lower in Cansim 380-0068 than in Cansim 031-0003 since 2000. In summary, except for investments in IPP and its components, it appears that quarterly-annual differences in investment do not have a clear pattern, suggesting that investment data are not consistent across different Cansim tables.

Capital stocks. To overcome the above differences of investment series between quarterly and annual data, assets are aggregated to 9 types in the quarterly estimates, including 4 types of ma-

Table E: Deviation of investment between Cansim 031-0003 and 380-0068, in percentage

Capital type	Current value		Chained dollar value	
	Mean	Maximum	Mean	Maximum
Non-residential structures	2.2	4.7	3.1	5.8
Buildings	5.2	20.9	4.9	20.3
Engineering	3.6	12.1	3.8	11.0
Intellectual property products	1.3	9.9	13.6	28.3
Mineral exploration and evaluation	0.9	21.6	3.5	18.7
Research and development	1.4	5.0	1.9	44.2
Software	2.9	20.4	18.1	35.3
Machinery and equipment	1.8	7.9	1.9	7.8
Computers	5.3	14.9	2.5	11.5
Telecommunication equipment	4.2	23.8	5.4	23.9
Furniture	15.6	55.7	20.3	79.1

chinery and equipment, 2 types of non-residential structures, and 3 types of IPP. Non-residential structures include non-residential buildings and engineering structures. IPP includes Mineral exploration and evaluation, Research and development, and Software. Machinery and equipment includes Computers, Communications equipment, Industrial machinery and equipment, and Transportation and furniture. This last asset is the aggregate of transportation equipment and furniture. Grouping machinery and equipment in this way brings current values of investments the closest between the quarterly and the annual data. Industrial machinery and equipment from Cansim 380-0068 consists of Industrial machinery and equipment and Other machinery and equipment, and from Cansim 031-0003 it consists of Industrial machinery, Agricultural machinery and Other machinery. Computers and Communications equipment respectively are the same assets by definition in Cansim 380-0068 and Cansim 031-0003. Investments in Transportation and furniture are obtained as the total investment subtracting investments in the first 3 types of machinery and equipment, calculated using the Fisher quantity index.

To calculate capital stocks with the perpetual inventory method, capital depreciation rates are first calculated from annual data of flows and stocks of capital (Cansim 031-0003). It is assumed that depreciation rates are constant in 4 quarters of a year. Capital depreciation rates are then smoothed using the Lowess non-parametric method. The 1990 capital stocks are used to start the recursion, and use seasonally unadjusted investments are used from quarterly investment data in CSNA12 (Cansim 380-0068).

Seasonally unadjusted real values of capital are available only for the aggregate capital of 3 types: non-residential structures, machinery and equipment, and IPP. It is assumed that, assets belonging to the same aggregate asset exhibit the same seasonality as the aggregate asset. The price

seasonality for the 3 aggregate capital types is obtained first, using current values from Cansim 380-0068 and real values from Cansim 380-0084. Seasonally adjusted prices for the 9 types of assets are obtained from Cansim 380-0068. Applying the prices seasonality to the seasonally adjusted prices gives the seasonally unadjusted prices of investments. With the seasonally unadjusted current values and prices, we can obtain the seasonally unadjusted real values of investments.

Table F reports the quarterly depreciation rates of capital. Depreciation rates for computers are again much larger than all other assets.

Table F: Quarterly capital depreciation rates

Asset	Average depreciation rate (%)
Non-residential structures	
Buildings	1.33
Engineering	2.45
Machinery and equipment	
Computers	18.17
Communications equipment	7.63
Industrial machinery	5.31
Transportation and furniture	7.59
Intellectual property products	
Software	13.43
Mineral exploration and evaluation	3.93
Research and development	7.72

Land. Quarterly quantities of land are obtained by interpolating the annual series for both the agricultural land and the business (non-agricultural) land, as well as the residential land (for calculating property tax rate).

Property tax rate. It is assumed that there are property taxes for five types of property: residential land, residential structure, agricultural land, non-agricultural business land, and non-residential structure. Of these properties, residential land and residential structure are not inputs of production. The effective property tax rate is the total property tax revenue divided by the total value of 5 types of properties.

The general business tax falling on the use of capital stocks is also needed, which is obtained as in [Diewert and Yu \(2012\)](#).

Real rate of return. The real rate of return is obtained by setting total revenue equal to total input costs, implied by the assumption of constant return to scale in production function. On the

output side, there are 10 output components; on the input side, there is the cost of labour for 12 types of workers and cost of capital for 14 types of assets.

The user cost of capital for each asset can be calculated using the obtained series of tax rates, real rate of return, and prices of capital.

B.4 Data for annual growth accounting

Data for the annual growth accounting is briefly described as the annual estimates are an update of [Diewert and Yu \(2012\)](#). Below only the differences from Diewert-Yu are explained

Annual output. There are several differences between the Diewert-Yu estimates and the current revision. First, in CSNA12, general government includes both government and NPISH. Second, research and development is a new type of asset, as a component of IPP. Third, exports and imports are revised and re-classified in CSNA12.

Annual inputs. Labour inputs are the same as measured in Diewert-Yu. The minor difference is that in the current revision, hours worked and compensation are extended from 2010 to 2012, using the LFS micro files. Capital services are also measured closely following Diewert-Yu. In CSNA12 and NBSA12, a new type of asset is introduced, IPP.

Table G: List of Cansim tables used for quarterly growth accounting

Variable	Table	Comments
Aggregate consumption expenditure	380-0064, 380-0084	
Consumption tax	380-0080, 380-0081	
Imputed rent	379-0027, 379-0031	Price is from annual estimates
Paid rent	380-0085	
Government spending	380-0064, 380-0084	
Non-business GDP at basic price	379-0027,379-0031	Price is from annual estimates
Government investment	380-0064, 380-0084	
Business investments	380-0064, 380-0084	
Business inventories	378-0121, 329-0056	Price is from annual estimates
Exports/imports	380-0064, 380-0070, 380-0080, 380-0084	
Labour inputs	383-0024,382-0021, LFS	
Capital services	380-0068, 031-0003	
property tax	378-0121, 380-0080	

C Appendix C: Figures

Figure 1: Real GDP, real input and TFP: 1981-2012

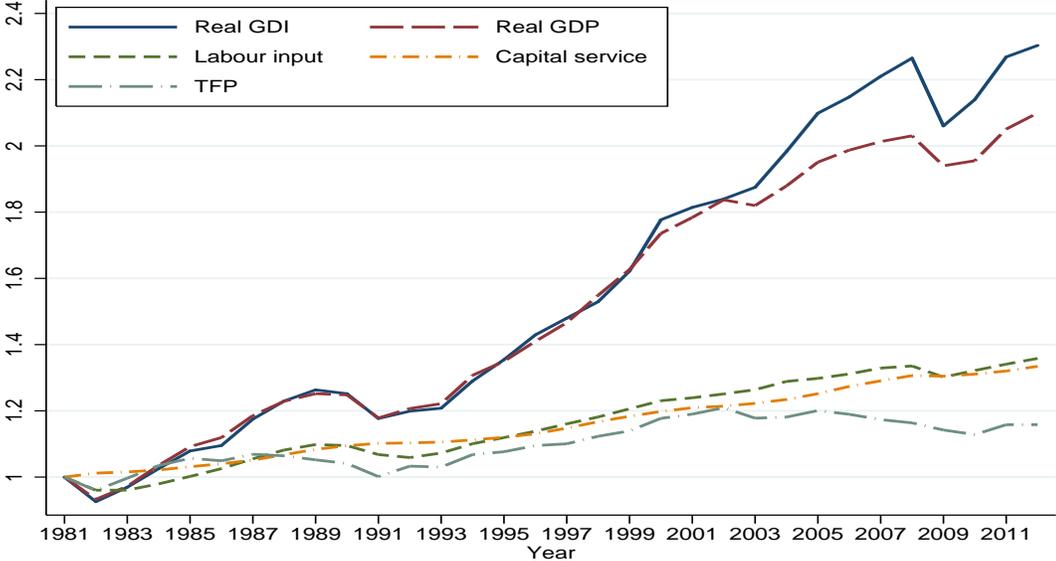


Figure 2: Role of R&D in TFP

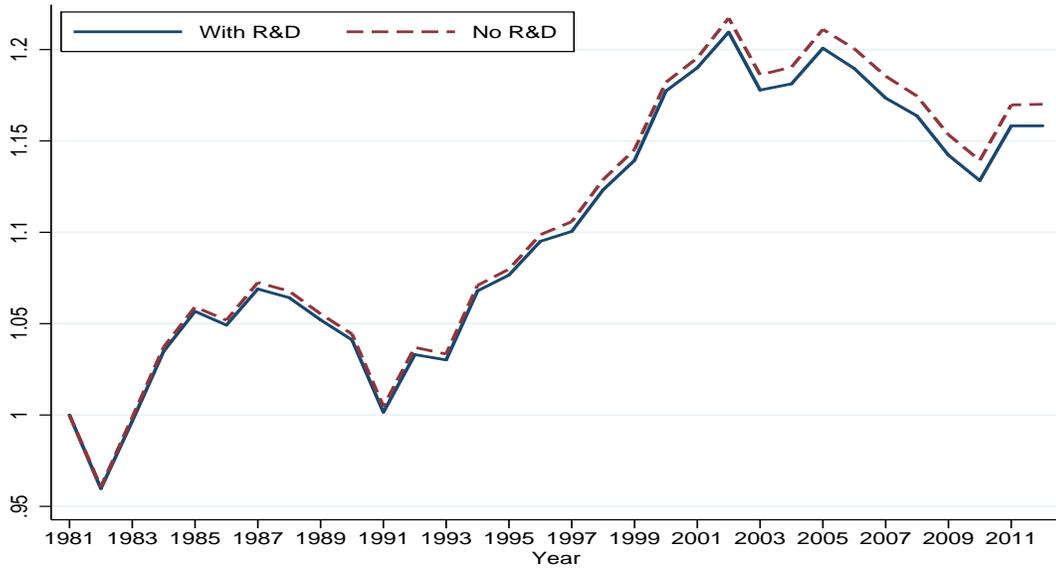


Figure 3: Comparison of TFP with Diewert-Yu estimates



Figure 4: Comparison of TFP with KLEMS

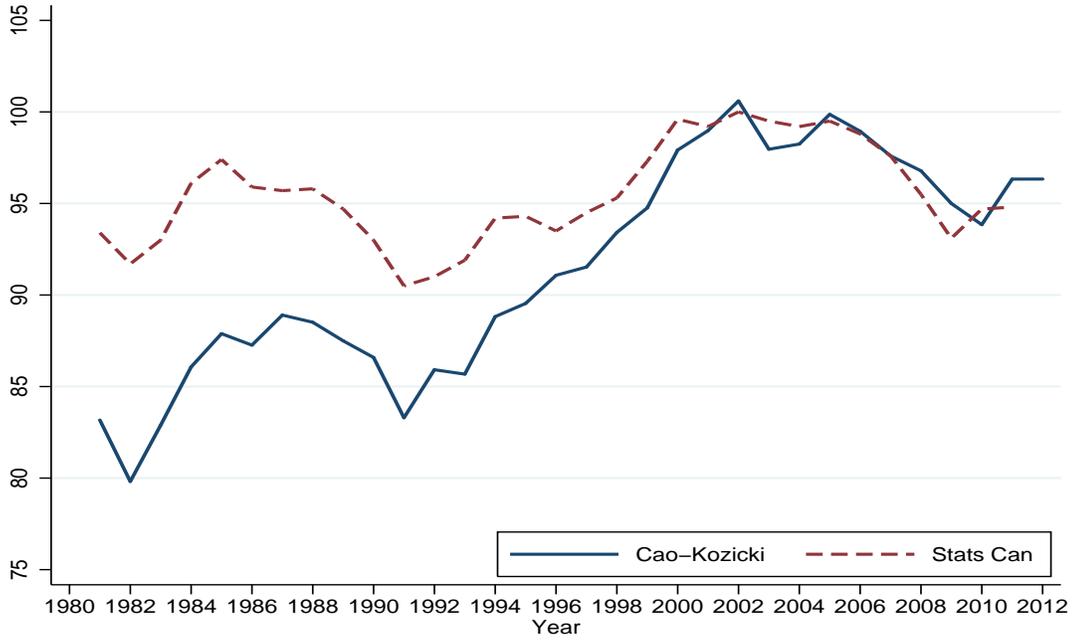


Figure 5: Real quarterly GDP and components, seasonally adjusted

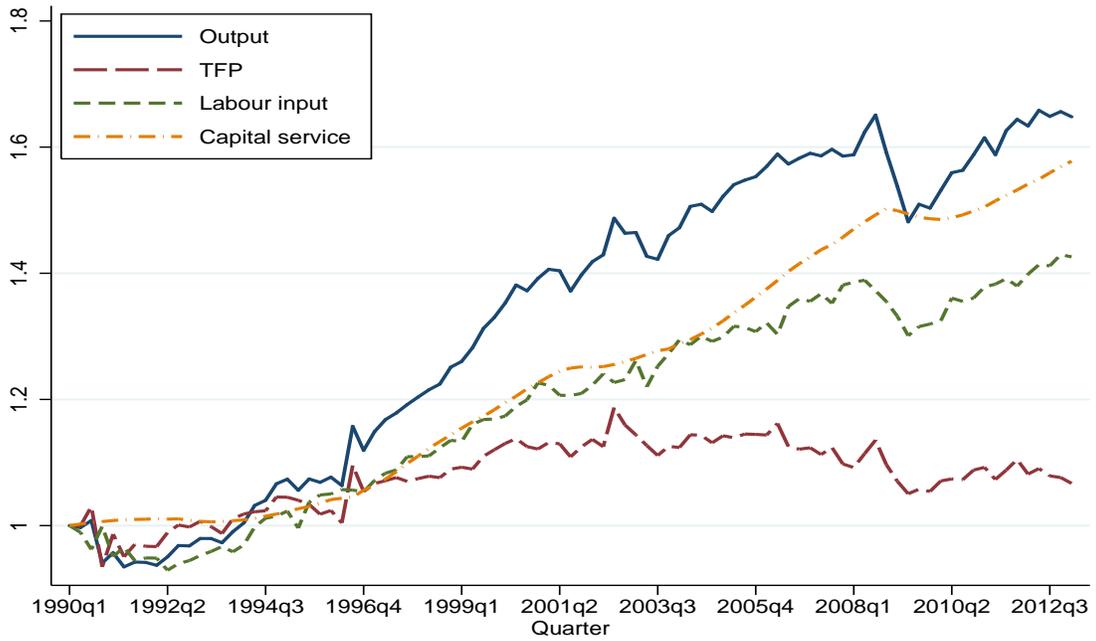


Figure 6: Comparing annual and quarterly TFP growths

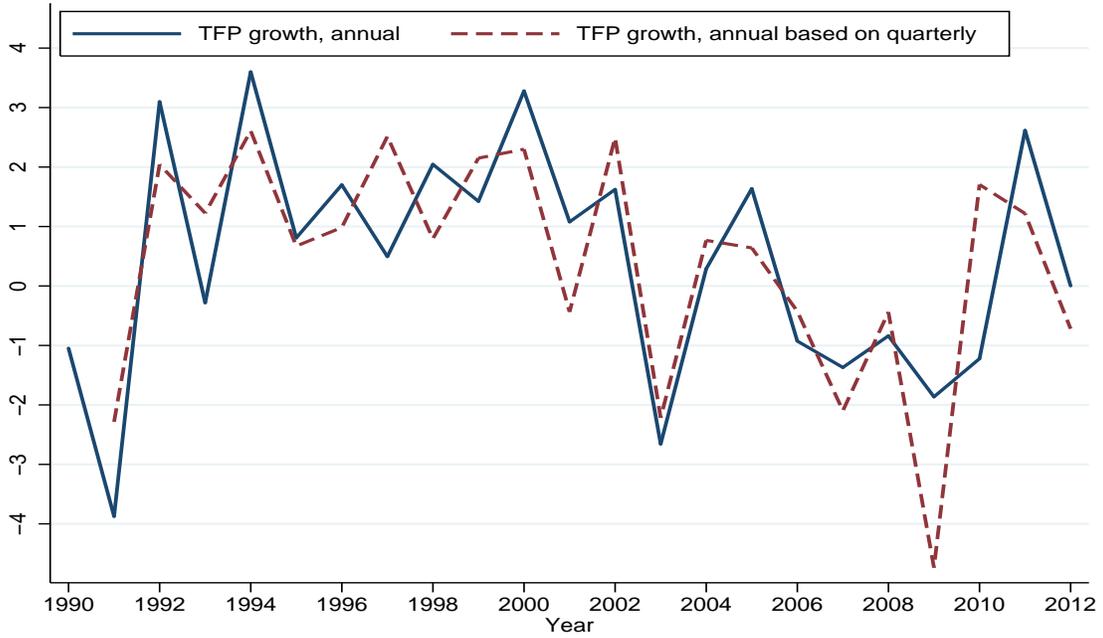
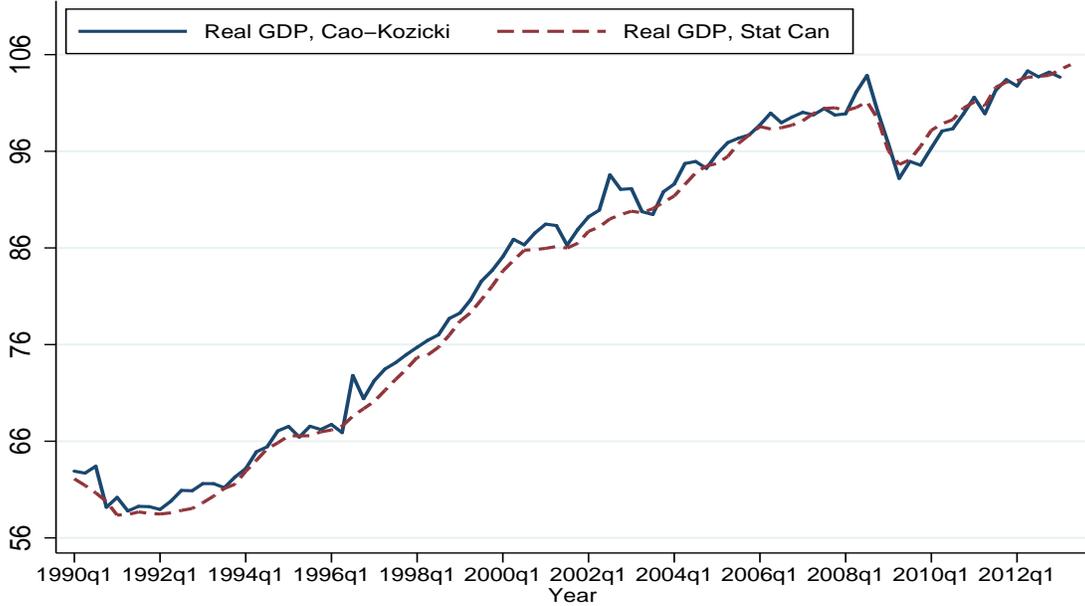
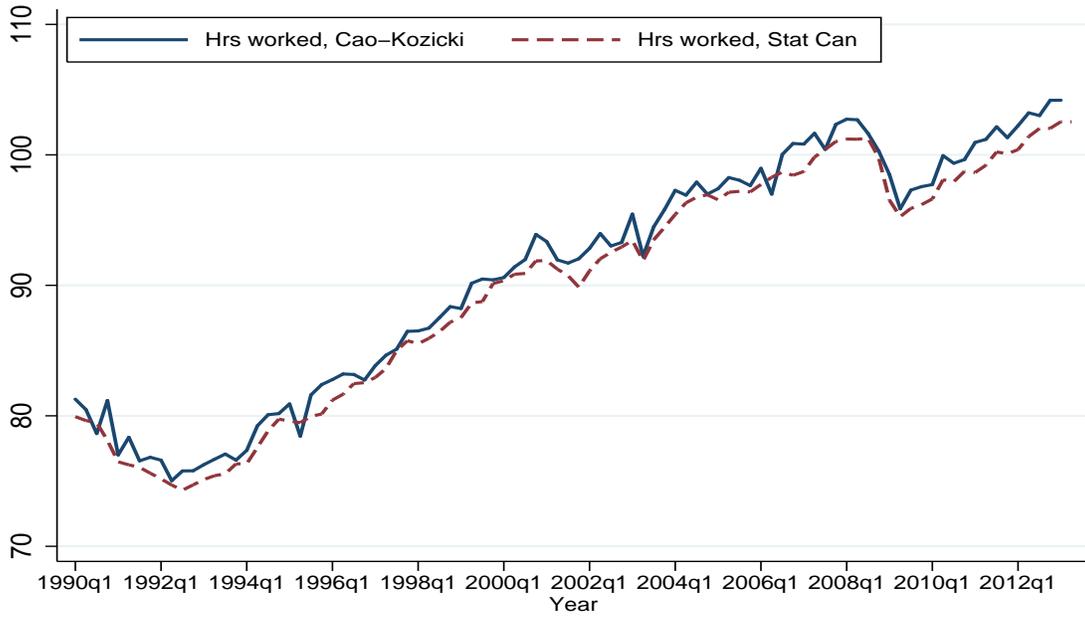


Figure 7: Real GDP in business sector



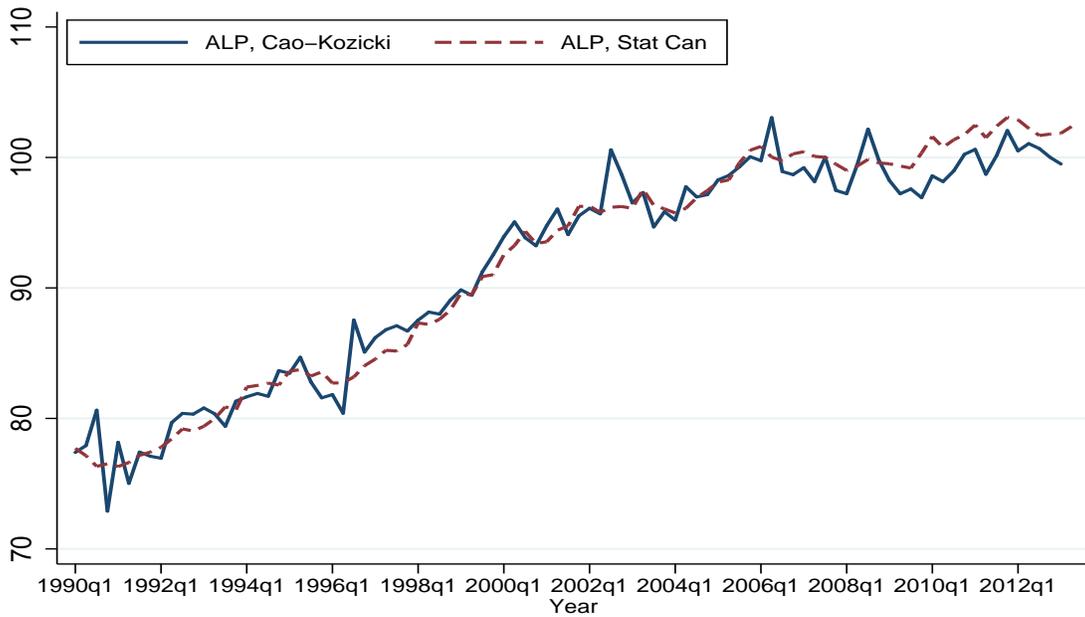
Source: Cansim 383-0008

Figure 8: Hours worked in business sector



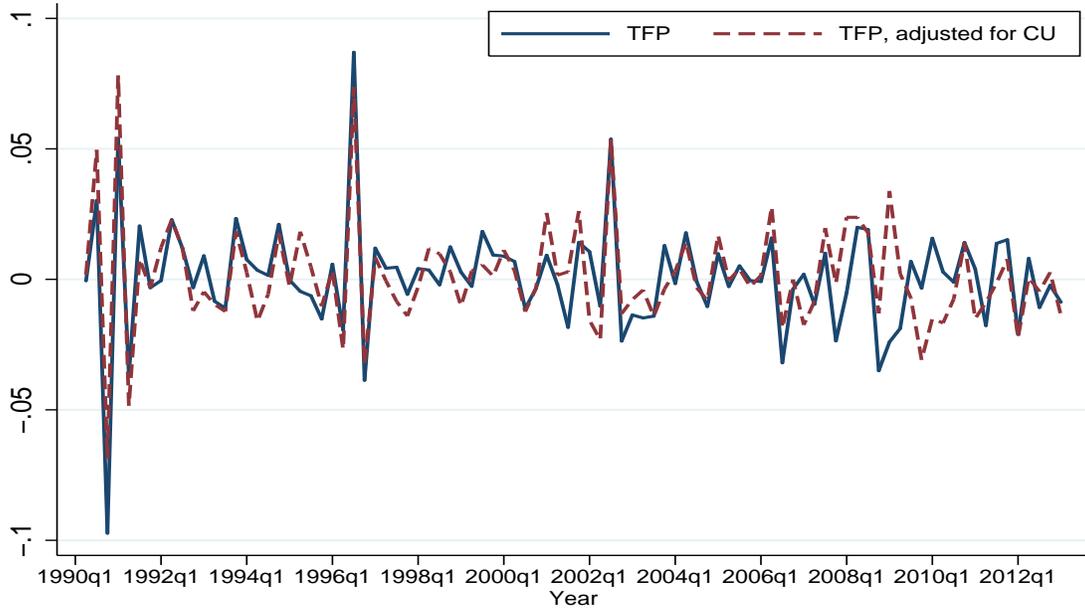
Source: Cansim 383-0008

Figure 9: Average labour productivity in business sector



Source: Cansim 383-0008

Figure 10: TFP and adjustment for capacity utilization



Source: authors' calculation; Cansim 028-0002

Figure 11: Real GDI and real GDP, 1981-2011

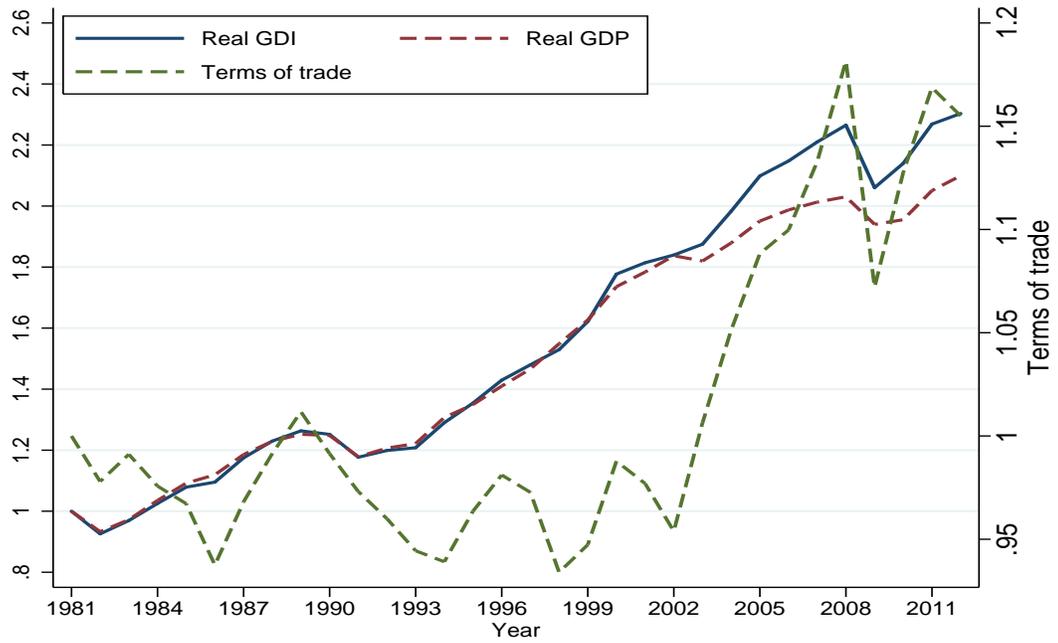


Figure 12: Decomposition of cumulative growth in real income

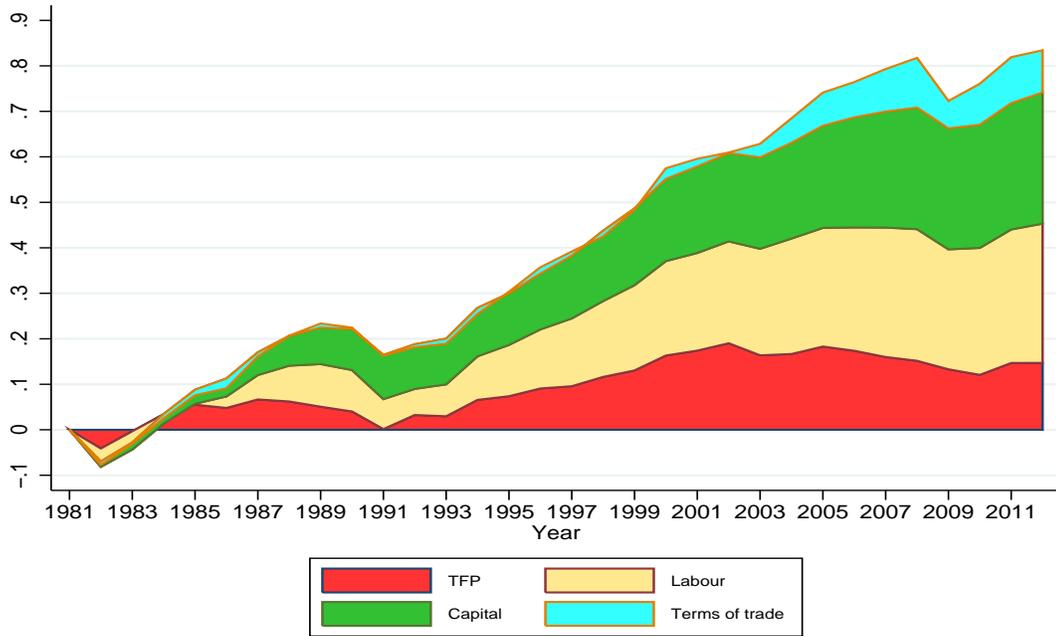


Figure 13: Decomposition of real income growth

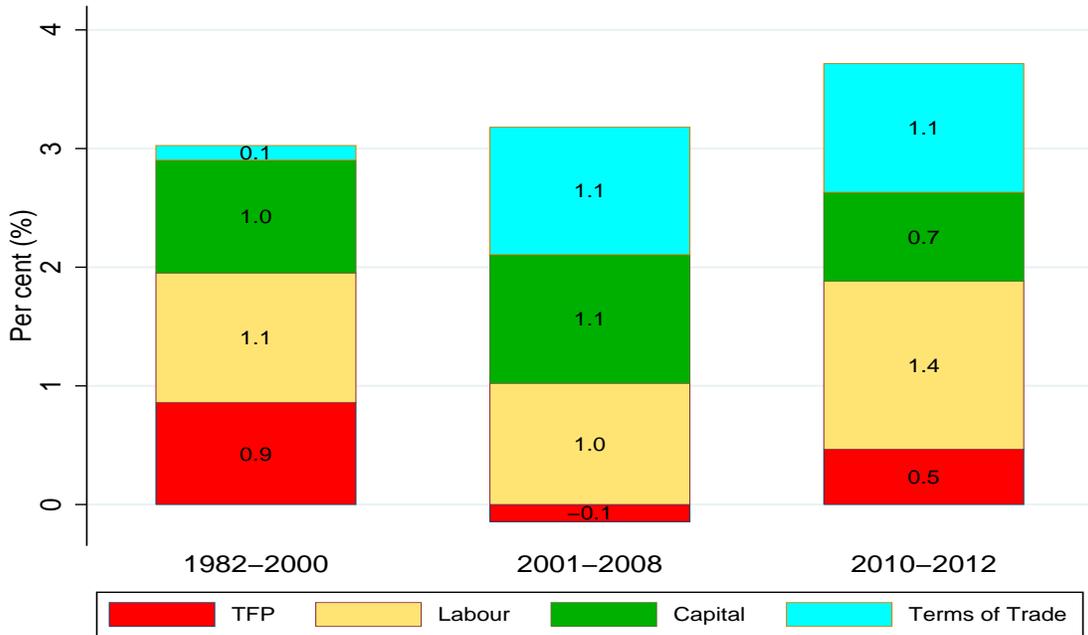


Figure 14: Real price indexes of exports and imports, 1981-2012



Figure 15: Share of export values in total export

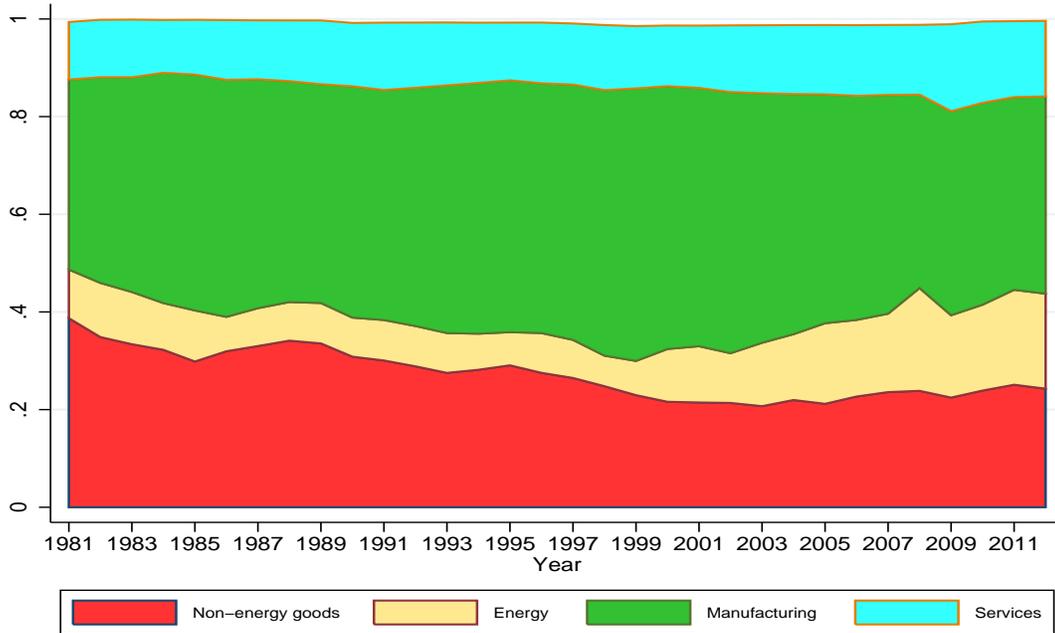


Figure 16: Share of import values in total import

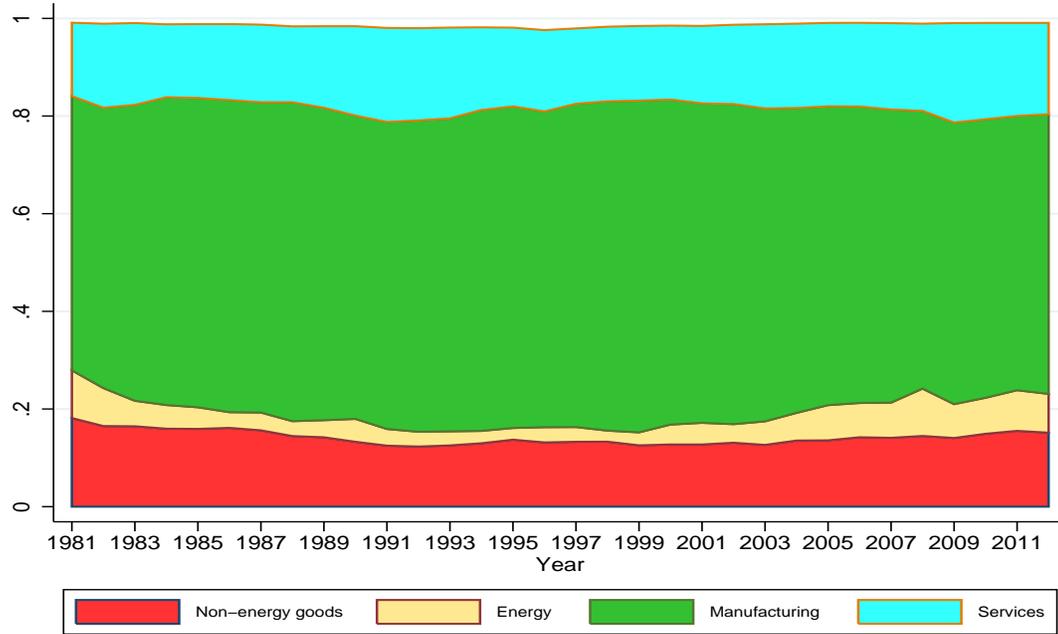


Figure 17: Decomposition of changes in aggregate terms of trade



Figure 18: Real price indexes of exports and imports by product, 1981-2012

