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Mobility Decomposition Over Several Periods: Theory and Evidence on Consumption Mobility from Peru

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Abstract

Decomposing economic mobility into structural and exchange components has long been a matter of concern among social scientists. In this paper I propose a decomposition of the multi-period mobility index proposed by Shorrocks (1978) into structural and exchange mobility components. The decomposition is applied to a study of consumption mobility in Peru.

Introduction

Decomposing economic mobility into structural and exchange components has long been a matter of concern among social scientists. While there seems to be consensus that exchange mobility is the component that captures the effect of re-rankings; the definition of structural mobility is less precise in the literature: the concept is thought to capture everything else, chiefly changes in the marginal distributions. Recently van Kerm has proposed understanding structural mobility in terms of a *growth* component and a *dispersion* component (van Kerm, 2004). His decomposition of mobility into structural and exchange components, and that of Ruiz-Castillo (2004), are the most significant recent contributions. Both work using counterfactual distributions that isolate the respective components. But these techniques have been devised for two-period analyses. An alternative decomposition has been proposed by Tsui (2009), which is the first (to my knowledge) that is applicable to several periods. However, Tsui's decomposition works with some benchmark situations, of complete mobility and complete immobility, that are at odds with those discussed in the rest of the literature.[1]

Considering this recent interest in mobility decomposition, this paper proposes a decomposition of mobility over several periods using the mobility concept and approach of Shorrocks (1978). Together with Maasoumi and Zandvakili (1986), they measure mobility over some continuous measure of wellbeing (typically income) as the degree of equalization of the whole aggregate "income" stream across a time period, relative to a weighted sum

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[1] A discussion on these differences is provided below.

of time-specific "income" inequalities. The proposed decomposition is an application of the counterfactual approach of Ruiz-Castillo and Van Kerm to the indices proposed by Shorrocks (1978). This proposal has the advantage of being applicable to multiple periods and of having benchmarks of complete mobility and perfect immobility, for the structural and exchange components, that are more in tune with the concepts discussed in the early literature. Extensions to alternative ways of computing aggregate "income" are also explored, based on the work of Maasoumi and Zandvakili (1986); and a comparison to the approach of Tsui (2009) is also provided.

Finally, the proposed decomposition is applied to the analysis of consumption mobility using two panel datasets from the Peruvian National Household Survey (ENAHO) spanning the periods 1998-2001 and 2002-2006; and using Shorrocks' index, combined with the square coefficient of variation for the inequality measurement. Mirroring the trends in income mobility conjectured, and confirmed, by Shorrocks, I find that, in both panel datasets, consumption mobility also increases with the length of the time period. Interestingly, the contribution of exchange mobility is never above 13%, and it *decreases* when the time period is lengthened.

The paper is organized as follows: The next section presents the decomposition of the Shorrock's mobility index into exchange and structural mobility components. The section ends with a subsection exploring extensions to alternative forms of aggregating the income stream. The following section compares the proposed decomposition with the approach developed by Tsui (2009). Then the empirical application to the analysis of consumption mobility and inequality in Peru proceeds, first with a subsection on data details and then with the results. The paper ends with some concluding remarks.

Mobility decomposition over multiple periods

Following the notation in Tsui (2009), let x_{it} ($\in \mathbb{R}_{++}$) be the value of a well-being attribute (to be called "income") for individual i in period t. With N individuals and T time periods, a matrix X with N rows and T columns summarizes the "income" trajectories for each individual (in the rows, each denoted by a row vector $x_{i.}$) and the "income" distributions in each time period (in the columns, each denoted by a column vector $x_{.t}$). The average "income" for individual i across time is: $\mu_i \equiv \frac{1}{T}\sum_{t=1}^{T} x_{it}$; whereas the average "income" in the population in period t is: $\mu_t \equiv \frac{1}{N}\sum_{i=1}^{N} x_{it}$. Respectively, the vector of average individual "incomes" from X is $V_N^{\mu}(X)$, and the vector of average time-specific "incomes" is $V_T^{\mu}(X)$. The total average "income across people and time is: $\mu \equiv \frac{1}{NT}\sum_{i=1}^{N}\sum_{t=1}^{T} x_{it}$. Shorrock's mobility index is the following:

$$M = 1 - \frac{I\left(V_N^{\mu}(X)\right)}{\sum_{t=1}^{T} w_t I\left(x_{.t}\right)}, \tag{1}$$

where $w_t = \frac{\mu_t}{\mu}$ and $I()$ is a strictly-convex, symmetric, scale-invariant inequality measure. As Shorrocks shows $M = 1$ if and only if $I\left(V_N^{\mu}(X)\right) = 0$, i.e. "when incomes are exactly equalized over the whole time interval" (Shorrocks, 1978, p. 386). This is the benchmark of complete mobility. Also $M = 0$ if and only if all the period-specific distributions of x have the same degree of relative inequality and no re-rankings or exchanges have taken place. Having the same degree of relative inequality means that any distribution, e.g. $x_{.2}$, can be obtained from any other distribution, e.g. $x_{.1}$, by multiplying each element of the latter by the same positive scalar. This is the benchmark of complete structural immobility. Coupled with the absence of any re-ranking, it yields the benchmark of complete immobility.

The decomposition

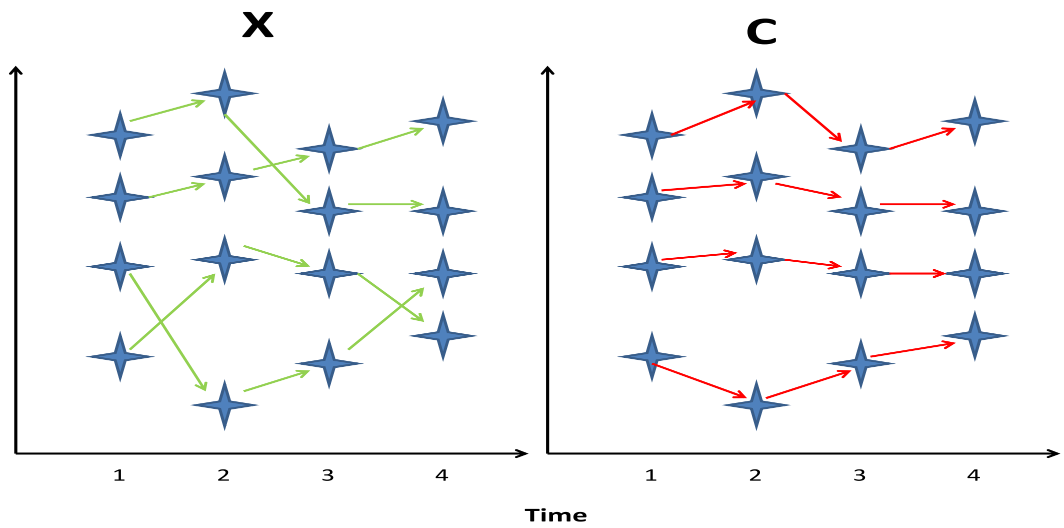
In their seminal articles, van Kerm (2004) and Ruiz-Castillo (2004) recommend isolating the exchange component from the structural component of mobility using counterfactual distributions, in a two-period setting. The idea is to consider exchange mobility as the consequence of re-rankings among individuals from one period to another. In this conception, a situation of distributional dynamics without exchange mobility is one in which each individual keeps his/her rank across all periods, from the poorest to the richest. Hence the counterfactual distributions are made of the same marginal distributions for each time period, but now the values of the poorest individual in each period are brought together to form one (counterfactual) trajectory (that of the poorest person without exchange mobility), and the same is done for each rank up to the formation of a (counterfactual) trajectory for the richest individual under the absence of exchange mobility.

Formally, the counterfactual distribution, C, can be generated by applying a series of *association increasing switches.*[2] These switches render the vectors $x_{.1}, x_{.2}, \ldots, x_{.T}$ relatively more similarly arranged.[3] Specifically, with an association-increasing switch a new matrix Y is derived from X by transfering "incomes" between individuals i and j in such a way that (weak) vector dominance of i over j is established. That is, $y_{i.} = x_{i.} \nabla x_{j.}$ and $y_{j.} = x_{i.} \triangle x_{j.}$, where $x_{i.} \nabla x_{j.} \equiv (\max[x_{i1}, x_{j1}], \ldots, \max[x_{iT}, x_{jT}])$ and $x_{i.} \triangle x_{j.} \equiv (\min[x_{i1}, x_{j1}], \ldots, \min[x_{iT}, x_{jT}])$. Meanwhile the other individuals are left untouched. Distribution C is attained once additional assocation-increasing switches are unable to generate any further changes in the original matrix X. Figure 1 illustrates the trajectories from X, along with the respective counterfactual trajectories from C, i.e. in the absence of exchange mobility.

2 The term "association-increasing switch" is taken from Seth (2011).

3 This expression follows a similar one used by Boland and Proschan (1988).

Figure 1: Left panel: Individual trajectories in X. Right panel: Counterfactual trajectories in C



Now, an element c_{rt} of C denotes the well-being attainment of the counterfactual individual r in period t. As before, $c_{r.}$ and $c_{.t}$ refer to (counterfactual) trajectories and distributions, respectively.[4] Given that in distribution C all re-rankings are purged out, the following index of multi-period structural mobility is proposed:

$$S = 1 - \frac{I\left(V_N^{\mu}\left(C\right)\right)}{\sum_{t=1}^{T} w_t I\left(c_{.t}\right)}. \tag{2}$$

It is easy to show that S also abides by Shorrock's theorem 1 (Shorrocks, 1978, p. 381), which implies: $0 \leq S$ and $S = 0$ if and only if all time-specific marginal distributions have the same level of relative inequality.[5] Interestingly, $S < 1$ because $I\left(V_N^{\mu}\right) = 0 \rightarrow I\left(c_{.t}\right) = 0 \forall t$, which renders S indeterminate. S fulfills the desirable properties of $I\left(\right)$ (e.g. scale invariance, population invariance) and, crucially, is insensitive to association-increasing switches (because C is not affected by them). Hence S is not "contaminated" by sources of exchange mobility.

Finally, the following index of multi-period exchange mobility is proposed, as the residual ensuing from subtracting structural mobility from total mobility:

$$E \equiv M - S = \frac{I\left(V_N^{\mu}\left(C\right)\right) - I\left(V_N^{\mu}\left(X\right)\right)}{\sum_{t=1}^{T} w_t I\left(c_{.t}\right)} \tag{3}$$

4 Note that the elements of $c_{.t}$ are the same as those of $x_{.t}$, but permutated.

5 This also means that, unlike the two-period case, the structural component based on the Shorrock's index, cannot distinguish between a growth-component and a dispersion component. An understandable result considering that with three, or more, periods the choice of a reference distribution, for the measurement of a growth component, becomes arbitrary.

Besides fulfilling properties of scale invariance and population invariance: $E \geq 0$. The proof is as follows: Firstly, note that $I\left(V_N^{\mu}(X)\right)$ can be expressed as $I\left(x_{.1} + x_{.2} + \ldots + x_{.T}\right)$, i.e. a function of the sum of time-specific vectors. Now Boland and Proschan (1988) state, in their proposition 2.5(a) (p. 289), that if and only if $I\left(x_{.1} + x_{.2} + \ldots + x_{.T}\right)$ is a Schur-convex function, then $I\left(V_N^{\mu}(X)\right)$ is *arrangement increasing.* A rigurous definition of an arrangement-increasing function requires introducing an arrangement-similarity ordering among vectors. The reader is referred to Boland and Proschan (1988) for the details. Here it suffices to note that if Y is obtained from X by an association-increasing switch, then $f(Y) \geq f(X)$ for any real-valued, arrangement-increasing function f. Also any convex and symmetric function is Schur-convex. Hence $I\left(x_{.1} + x_{.2} + \ldots + x_{.T}\right)$ is Schur-convex and arrangement increasing. Finally, since C is always obtainable from X by a sequence of assocation-increasing switches, it follows that $I\left(V_N^{\mu}(C)\right) \geq I\left(V_N^{\mu}(X)\right)$, which implies $E \geq 0$.

For the same reasons, $E = 0$ if and only if $C = X$, i.e. when there is no exchange mobility. In the other extreme, $E = 1$ if and only if $I\left(V_N^{\mu}(X)\right) = 0$ and all time-specific marginal distributions have the same level of relative inequality. In other words, $E = 1$ if and only if there is complete mobility and it is all attributable to the exchange component.

Extensions

Maasoumi and Zandvakili (1986) noted that Shorrocks (1978) only considered the choice: $w_t = \frac{\mu_t}{\mu}$ for his index, M. Drawing from this insight, in this subsection I first show that the indices M, E and S can be used with several other choices for the weights of the time-specific inequality measures. Then I discuss the suitability of other options explored by Maasoumi and Zandvakili (1986), for decomposability purposes.

Alternatively to μ_i, the individual aggregate "income" can also be computed assigning general weights to the different years; for instance: $\overline{\mu}_i \equiv \sum_{t=1}^{T} \delta_t x_{it}$, where $\delta_t > 0$ and $\sum_{t=1}^{T} \delta_t = 1$. Then it is straightforward to replicate the proof for Theorem 1 part (a) in Shorrocks (1978, p. 381) in order to show that:

$$I\left(V_N^{\overline{\mu}}(X)\right) \leq \sum_{t=1}^{T} \overline{w}_t I\left(x_{.t}\right), \tag{4}$$

where $V_N^{\overline{\mu}}(X) := (\,\overline{\mu}_1,\, \overline{\mu}_2, \ldots,\, \overline{\mu}_N)$ and $\overline{w}_t = \frac{\delta_t \mu_t}{\sum_{i=1}^{N} \overline{\mu}_i}$.[6] Hence, not only different Schur-convex inequality indices can be chosen for $I()$, but also different choices of aggregate individual "income" and intertemporal weights are amenable for use in the construction of Shorrocks mobility indices decomposable into structural and exchange components, just as in the previous sub-section. The only requirement is that the weights, δ_t, used in $\overline{w}_t$, should be the same as those used in $\overline{\mu}_i$. The particular choice of the previous sub-section is, simply: $\delta_t = \frac{1}{T}$. An immediate benefit of this extension is that the weights, δ_t, can be used explicitly as time-discount factors.

Now, relying on the work of Maasoumi (1986), Maasoumi and Zandvakili (1986) proposed to extend Shorrock's concept of mobility as equalization of aggregate "incomes" to cases where the aggregate individual "income" is a generalized mean of time-specific "incomes",

[6] This proof, as well as any of the other ommitted ones in the paper, is available from the author upon request.

i.e. not just the arithmetic mean case considered by Shorrocks (1978), but in the context of *generalized entropy measures of inequality*. In the particular case of *the two Theil measures*, it is easy to deduce from Maasoumi (1986) that:

$$I_{\gamma}^{GE}\left(V_N^{\mu,\gamma}(X)\right) \leq \sum_{t=1}^{T} \delta_t \left[\frac{\mu_t}{\sum_{i=1}^{N} \overline{\mu}_i}\right]^{1+\gamma} I_{\gamma}^{GE}(c_{.t}), \forall \gamma = -1, 0, \tag{5}$$

where I_0^{GE} and I_{-1}^{GE} are the two Theil measures (see Maasoumi, 1986, p. 992), $V_N^{\mu,\gamma}(X) := \left(\mu_1^{1+\gamma}, \ldots, \mu_i^{1+\gamma}, \ldots, \mu_N^{1+\gamma}\right)$ and $\mu_i^{1+\gamma}$ is a weighted generalized mean such that: $\mu_i^{1+\gamma} \equiv \left[\sum_{t=1}^{T} \delta_t x_{it}^{1+\gamma}\right]^{\frac{1}{1+\gamma}}$.[7] It is easy to show, by looking at the proposition 2 of Maasoumi (1986), that $I_{\gamma}^{GE}\left(V_N^{\mu,\gamma}(X)\right) = \sum_{t=1}^{T} \delta_t I_{\gamma}^{GE}(c_{.t})$ if and only if all the time-specific marginal distributions have the same relative inequality, i.e. one distribution can be obtained from any other one by a positive scalar multiplication.

The decomposition proposed in the previous sub-section is also applicable in this setting characterized by aggregate individual "income" measures of the form $\mu_i^{1+\gamma}$ and the use of the two Theil measures. The respective mobility, structural mobility and exchange mobility measures are:

$$M_{\gamma}^{MZ} = 1 - \frac{I_{\gamma}^{GE}\left(V_N^{\mu,\gamma}(X)\right)}{\sum_{t=1}^{T} \delta_t \left[\frac{\mu_t}{\sum_{i=1}^{N} \overline{\mu}_i}\right]^{1+\gamma} I_{\gamma}^{GE}(c_{.t})}, \forall \gamma = -1, 0, \tag{6}$$

$$S_{\gamma}^{MZ} = 1 - \frac{I_{\gamma}^{GE}\left(V_N^{\mu,\gamma}(C)\right)}{\sum_{t=1}^{T} \delta_t \left[\frac{\mu_t}{\sum_{i=1}^{N} \overline{\mu}_i}\right]^{1+\gamma} I_{\gamma}^{GE}(c_{.t})}, \forall \gamma = -1, 0, \tag{7}$$

$$E_{\gamma}^{MZ} \equiv M_{\gamma}^{MZ} - S_{\gamma}^{MZ} = \frac{I_{\gamma}^{GE}\left(V_N^{\mu,\gamma}(C)\right) - I_{\gamma}^{GE}\left(V_N^{\mu,\gamma}(X)\right)}{\sum_{t=1}^{T} \delta_t \left[\frac{\mu_t}{\sum_{i=1}^{N} \overline{\mu}_i}\right]^{1+\gamma} I_{\gamma}^{GE}(c_{.t})}, \forall \gamma = -1, 0 \tag{8}$$

Note that M_{γ}^{MZ} is basically the mobility measure proposed by Maasoumi and Zandvakili (1986). Applying the results of Boland and Proschan (1988), only one of the two Theil measures, i.e. $I_0^{GE}\left(V_N^{\mu,\gamma}(X)\right)$ is found to be arrangement-increasing since it is easy to show its Schur-convexity. Therefore proposition 2.5(a) is applicable. This case, for $I_0^{GE}\left(V_N^{\mu,\gamma}(X)\right)$, is a specific example of the generalization of Shorrock's index proposed above.

On the other hand, $I_{-1}^{GE}\left(V_N^{\mu,\gamma}(X)\right) = \frac{1}{N}\sum_{i=1}^{N} \ln\left[\frac{\frac{1}{N}\sum_{i=1}^{N}\prod_{t=1}^{T} x_{it}^{\delta_t}}{\prod_{t=1}^{T} x_{it}^{\delta_t}}\right]$. By showing that $\ln\left[\frac{\frac{1}{N}\sum_{i=1}^{N}\prod_{t=1}^{T} x_{it}^{\delta_t}}{\prod_{t=1}^{T} x_{it}^{\delta_t}}\right]$ is *not* L-superadditive, proposition 2.5(b) of Boland and Proschan (1988) can be used to show that $I_{-1}^{GE}\left(V_N^{\mu,\gamma}(X)\right)$ is not arrangement increasing (Boland and Proschan, 1988, p. 289). It turns out that $I_{-1}^{GE}\left(V_N^{\mu,\gamma}(X)\right)$ is actually insensitive to association-increasing switches. Hence $I_{\gamma}^{GE}\left(V_N^{\mu,\gamma}(C)\right) \geq I_{\gamma}^{GE}\left(V_N^{\mu,\gamma}(X)\right)$ for $\gamma = 0$, but not for $\gamma = -1$.

[7] Maasoumi (1986) also derived a useful decomposition of $I_{\gamma}^{GE}\left(V_N^{\mu,\gamma}(X)\right)$, for the case when $\gamma \neq -1, 0$, into two components, one which depends on the $I_{\gamma}^{GE}(c_{.t})$ from each period t. However, by looking at his proposition 1, it is not clear a priori that $I_{\gamma}^{GE}\left(V_N^{\mu,\gamma}(X)\right)$ is always lower than, or equal to, a weighted sum of $I_{\gamma}^{GE}(c_{.t})$ when $\gamma \neq -1, 0$.

Therefore, for $\gamma = 0$ the following results hold: $E_\gamma^{MZ} \geq 0$, and, just as before, $E_\gamma^{MZ} = 0$ if and only if there are no re-rankings; whereas $E_\gamma^{MZ} = 1$ if there is complete mobility and it is only attributable to the exchange component (in which case, also $S_\gamma^{MZ} = 0$). Finally, just as in the previous section, $S_\gamma^{MZ} < 1$ because $I_\gamma^{GE}\left(V_N^{\mu,\gamma}(C)\right) = 0 \rightarrow I_\gamma^{GE}(c_{.t}) = 0 \forall t$, which renders S_γ^{MZ} indeterminate.

Comparison with other approaches: The mobility decomposition of Tsui (2009)

In a very interesting contribution, Tsui (2009) proposes the first use of counterfactual distributions for the decomposition of mobility over several periods. However the conceptual approach taken is different from the older tradition developed by Shorrocks (1978) and Maasoumi and Zandvakili (1986). Instead of relying on a comparison of aggregate "incomes" across individuals, Tsui (2009) departs from an inequality index that is axiomatically characterized, and decomposable into two components, the following way:

$$\begin{aligned} \overline{I}(X) &= \sum_{t=1}^{T} J_t(x_{.t}) + \overline{E}(X) \\ &= \sum_{t=1}^{T} \frac{\rho}{N} \sum_{i=1}^{N} \left[\left(\frac{x_{it}}{\mu_t} \right)^{c_t} - 1 \right] + \frac{\rho}{N} \sum_{i=1}^{N} \prod_{t=1}^{T} \left[\left(\frac{x_{it}}{\mu_t} \right)^{c_t} - 1 \right] \end{aligned} \tag{9}$$

where $J_t(x_{.t}) \equiv \frac{\rho}{N} \sum_{i=1}^{N} \left[\left(\frac{x_{it}}{\mu_t} \right)^{c_t} - 1 \right]$ and $\overline{E}(X) \equiv \frac{\rho}{N} \sum_{i=1}^{N} \prod_{t=1}^{T} \left[\left(\frac{x_{it}}{\mu_t} \right)^{c_t} - 1 \right]$. ρ and c_t are scalars chosen so that the desirable axioms are fulfilled. As Tsui (2009) explains, the first component, $\sum_{t=1}^{T} J_t(x_{.t})$, is insensitive to association-increasing switches. Hence it cannot capture exchange mobility, unlike the second component, $\overline{E}(X)$. Then, relying on the counterfactual distribution idea, and on $\overline{E}(X)$, Tsui (2009) proposes the following multi-period mobility index:

$$\overline{M}(X) = \frac{\overline{E}(C) - \overline{E}(X)}{\overline{E}(C)} \tag{10}$$

The index satisfies several desirable properties (see Tsui, 2009), but here it is worth highlighting that the situations of complete mobility and complete immobility are different, compared to those of Shorrocks (1978), Maasoumi and Zandvakili (1986) , and this paper's decomposition. For instance, $\overline{M}(X) = 0 \leftrightarrow \overline{E}(C) = \overline{E}(X)$. In other words, complete immobility is declared under the absence of re-rankings, i.e. exchange mobility. By contrast, the mobility indices in the previous section allow for structural mobility (i.e. any other changes in the marginal distributions, except re-rankings) to affect total mobility. On the other extreme, $\overline{M}(X) = 1 \leftrightarrow \overline{E}(X) = 0$. But $\overline{E}(X) = 0$ can occur under some sufficient conditions that would not guarantee complete mobility according to the indices of the previous section. For instance, for $\overline{E}(X) = 0$ the following condition is sufficient: $\forall i : \exists t \mid x_{it} = \mu_t$. That is, if every individual has his/her "income" equal to the population mean in at least one time period (whichever), then $\overline{M}(X)$ declares complete mobility. One can easily show with examples that this is not the case with the indices of the previous section.

Empirical application

The empirical application studies the mobility of per-capita household consumption with data from Peru. Shorrocks (1978) made two important points about the relationship between static and dynamic assessments of income inequality. Firstly, he suggested that static measures of inequality may not "reflect the differences between individuals" (p. 1) properly, since these may vary over time, thereby justifying the use of measures that incorporate notions of mobility. Secondly, by explaining that inequality measures depend on the accounting period considered for the construction of the income statistics, Shorrocks posited that in longer accounting periods inequality should decrease, as people at the poorest and richest social echelons rarely remain there permanently. Would these behaviours also hold for household consumption? What is the extent of household consumption mobility that is due to exchanges? How much of consumption mobility is due, instead, to the structural component? In this section, answers to these empirical questions are explored using data from Peru. The mobility indices are computed using the square coefficient of variation, denoted as GE1 below, since it is a member of the Generalized Entropy class of inequality measures for which $\gamma = 1$.

Data

The data for Peru comes from two panel-datasets generated and assembled by the Peruvian Institute of Statistics and Information Technology (INEI). The first dataset covers the 4-year period 1998-2001. This period was characterized by economic recession. GDP growth rates for the period were, from 1998 until 2001, respectively: -0.5, 0.9, 3.1, 0.6; whereas private consumption from national accounts experienced the following growth rates: -0.8, -0.4, 3.9, 1.8.[8] The second dataset covers the 5-year period 2002-2006. Unlike the previous period, this one witnessed the beginning of a commodity boom and ensuing economic bonanza. GDP growth rates for the period were, from 2002 until 2006, respectively: 4.9, 4.0, 5.2, 6.4, 8.0; whereas private consumption from national accounts grew at the following rates: 4.6, 3.1, 3.5, 4.4, 6.5.[9] Both datasets provide, then, interesting contrasting scenarios for the study of short-term consumption mobility.

Sample sizes for the 1998-2001 and 2002-2006 panel datasets are, respectively: 1188 and 2096 households. In both cases household consumption is measured monthly and per capita. For the 1998-2001 panel dataset a regional price deflator for 2001 is available. Hence using that deflator and the Peruvian consumer price indices, all the consumption values are expressed in prices of Lima in 2001. A similar regional deflator for 2002-2006 is unavailable. However regional poverty lines are available. Hence these were used as price deflators for the 2002-2006 period.

Results

Table 1 shows the computation of Shorrocks' mobility index, i.e. (1), using GE1, for Peru 1998-2001. Mean per-capita consumption in the panel sample decreased during that

[8] The temporary spike in 2000 can be attributed to the presidential election in which Fujimori sought re-election for a third term.

[9] These data come from several volumes of the Peruvian Central Bank Annual Reports, which can be found online, in English, at: http://www.bcrp.gob.pe/publications/annual-report/.

period, along with yearly measures of inequality (although inequality experienced a spike in 1999). Just like Shorrocks conjectured for income, consumption mobility in the first Peruvian panel dataset also increases along with the number of periods considered. With 2 periods (1998 and 1999) the value is 0.570, with 3 (1989-2000) it is 0.735, and finally, with 4 (1989-2001) it raises to 0.806. Underlying the trend is both an increase in the weighted sum of inequalities (denominator of the subtracted term in M) and a decrease in GE1 for the household consumption means (numerator of the subtracted term in M).

Table 1: Mobility of per-capita consumption: Peru, 1998-2001

Year	1998	1999	2000	2001
Period	1	2	3	4
Mean	299.600	301.951	277.099	272.912
GE1	0.379	0.477	0.362	0.354
Weighted sum of inequalities	—	0.856	1.222	1.579
GE1 for individual means	—	0.368	0.324	0.306
Shorrock's index (M)	—	0.570	0.735	0.806

Table 2 shows the decomposition results for M and the first peruvian dataset. Two important features come up. Firstly, for every period length, exchange mobility is a relatively minor contributor to total mobility (11.6%, 8.8% and 6.6% for 2, 3, and 4 periods, respectively). Most of the observed multi-period mobility is due to changes in the shape of the marginal distributions, excluding transformations based on multiplying every consumption value by the same scalar. Secondly, as the period length increases, the contribution of exchange mobility decreases; even though one would expect that a longer period would allow for more potential re-rankings among households. However, as shown in the table, structural mobility increases faster than total mobility as the accounting period is stretched. Hence the (residual) contribution of exchange mobility decreases.

Table 2: Mobility decomposition of per-capita consumption: Peru, 1998-2001

Year	1998	1999	2000	2001
Period	1	2	3	4
Mobility (M)	—	0.570	0.735	0.806
Structural mobility (counterfactual) (S)	—	0.504	0.670	0.753
Exchange mobility (residual) (E)	—	0.066	0.065	0.053

Table 3 shows the computation of Shorrocks' mobility index, with the same settings as before, but for Peru 2002-2006. Mean per-capita consumption in the panel sample increased during that period, while yearly measures of inequality exhibit an erratic pattern, although they are lower at the end of the period. Again, just like in the first panel, consumption mobility in the second Peruvian panel dataset also increases along with the number of periods considered. Remarkably, the mobility values of the two datasets, for comparable periods, are very similar; even though, mobility values are slightly higher in the second panel dataset. With 2 periods (2002 and 2003) the value is 0.576 (compared to 0.570), with 3 (2002-2004) it is 0.746 (compared to 0.735), and with 4 (2002-2005) it is 0.815 (compared to 0.806). Finally, with 5 periods mobility reaches 0.854. Like before, underlying the trend is both an increase in the weighted sum of inequalities (denominator of the subtracted term in M) and a decrease in GE1 for the household consumption means (numerator of the subtracted term in M).

Table 3: Mobility of per-capita consumption: Peru, 2002-2006

Year	2002	2003	2004	2005	2006
Period	1	2	3	4	5
Mean	204.588	279.339	284.162	288.977	314.590
GE1	0.498	0.446	0.328	0.366	0.356
Weighted sum of inequalities	—	0.937	1.250	1.611	1.960
GE1 for individual means	—	0.397	0.318	0.298	0.287
Shorrock's index (M)	—	0.576	0.746	0.815	0.854

Table 4 shows the decomposition results for M and the second peruvian dataset. The two features of the previous decomposition also turn up: exchange mobility is a relativey minor contributor to total mobility (12.8%, 9.5%, 7.3% and 5.9% for 2, 3, 4, and 5 periods, respectively); and, as the period length increases, the contribution of exchange mobility decreases. Interestingly, for comparable periods, exchange mobility in the second dataset is larger than in the first dataset. In other words, in both absolute and relative terms, re-rankings among households were a more important source of mobility during the beginning of the economic boom period than during the last major recession, in Peru.

Table 4: Mobility decomposition of per-capita consumption: Peru, 2002-2006

Year	2002	2003	2004	2005	2006
Period	1	2	3	4	5
Mobility (M)	—	0.576	0.746	0.815	0.854
Structural mobility (counterfactual) (S)	—	0.502	0.674	0.755	0.804
Exchange mobility (residual) (E)	—	0.074	0.071	0.060	0.050

Concluding remarks

This paper proposed a most natural way of decomposing multiple-period mobility indices based on Shorrocks' mobility concept, into exchange and structural components, using the counterfactual distribution ideas proposed by van Kerm and Ruiz-Castillo (in the context of two-period analysis). Tsui's approach has been the first to apply the counterfactual-distribution ideas to a multiple-period setting, but using mobility concepts and indices that differ in their appraisals of benchmark situations of complete mobility and immobility, from those originally put forward by Shorrocks, and Maasoumi and Zandvakili. In this context, this paper provides the first decomposition of multiple-period indices based on the latter paradigm. The discussion of differences between the two multiple-period measurements found that: 1) For Tsui's mobility index the absence of re-rankings is a necessary and sufficient condition to declare complete immobility, whereas the Shorrocks indices still allow for structural changes to generate mobility even in the absence of such re-rankings, i.e. the latter are not sufficient for the identification of complete immobility; 2) the situation of complete mobility in each approach is attained under very different conditions. Whereas the Shorrrocks approach requires the complete equalization of aggregate "incomes" for complete mobility to hold; in the Tsui approach it suffices that, for every individual, "income" is equal to the time-specific average at least once. Neither condition secures complete mobility in the alternative mobility approach, i.e. "outside their own approach". The paper also attempted to extend the decomposition to more general aggregations of "incomes", in the spirit of Maasoumi and Zandvakili. It was shown that any general, linear weighted sum of time-specific incomes is fit for the decomposition exercise. This confirms that time-discount factors can be considered explicitly in the multiple-period mobility analysis. However, the use of alternative aggregation methods, particularly among the family of generalized means, proved difficult. Conditions (4) and (5) cannot be derived for inequality measures other than the two Theil measures. Then, while one of the Theil measures is amenable for the decomposition; the other one, i.e. the mean log deviation, is not, due to its insensitivity to re-rankings.

The empirical application to the analysis of consumption mobility using two panel datasets from Peru reveals three interesting features: 1) a behaviour of per-capita household consumption similar to that of income, whereby lengthening the time period leads to higher mobility; 2) a relatively low contribution of exchange mobility to overall consumption mobility; and 3) a reduction in the latter contribution as the time period is lengthened. Further research should test whether these patterns are robust across different datasets and choices of inequality indices. If confirmed, these trends may warrant further theoretical and empirical inquiry into their causes.

The main contribution of this paper is a decomposition of mobility indices based on Shorrock's approach, into structural and exchange components. A question for future research is whether alternative ways of measuring exchange and structural mobility *over several periods* are worth considering; that is, above and beyond the decomposition approaches put forward by Tsui and this paper. In this respect a key observation is that all current decomposition approaches isolate a structural component that is insensitive to re-rankings (as it should be), but their exchange component is sensitive to both re-rankings (as desired) and to structural changes. One possible alternative is not a decomposition of an index into these two components, but rather measuring exchange mobility over several periods using indices of exchange mobility that are only sensitive to re-rankings while being insensitive to any other distributional change. For this *pure* exchange component, either intragroup rank concordance indices or copula-based indices are good candidates. Likewise, in this possible alternative, structural mobility could be measured with indices that are insensitive to re-rankings while being sensitive to other distributional changes. For the structural mobility indices, the rank-insensitive indices proposed in the first part of this paper, or a normalized version of $\sum_{t=1}^{T} J_t\left(x_{.t}\right)$ in Tsui (2009, equation 5), are suitable examples. Future research could ponder the pros and cons of using these pairs of indices separately, as in a *dashboard* approach, or together, as part of a composite index.

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