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Fuzzy Chronic Poverty:
A Proposed Response to Measurement Error for Intertemporal Poverty Measurement

Catherine Porter and Gaston Yalonetzky

For additional information please contact:

Name: Catherine Porter
Affiliation: University of Oxford

Email Address: catherine.porter@economics.ox.ac.uk

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Catherine Porter
University of Oxford

Gaston Yalonetzky
University of Leeds, Oxford Poverty and Human Development Initiative

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Abstract

A number of chronic poverty measures are now being used by policymakers to quantify the prevalence and intensity of chronic poverty, vis-a-vis transient experiences. For this exercise, data on multiple time periods is used to assess the welfare trajectories of individuals, or households, over time, in order to distinguish the intertemporally poor from the non-poor, or the transient poor. The extent of intertemporal poverty can also be compared and profiled in the same way as static poverty. Our research examines the implications of measurement error for discontinuous chronic poverty measures, and proposes an alternative approach to measure chronic poverty that seeks to minimize the consequences of measurement error. The approach is based on a novel criterion for the identification of poverty that draws from fuzzy set theory.

1 Introduction

Measuring poverty over time is a subject that has grown in interest over the past 10 years in particular, not least due to the increasing availability of panel datasets over time. Several intertemporal poverty measures have now been proposed and are in use in empirical applications around the world. No one measure has yet become the standard, however, several extensions of the Foster-Greer-Thorbecke (1984) static model are currently proposed. Porter and Quinn (2012) review the intertemporal poverty measurement literature, and show that some of the well-established properties of static poverty measurement are not easy to extend to the intertemporal context. We do not review all of them here, but we note that several options are available to the poverty analyst. Amongst others, measures have been proposed by Jalan and Ravallion (2000); Porter and Quinn (2008); Hoy and Zheng (2007); Calvo and Dercon (2009); Foster (2009); Bossert et al. (2012); Gradin et al. (2011); Foster and Santos (2012).

In particular, there has been a particular policy interest in trying to capture duration of poverty. In parallel, chronicity is a concept which many authors in the intertemporal poverty literature have attempted to incorporate. This is an appealing concept in the intertemporal context - all other things equal, the length of time spent in poverty may have a more than one-for-one impact on the underlying wellbeing of a person.[1] There is also a direct analogy to the unemployment literature, that shows, in particular, that spending longer time in unemployment (reference) may also decrease the chances of exiting unemployment. The same may be said of poverty.

Although they are not incompatible properties in theory, nobody in the literature has yet managed to design a measure that incorporates an appropriate concept of DURATION SENSITIVITY as well as being continuous. Continuity is an important and desirable property of any poverty measure, given that any discontinuity would render the measure excessively sensitive to small changes in the wellbeing measure being used. Of particular concern, the measure would also be sensitive to measurement error generating spurious fluctuations around poverty lines; in turn leading to misclassifications of people as either non (chronic poor) or poor. In the static and multidimensional context, similar concerns have motivated the incorporation of insights from fuzzy set theory (see e.g. (Lemmi and Betti 2006), in order to better identify the poor, and to avoid the problem of setting a poverty line that then classifies people as poor or non-poor, with nothing in between. Our focus in this paper is somewhat pragmatic, building on these insights, in order to create a “thick” poverty line enabling us to mitigate the potentially excessive sensitivity of intertemporal poverty measures, that are discontinuous, to spurious transitions across the poverty line. Specifically we propose a generalization of two popular intertemporal poverty measures: the measure of Foster (2009) and the more recent measure of Gradin

[1] See for example several qualitative research papers and summaries from the Chronic Poverty Research Centre, www.chronicpoverty.org.

et al. (2012). The two new proposals are characterized by a lower sensitivity to transitions around the poverty lines. On accordance to fuzzy set theory applied to poverty measurement, our measures allow *some* people to have a fuzzy poverty status, somewhere between being poor and non-poor. We explore these measures' empirical implications with the Ethiopian Panel Household Survey.

The rest of the paper proceeds as follows. Firstly, we briefly introduce a few ideas about intertemporal poverty measurement, followed by a basic notion of poverty identification with fuzzy sets. Then we dedicate two sections, respectively, for the new proposals generalizing the measures of Foster (2009) and Gradin et al. (2012). The empirical illustration follows; and, finally, the paper ends with some concluding remarks.

2 Intertemporal Poverty Measurement

Consider a matrix X, whose N rows have information on the wellbeing attainments of N individuals across a time span. Each column, therefore, hosts the distribution of the attainment across the population in a specific time period. The number of columns/periods is T.

$$X = \begin{pmatrix} x_{11} & x_{12} & \cdots & x_{1T} \\ x_{21} & x_{22} & \cdots & x_{2T} \\ \vdots & \vdots & \ddots & \vdots \\ x_{n1} & x_{n2} & \cdots & x_{nT} \end{pmatrix} \tag{1}$$

A typical attainment element of the matrix is: $x_{nt}\,(\in \mathbb{R}_+)$, that is, the attainment of individual n in period t. The poverty lines, specific to each period, are denoted by z_t;[2] and a person is deemed poor in period t if: $x_{nt} < z_t$. When conceptualising poverty over time, it is useful to think about the *sequence* of wellbeings experienced by an individual n, that is, the nth row of the data matrix $\mathbf{x}_n = (x_{n1}, x_{n2}, \ldots, x_{nT})$. This can be understood as n's **trajectory** of wellbeings (Porter and Quinn, 2008).

Some earlier measures developed in the literature (Jalan and Ravallion, 1998 which is an extension of Rogers and Rogers 1993) are also extensions of the FGT (1984) mea-

[2] From a vector of poverty lines, $Z : (z_1, \ldots, z_t, \ldots, z_T)$.

sures, based on an averaging of the income stream over time. The methodology is fairly simple, and intuitively appealing - if on average over the time period under consideration, a person's income lies below the poverty line, then they can be considered as chronic poor. Their poverty gap (and further poverty measures) can be calculated also using this average, and the FGT formula above can be applied to mean consumption or income. The methodology has been criticised partly because it allows a period of high income to compensate for a period in severe poverty. This led several authors (Calvo and Dercon, 2009; Foster 2009) to propose an extension of the static concept of FOCUS (Foster and Shorrocks, 1991) to the intertemporal context. They propose that the principle of STRONG FOCUS should apply to any chronic poverty measure; that is, the poverty measure should not be sensitive to changes in wellbeing, *in any time period when wellbeing is above the poverty line.*

The concept of STRONG FOCUS is not enough to distinguish between the group of static poor and intertemporal poor, which led Foster (2009) to introduce the concept of DURATION SENSITIVITY, which is at the heart of the identification strategy in his measure: only those people who are poor for at least a certain proportion of time qualify as chronic poor. Porter and Quinn (2008) show that this property is incompatible with another property, that penalises depth of poverty and allows a non-zero elasticity of substitution of wellbeing between periods (INTERTEMPORAL TRANSFER). Which of these properties one wishes to incorporate in the analysis is a normative choice, and depends on the policy context and the data under consideration.

Two methods of capturing chronicity have been proposed so far in the literature: the first relates to the total number of time periods spent in poverty, regardless of their order in time. This has been termed DURATION SENSITIVITY by Porter and Quinn (2012) and TIME MONOTONICITY by Foster (2009). The second is contiguity of poverty (introduced by Chakravarty et al 2011, whose measure is generalised by Gradin et al 2012). CONTIGUITY refers to the concept that *consecutive* spells of poverty without any recovery time in between may be more damaging to wellbeing than when there is some recovery time between. So, for example in a three-period panel, a sequence (poor, poor, non-poor) would be ranked as worse off than (poor, non-poor, poor). Both these are appealing normative properties.

However, another property that is extremely desirable is CONTINUITY. That is, an infinitesimal change in wellbeing in any period should lead to no more than an infinitesimal change in the value of the trajectory measure. (Porter and Quinn, 2012). A motivation for continuity is that marginal changes in wellbeing should have a marginal effect on the evaluation of poverty. If the trajectory ordering is not continuous then we may find trajectories which are ordered in a perverse way. For empirical applications this is also extremely important: a discontinuous measure would be excessively sensitive to measurement error, at any point of discontinuity.

3 Duration Sensitive Poverty Measures

We first tackle one of the most popular measures, that proposed by Foster (2009), which has increasingly been adopted in policy applications (e.g. Perez-Mayo (2009), Nunez Velasquez (2009)).

Foster proposes a property of **Time Monotonicity** "If x is obtained from y by a duration enhancing decrement to a chronically poor person, then $K(x; z, \ddot{I}, ,) > K(y; z, \ddot{I}, ,)$." (2007, pX)

During a period in which a chronically poor person happens to be having a spell outside of poverty, if the income level falls below the poverty line (thus raising the number of duration of poverty experienced by this person), then poverty should rise."

The measure includes a 'double cutoff': 1) A poverty line indicating material deprivation in one time period, and 2) a duration cutoff denoting the number of periods in poverty experienced by one individual, or household, that categorises them as "chronic poor". If a person is deprived for a period longer than the duration poverty line, then the person is considered chronically poor. This measure's focus axiom is insensitive to any deprivations from people who are not deemed chronically poor in the identification stage. Given this duration cutoff, measurement error has been shown to have quite serious consequences around the discontinuity (see Porter (2010)). We recall that Foster et al. (1984) introduced a class of measures, known as p-alpha or FGT measures,

$$P_\alpha(\mathbf{x}) = \frac{1}{N}\sum_{n=1}^{N}\left(1-\frac{x_n}{z}\right)^\alpha \mathbb{I}(x_n \leq z), \tag{2}$$

for $\alpha \geq 0$ which satisfy Focus, Anonymity, Weak Monotonicity, Weak Transfer, Subset Consistency and Population Size Neutrality. In addition they satisfy Strict Monotonicity and Continuity for $\alpha > 0$ and Strict Transfer for $\alpha > 1$.[3] They have become extremely well-known and widely applied measures.

As outlined above, then, any person n in the set is deemed poor in any period t if: $x_{nt} < z_t$. Foster's (2009) chronic poverty measure is based on a deprivation count which has a very simple and intuitive understanding. A person is chronic poor (as opposed to transient poor, or non poor) if they are poor for a minimum number of time periods relative to the period under consideration. A count of deprivation periods is computed weighting each deprivation period with weights, w_t, such that: $w_t \in \mathbb{R}_{++} \wedge \sum_t^T w_t = T$. [4] Hence the weighted number of deprivation periods suffered by individual n is: $c_n \equiv \sum_{t=1}^{T} w_t \mathbb{I}\left(z_t > x_{nt}\right)$.

Foster (2009) identifies the chronically poor as those individuals whose weighted count of deprivation is above (or equal to) a certain cutoff, $\tau \in [0, T]$. The poverty identification function is thus:

$$\varphi\left(c_n;\tau\right) \equiv \mathbb{I}\left(c_n \geq \tau\right) \tag{3}$$

Then, for an individual poverty function Foster proposes a weighted sum of the powered and censored normalized poverty gaps in every period, i.e. the FGT metric Foster et al. (1984):

$$p\left(x_{n.}; Z, W, \tau, \alpha\right) \equiv \varphi\left(c_n;\tau\right)\sum_{t=1}^{T} w_t\left(1-\frac{x_{nt}}{z_t}\right)\mathbb{I}(z_t > x_{nt}), \tag{4}$$

[3] The FGT measures are not the only measures to satisfy these properties; the measure suggested by Chakravarty (1983) also does, for example

[4] From a vector of weights, $W : (w_1, \ldots, w_t, \ldots, w_T)$.

where $x_{n.}$ is the row vector of individual n's attainments throughout time. Note that in this approach to chronic poverty measurement the sequence and timing of poverty spells does not impact the individual measure. Foster calls this property "time anonymity".

Finally, the social poverty measure, P, has a functional form satisfying desirable properties like individual anonymity, population replication invariance and additive decomposability:

$$P \equiv \frac{1}{N} \sum_{n=1}^{N} p\left(x_{n.}; Z, W, \tau, \alpha\right) \tag{5}$$

The second measure under consideration is the one by Gradin et al (2011), which is a generalisation of Bossert et al. (2012). This measure has a slightly different duration property, in that consecutive spells of poverty are weighted more heavily. Bossert et al. (2012) observe that: "[t]he negative effects of being in poverty are cumulative, hence a two-period poverty spell is much harder to handle than two one-period spells that are interrupted by one (or more) period(s) out of poverty." (p1).

Gradin et al. (2011) take a similar approach to that of Foster, in that the measure is an intertemporal sum of FGT per-period poverty measures. However, they do not incorporate the duration cutoff for identification. This means that anyone with any period of poverty at all is included in the group of intertemporally poor. [5]In order to penalise contiguous periods of poverty the authors introduce a weight multiplying the FGT normalized poverty gap (in 6). This weight,w_{it},depends on the length of a contiguous poverty spell, denoted by s_{it}. Thereby the same poverty shortfall gets weighted more heavily if it belongs in a longer experience of uninterrupted poverty.

$$p_{\mathrm{G}}(x_{n.}; Z, S, \alpha) = \frac{1}{T} \sum_{t=1}^{T} \left(1 - \frac{x_{nt}}{z}\right)^{\alpha} \mathbb{I}(z > x_{nt}) w_{nt} \tag{6}$$

where

$$w_{nt} = \left(\frac{s_{nt}}{T}\right)^{\beta}, \tag{7}$$

and S is the vector of poverty spells, s_{nt}. So for example, a single period in poverty

[5]In the literature this would be a union identification approach.

enters with a weighting of $(1/T)^{\beta}$; both periods in a two period spell would be weighted by $(2/T)^{\beta}$ as in formula 7 above. As noted by Porter and Quinn (2012) the Gradin *et. al.* measure satisfies WEAK IDENTIFICATION, GENERAL FOCUS, WEAK MONOTONICITY, STRONG FOCUS, RESTRICTED STRICT MONOTONICITY (if $\alpha > 0$) and CONTINGUITY but not STRICT MONOTONICITY, CONTINUITY, NON-DECREASING COMPENSATION or TIME SYMMETRY. Its discontinuities mean that it does not satisfy INTERTEMPORAL TRANSFER or DURATION SENSITIVITY although it does satisfy each of these for certain poor trajectories. Gradin *et. al.* note that Foster (2009) is a special case of their measure if $\beta = 0$ and $\tau = 1/T$. Finally, the social poverty measure, P, can be constructed by inserting (6) into the general form (5).

4 Introduction of poverty identification with fuzzy sets

In order to compensate for the potential effects of measurement error on duration-sensitive chronic poverty measures, we propose a generalization of the two measures outlined above, building on the fuzzy set literature. Fuzzy set theory has been used extensively in the social sciences for some time (e.g. see Ragin (2000), Smithson and Verkuilen (2006)). In the poverty literature, fuzzy set theory was introduced as an alternative identification criterion by researchers who were unhappy with the blunt dichotomy posed by traditional poverty lines for the identification of the poor. Instead they opted for the membership functions used in fuzzy set theory (see e.g. Lemmi and Betti (2006)). While we do not intend to contest the practice of setting a poverty line for identification purposes, we do worry about the consequences of using a traditional poverty line in chronic poverty assessments based on duration-sensitive measures, when transitions across the line may be taking place spuriously due to measurement error. Since traditional measurement error corrections are usually not readily available (for a comprehensive treatment ,see Bound et al. (2001)), we propose a fuzzy-style adjustment to the period-specific poverty lines, and then to the identification criteria of both the time-specific poor and the chronically poor. This adjustment smooths out the impact of (potentially spurious) transitions that take place across, and to close to, the poverty lines. Thereby we generalize several of the proposed duration-sensitive measures of chronic poverty.

An illustration of our proposed identification adjustment is in Figure 1 where a traditional poverty line, z, is compared against a "thick" poverty line bounded by z_1 and z_2 such that $z_1 < z < z_2$. In a traditional identification approach, a person is deemed poor if his/her income is below z, and non-poor otherwise. Under our approach poverty status ceases to be dichotomic if a person's income is in the interval $[z_1, z_2]$. Two important features of our identification approach stand out: 1) transitions across z, in its vicinity, do not generate abrupt changes in poverty status when the "thick" poverty line is used. For big changes in poverty status to happen, the magnitude of the transition has to be large enough to cross from z_1 to z_2 (or the other way around). In those cases we assume that the transition is less likely to be spurious (e.g. driven by measurement error). 2) Our fuzzy identification approach can be fine-tuned by either changing the values of $[z_1, z_2]$ or by changing the parameters that control the shape of the membership function (e.g. its convexity).

4.1 Poverty identification with fuzzy sets: the case of the measures by Foster (2009)

As it is clear from (1), a change in x_{nt} that changes the deprivation status in period t, i.e. a transit across z_t, increases, or reduces, c_n in the amount w_t. In turn such a perturbation may or may not change $\varphi(c_n; \tau)$ from 1 to 0 (or viceversa). As long as there is transit across z_t, a change in individual poverty status is possible, *irrespective of the magnitude of the change in* x_{nt} *that caused the transit.* However we do not want small, and potentially spurious, changes around z_t to have a significant effect on chronic poverty status. In order to reduce the likelihood of such occurrence, we propose an alternative poverty identification function, which is very similar to (1), with the exception that now deprivation in one particular period is determined by a fuzzy poverty line the following way:

$$\pi_{nt} = \left\{ \begin{array}{ccc} 1 & if & x_{nt} < z_{1t} \\ \left(\frac{z_{2t}-x_{nt}}{z_{2t}-z_{1t}}\right)^{\theta} & if & z_{1t} \leq x_{nt} \leq z_{2t} \\ 0 & if & x_{nt} > z_{2t} \end{array} \right\} \tag{8}$$

where $z_{1t} \leq z_t \leq z_{2t}$, i.e. there is now a "thick" poverty line, and $\theta \in \mathbb{R}_{++}$. Depending on the values that the parameter θ three deprivation identification functions emerge. These are illustrated in figure 2. We note here also, that if we are concerned with errors of exclusion, we may wish to set the lower bound of the thick poverty line at z, and an upper bound somewhere above it.

Figure 1: Fuzzy identification of the poor

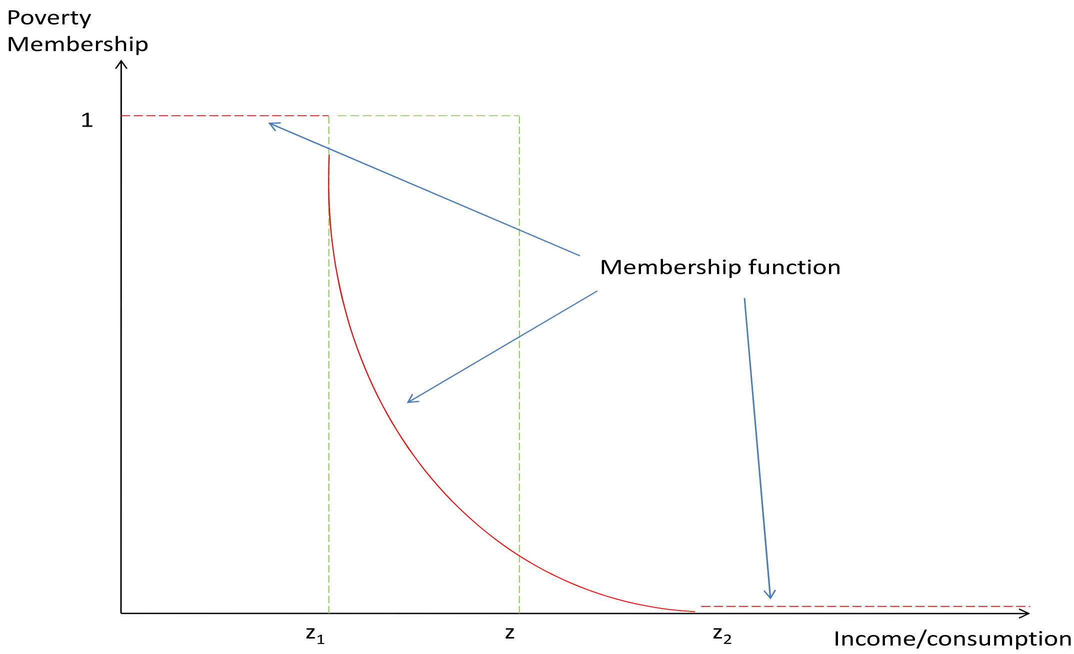
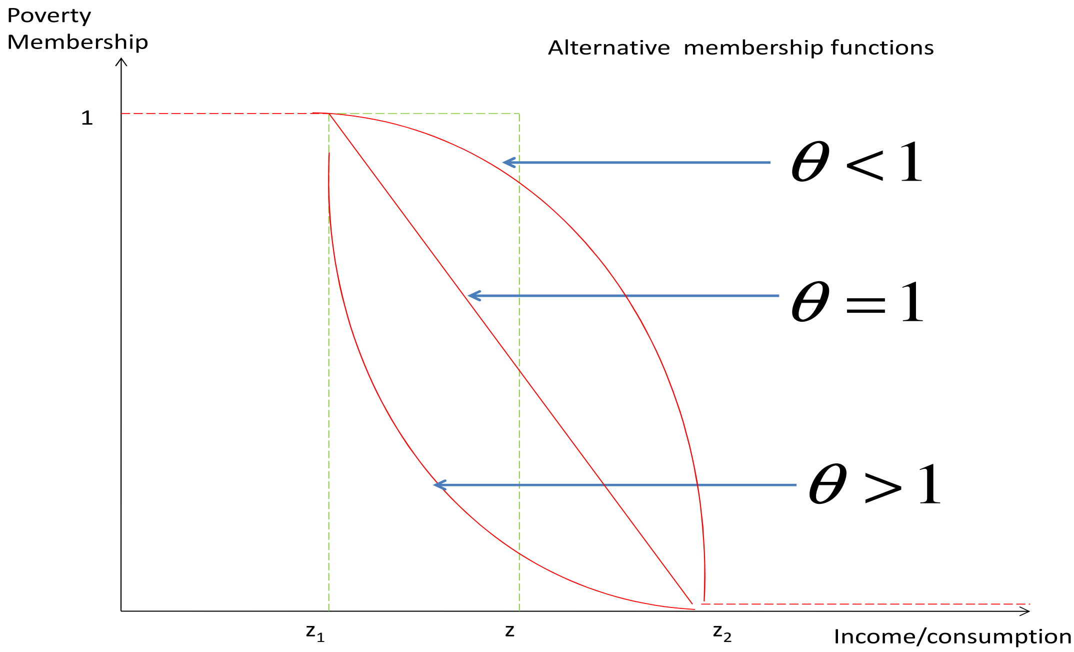


Figure 2: Fuzzy identification of deprivation status in period t



The next step in the proposal is to redefine the deprivation count: $c_n^{\pi} \equiv \sum_{t=1}^{T} w_t \pi_{nt}$. Then the new individual poverty function is:

$$p_{\pi}\left(x_{n.}; Z_{\pi}, W, \tau, \alpha\right) \equiv \varphi\left(c_n^{\pi}; \tau\right) \sum_{t=1}^{T} w_t \left[1 - \frac{x_{nt}}{z_t}\right]^{\alpha} \mathbb{I}(z_t > x_{nt}), \alpha \geq 0, \tag{9}$$

where $\varphi\left(c_n^{\pi}; \tau\right) = \mathbb{I}\left(c_n \geq \tau\right)$, and the vector Z_{π} is now made of trios of poverty lines, one per time period, $Z_{\pi} := \{z_{11}, z_1, z_{21}; \ldots; z_{1t}, z_t, z_{2t}; \ldots, z_{1T}, z_T, z_{2T}\}$. Finally, the new social poverty function is:

$$P_{\pi} \equiv \frac{1}{N} \sum_{n=1}^{N} p_{\pi}\left(x_{n.}; Z_{\pi}, W, \tau, \alpha\right) \tag{10}$$

Two interesting differences between the two families of measures are worth highlighting. Firstly, a transit across z_t is less likely to change c_n^{π} by a full amount of w_t. The change, Δc_n^{π} depends now on the magnitude of the change in x_{nt}, Δx_{nt} :

$$\Delta c_n^{\pi} = \left[\pi_{nt}\left(x_{nt} - \Delta x_{nt}\right) - \pi_{nt}\left(x_{nt}\right)\right] w_t \tag{11}$$

The lower sensitivity of c_n^{π} to the same change in x_{nt}, as reflected in (11), is the main feature rendering P_{π} better protected from drastic changes in deprivation status, and chronic poverty status, due to small and potentially spurious transits across z_t. However this new specification has other consequences. A second interesting difference is that the baseline number of chronically poor people according to P_{π} need not coincide with that according to P. For the case of deprived people in period t, the following condition, for continuous variables, establishes the circumstances under which P_{π} *overstates* the proportion of deprived people in period t:

$$\int_{z_{1t}}^{z_t} [1 - g(x)]\, dF(x) < \int_{z_t}^{z_{2t}} g(x)\, dF(x) \tag{12}$$

where $F(x)$ is the cumulative distribution function of x and $g(x)$ is the membership function with support in the range $[z_{1t}, z_{2t}]$. For instance, our proposed membership function is: $g(x) = \left(\frac{z_{2t}-x_{nt}}{z_{2t}-z_{1t}}\right)^{\theta}$. The left-hand side of (12) measures the loss in full deprivation status experienced by those who still have partial deprivation status, i.e. individuals for whom $z_{1t} \leq x_{nt} \leq z_t$. The right-hand side measures the acquired partial deprivation status among individuals who, otherwise, would not be considered deprived in period t, i.e. people for whom $z_t \leq x_{nt} \leq z_{2t}$. Whenever the latter is greater than the former, the social poverty headcount is greater according to P_{π}.

4.2 Poverty identification with fuzzy sets: the case of the measures by Gradin et al. (2012)

In the case of the measures by Gradin et al. (2012), the concern with a small perturbation generating a transit across z_t that changes the deprivation status in period t, is not that the chronic poor status may be affected, since in these measures a union approach for the identification is considered, i.e. $\tau = 0$. However, as is clear from 7, the small perturbations just described can generate significant changes in several of the spell variables, i.e. s_{nt}, which in turn affect the weights. This becomes apparent by examining the formula for

s_{nt}:

$$s_{nt} = [\sum_{i=t-m}^{t+n} \mathbb{I}(z_i > x_{ni})][\prod_{i=t-m}^{t+n} \mathbb{I}(z_i > x_{ni})]\mathbb{I}(z_i \leq x_{n,t-m-1})\mathbb{I}(z_i \leq x_{n,t+n+1}) \qquad (13)$$

As is clear in (13), changes in period poverty status, both within $t-m$ and $t+n$, as well as in the immediately adjacent periods ($t-m-1$, $t+n+1$, can generate discontinuous changes in s_{nt}. Our proposal attempts to reduce this sensitivity to small changes in x_{nt} generating transit across z_t, by introducing π_{nt}, from (8), into (13), and "thickening" the poverty lines. This yields the following spell value function:

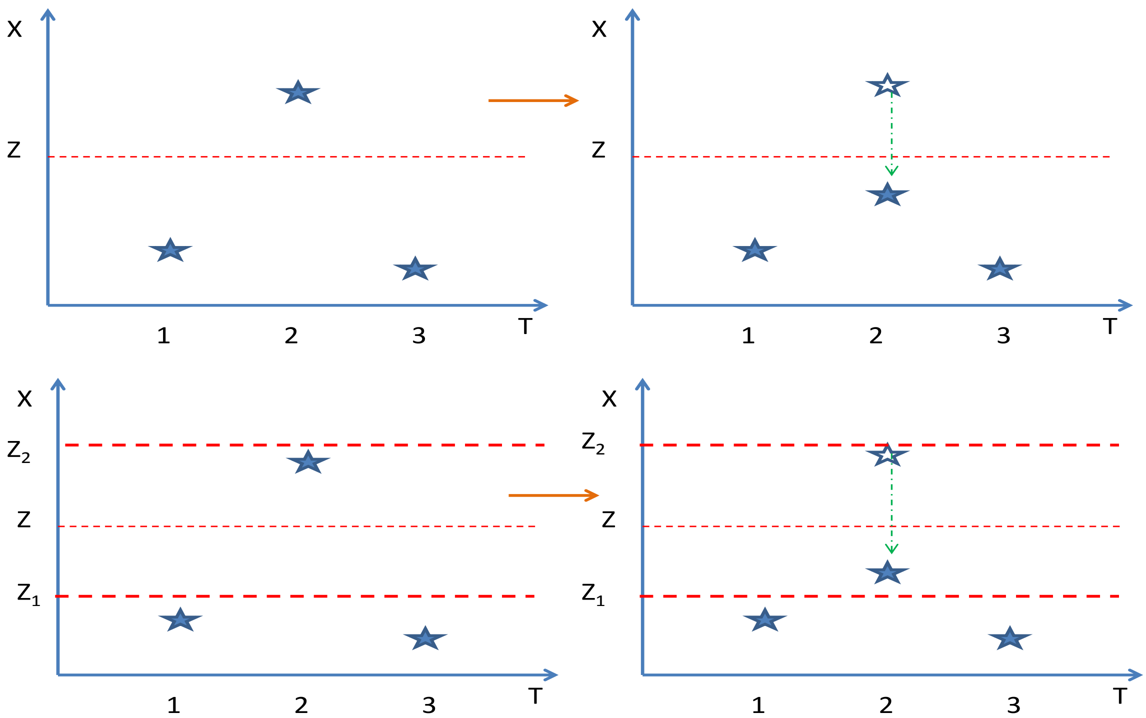
$$s_{nt}^f = [\sum_{i=t-m}^{t+n} \pi_{ni}][\prod_{i=t-m}^{t+n} \pi_{ni}]\mathbb{I}(z_{2i} \leq x_{n,t-m-1})\mathbb{I}(z_{2i} \leq x_{n,t+n+1}) \qquad (14)$$

An illustration

In this section we provide one illustration of the impact of "thickening" the poverty lines in the context of the Gradin et al. poverty measures. The four panels of figure (3) shows the income profiles of an individual over three periods. According to the top left panel, the individual is poor in periods 1 and 3 if poverty line z is used. In the top right panel, the individual's income in period 2 is lower enough to render him/her poor. Comparing the poverty spells of the two top panels it turns out that: $S^{tl} := (1,0,1)$, while $S^{tr} := (3,3,3)$ (where "tl" and "tr" denote, respectively, the top left and the top right panels). Let g_t^α be the (FGT) normalized poverty gap in period t. Then $p_{\mathrm{G}}^{tl} = (g_1^\alpha + g_3^\alpha)(\frac{1}{3})^\beta$, $p_{\mathrm{G}}^{tr} = (\sum_{t=1}^3 g_t^\alpha)(\frac{3}{3})^\beta$; and the difference between the two is:

$$\Delta p_{\mathrm{G}}^{top} \equiv p_{\mathrm{G}}^{tr} - p_{\mathrm{G}}^{tl} = 3^{-\beta}[g_2^\alpha(3)^\beta + (g_1^\alpha + g_3^\alpha)(3^\beta - 1^\beta)] \qquad (15)$$

Figure 3: Fuzzy poverty spells



By contrast, the two bottom panels perform the same comparison but using a "thick" poverty line, between z_1 and z_2, for period poverty identification, and z for the normalized poverty gaps. Using π_{nt} it turns out that: $s_t^{bl} = 2 + (\frac{z_2 - x_2}{z_2 - z_1})^\theta \; \forall t = 1,2,3$, while $s_t^{br} = 2 + (\frac{z_2 - x_2 + \epsilon}{z_2 - z_1})^\theta \; \forall t = 1,2,3$ (where "tl" and "br" denote, respectively, the bottom left and the top right panels, and ϵ represents the drop in income on the right-half panels). Then $p_{\mathrm{G}}^{bl} = 3^{-\beta} \sum_{t=1}^{3} g_t^\alpha (2 + [\frac{z_2 - x_2}{z_2 - z_1}]^\theta)^\beta$, $p_{\mathrm{G}}^{br} = 3^{-\beta} \sum_{t=1}^{3} g_t^\alpha (2 + [\frac{z_2 - x_2 + \epsilon}{z_2 - z_1}]^\theta)^\beta$; and the difference between the two is:

$$\Delta p_{\mathrm{G}}^{bot} \equiv p_{\mathrm{G}}^{br} - p_{\mathrm{G}}^{bl} = 3^{-\beta} \sum_{t=1}^{3} g_t^\alpha [(2 + [\frac{z_2 - x_2 + \epsilon}{z_2 - z_1}]^\theta)^\beta - (2 + [\frac{z_2 - x_2}{z_2 - z_1}]^\theta)^\beta] \tag{16}$$

Comparing (15) against (16) it is clear that the impact of ϵ, should be milder on $\Delta p_{\mathrm{G}}^{bot}$ than on $\Delta p_{\mathrm{G}}^{top}$ as long as: $\theta > 0$, $\beta \geq 1$, $x_2 - \epsilon > z_1$ and $z_2 > x_2$. For instance, when $\beta = \theta = 1$, as in (17):

$$\Delta p_{\mathrm{G}}^{bot}(\beta = \theta = 1) = \frac{\epsilon}{3(z_2 - z_1)} \sum_{t=1}^{3} g_t^\alpha < \frac{2}{3}(g_1^\alpha + g_3^\alpha) + g_2^\alpha = \Delta p_{\mathrm{G}}^{top}(\beta = \theta = 1) \tag{17}$$

Empirical application

We explore the empirical implications of this generalization using the Ethiopian Panel Household Survey (ERHS). The ERHS is a well known panel dataset from a developing country. The ERHS contains data on just over 1100 households in 15 villages, observed at six points in time over a ten year period, 1994 – 2004. The timing of the rounds is not even, with fieldwork in 1994, 1995, 1997, 1999 and 2004. We use information on household consumption, from detailed diaries that the households were asked to keep in the two weeks prior to the survey, including food that was home grown, bought at market, and received as a gift or benefit from government.

The poverty line is village-specific, and represents the amount needed to consume 2000 calories per day plus some basic non-food items (such as firewood to cook). It is thus an extremely austere poverty line, around one-third of the commonly used "dollar a day" line. In each round we also deflated consumption and the poverty line by a village specific food price index that was collected at the community level, and construct a measure of consumption per adult equivalent. For more details on this survey and the calculation of the by now quite widely used consumption basket, please see Dercon and Krishnan (1998). The poverty line is on average 43 Ethiopian Birr (1994 prices) per adult equivalent in the household.

Several authors have analysed wellbeing based on consumption measures in the ERHS, including most recently Dercon and Porter (2011) and Dercon et al. (2012). Cross sectionally, poverty has fallen in the study villages between 1994 and 2004, with the headcount reducing from just under 40% to just over 20%. Table 1 shows the tabulation of number of periods spent in poverty. Looking at households over time, there is a lot of movement in and out of poverty, and only around a fifth of households have never experienced any poverty at all. However, only 2.5% had consumption below the poverty line in every visit over the ten year period. Hence we are faced with exactly the kind of exercise that was outlined in theory above. Some households have longer periods in poverty, but do not fall very much below the poverty line, some have fewer episodes of poverty but those are very severe.

In terms of calculating the Foster (2009) measure, we can see immediately from the table that it depends on the value of choice parameter τ as to who is included in the measure. If $\tau = 0.6$, or 3 periods, then 22% of the sample will be classified as poor. If we decide to set $\tau = 0.8$ then only 10% will be poor. Recall that the other measures are calculated based on this identification step (poverty gap, squared poverty gap). We summarise the Foster measures for the case of $\tau = 3$ in table 2 below. As seen in the previous table, 22% of households are classified as poor, and the poverty gap is 3% and the squared poverty gap is 8%.

We now calculate the "fuzzy" Foster measure, by taking an upper and lower bound around the poverty line. The first is to set the upper bound (z_2) at 10% above the poverty line, and symmetrically with the lower bound (i.e. $z_1 = 0.9Z$). We note that this means that $p = 1$ only if consumption is below the lower bound poverty line z_1, and $p = 0$ for consumption above the upper bound z_2. Consumption values between z_1 and z_2 receive a value between zero and one. We also have to choose the shape of the fuzzy set. As with the setting of the line, this is an arbitrary choice, so we show several options. The simplest we apply firstly is to set $\theta = 1$ as shown above, this gives a straight line between z_1 and z_2. The headcount according to the Foster measure includes 255 households. When we create the fuzzy poverty line at 10% above and below, 213 households are now poor. We show in table 3 below the effect of increasing the bandwidth of the fuzzy line - at 20% bandwidth, 181 households (or 18% of households) are considered chronic poor, and at 30% bandwidth 167 households. The P1 and P2 poverty measures also decrease correspondingly.

We present in table 4 below the effect of reducing our theta parameter to 0.5. This means that the poverty values are higher than previously- for example at the poverty line, z, $p = 0.5$ when $\theta = 1$ but $p = 0.7$ when $\theta = 0.5$. The headcount measure in this case doesn't change too much - it is also lower than that of the original measure, at 18.4% when we consider a 10% bandwidth. For the other two poverty measures, again the measure is very slightly higher. As discussed in the theory section, the choice of the theta parameter is a normative decision as it determines the shape of the fuzzy set - in this particular dataset, we may tentatively conclude that the choice of theta is less important than the bandwidth or "thickness" of the fuzzy poverty line.

We also discussed above that this methodology would show that the Foster measure is likely to overestimate chronic poverty in the presence of measurement error, when there are transitions of a small amount above and below the poverty line that may be spurious due to measurement error. We may wish to make a slightly different normative choice, which is to set the thick poverty line at z as a minimum, and allow periods in which consumption is just above the poverty line to still be considered poor. The assumption here would be that we care more about measurement error that misclassifies a household just above the poverty line, rather than just below it as we wish to penalise errors of exclusion more heavily than those of inclusion. A summary table below (5) shows headcount measures for all of our different assumptions. The last two lines consider the poverty line z as the lower bound. This by design, would increase the poverty measures - the headcount from 21% to just under 24%. The change may seem minor, but in terms of targeting could be important.

Further empirical work in this section will include the calculation of the Gradin et al "fuzzy measures". *TO DO.*

5 Conclusions

This paper has presented an empirical adjustment for some well-known chronic or longitudinal poverty measures that have desirable normative properties, but may be excessively sensitive to measurement error, due to the discontinuity inherent in their calculation. The adjustment is fairly simple, drawing on fuzzy set theory, we construct a "thick" poverty line that enters into the identification step of the poverty measures. The empirical section presents some results for rural Ethiopia *[for Foster 2009 measure and for Gradin et al this is still work in progress]*, and shows that in this case the choice of theta (the shape of the fuzzy set) is less important than the bandwidth (or how thick the poverty line is). The fuzzy adjustment around the poverty line shows that the Foster (2009) measure may overestimate chronic poverty. However, adjusting the measure to penalise only errors of exclusion, increases the chronic poverty measure. The poverty analyst should make choices on these sensitivity tests based on the appropriate objectives of the measurement exercise, we hope that this adjustment adds another tool to the measures and sensitivity tests available.

6 Tables

Table 1: **Number of periods in poverty**

Item	Number	Per cent
Never Poor	424	36
Once	297	25
Twice	203	17
Three times	143	12
Four times	82	7
In every period	30	3
Total	1,179	100

Source: ERHS Data

Table 2: Foster (2009) p-alpha measures

Variable	**Mean**	**Std. Dev.**
Headcount	0.216	0.412
Poverty Gap	0.062	0.133
Squared Poverty Gap	0.034	0.081
N	1179	

Table 3: Fuzzy Foster measures (theta=1)

Variable	**Mean**	**Std. Dev.**
Fuzzy Foster P0, 10 percent	0.181	0.385
Fuzzy Foster P1, 10 percent	0.057	0.132
Fuzzy Foster P1, 10 percent	0.032	0.081
Fuzzy Foster P0, 20 percent	0.154	0.361
Fuzzy Foster P1, 20 percent	0.052	0.129
Fuzzy Foster P1, 20 percent	0.029	0.079
Fuzzy Foster P0, 30 percent	0.142	0.349
Fuzzy Foster P1, 30 percent	0.049	0.128
Fuzzy Foster P1, 30 percent	0.028	0.079
N	1179	

Table 4: Fuzzy Foster measures (theta=0.5)

Variable	Mean	Std. Dev.
Fuzzy Foster P0, 10 percent	0.185	0.388
Fuzzy Foster P1, 10 percent	0.058	0.132
Fuzzy Foster P2, 10 percent	0.032	0.081
Fuzzy Foster P0, 10 percent	0.168	0.374
Fuzzy Foster P1, 20 percent	0.054	0.13
Fuzzy Foster P2, 20 percent	0.03	0.079
Fuzzy Foster P0, 10 percent	0.165	0.371
Fuzzy Foster P1, 30 percent	0.052	0.129
Fuzzy Foster P2, 30 percent	0.029	0.079
N	1179	

Table 5: Proportion of poor, Foster and Fuzzy Identification

Variable	Mean	Std. Dev.
Foster	0.216	0.412
Fuzzy(10%, theta 1)	0.181	0.385
Fuzzy(20%, theta 1)	0.154	0.361
Fuzzy(30%, theta 1)	0.142	0.349
Fuzzy(10%, theta 0.5)	0.185	0.388
Fuzzy(20%, theta 0.5)	0.168	0.374
Fuzzy(23%, theta 0.5)	0.165	0.371
Fuzzy (z=min, plus20%, theta1)	0.229	0.42
Fuzzy (z=min, plus20%, theta0.5)	0.237	0.426
N	1179	

Notes: For example Fuzzy 10% means that the upper bound of the "thick" poverty line is at $1.1z$ and the lower bound at $0.9z$, similarly with 20 and 30% respectively. Theta is the parameter referred to in equation X above. The bottom two lines include the poverty line as the lower bound of the thick poverty line with the upper bound at $1.2z$.

7 Bibliography

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