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Chain Linked Quantity Indices When the Quantity has Been Zero

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Chain linked quantity indices when the quantity has been zero*

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Abstract

What meaning may be ascribed to chain linked quantity indices when the quantity has been zero? Is it possible to construct an extension which does not have the zero-quantity problem which the usual definition yields?

Underlying the problems for chain-linked quantity index variables seems to be the lack of price information when statistics in current year prices and previous year prices only are available.

The paper recommends that price indices need to be released (variable for variable) as well as the values in current year prices (and previous year prices) in order for users to be able to handle quantities passing a zero as a routine matter.

Examples of how to reconstruct these price indices are given. A description of how to reconstruct these price indices under more generalized circumstances is a topic of future research.

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1 Introduction

Over the last twenty years there has been a trend from fixed basket price indices to chain linked price and quantity indices in official statistics. Most notable was the change in recommended price index in the National Accounts manuals from SNA1968[12] to SNA1993[3].

The 1996 Boskin Report[1] is sometimes mentioned as the final drop which spawned change in the price index principle of major National Statistical Institutes the World over.[1]

In the recent practical implementations of this recommendation of statistical price index principle, an occational source of frustration has been time series which happen to become zero at least once over the period considered.

Basically the index formula "breaks down" since the chain linked price index is multiplicative and the quantity is used to weigh the corresponding component price index. The value of the price index for each period away from the base year, beyond the zero observed value, automatically is zero.

This note addresses this and associated problems from primarily a practical perspective with a few theoretical observations. It also ends up addressing the curious case of changes in inventory figures in current year prices and in real terms with opposite signs, which "seems logically inconsistent" to Diewert (2009) [2].[2]

1.1 Standard representation of the fundamental formulas:[3]

Assume you have access to figures for two valuations of every variable, Y :

1. valued at current prices, Y_t
2. valued at previous year prices, sY_t

There are two identities which always hold for linear combinations of variables for which we need chain linked quantity indices, viz. in current year prices and in previous year prices.[4]

From the values of these time series $\{(Y_t, sY_t) \mid t\epsilon T\}$ you derive the values in chain-linked prices, with base year at period zero, of the quantity index, fY_t, and the price index, pY_t, using the following recursive formulas:

1. $pY_t = pY_{t-1} \cdot (Y_t/sY_t)$, for $t > 0$,

[1] Cf. Triplett (2006)[11]. In Denmark the change was effective for the National Accounts in publications as of the revision in 2005, cf. DST2005[8]. Some years prior to the release in 2005 figures were made available to users of the publicly accessible databank: www.statistikbanken.dk.

[2] Please note that the problems dealt with in this paper, of a quantity passing a zero is also relevant for "real" quantities derived from a fixed basket price index too, when that quantity is zero during the base year.

[3] This is the representation which is extensively used in the work on the databank for the macroeconometric model ADAM, which is developed and maintained by the division Economic Models, Statistics Denmark.

[4] If Y is an aggregate of n components Y_i, for i=1,...,n, and $Y_t = \sum_{i=1}^{n} Y_{i,t}$, then $sY_t = \sum_{i=1}^{n} sY_{i,t}$.

2. $pY_t = pY_{t+1} \cdot (sY_{t+1}/Y_{t+1})$, for $t < 0$,
3. $fY_t = Y_t/pY_t$

given the initial conditions:

4. $pY_{t=0} = 1$
5. $fY_{t=0} = Y_{t=0}$

Defined in this way the quantity index is a chained Laspeyres quantity index, and the price index is a chained Paasche prices index, according to CPI2004, §§15.12 and 15.75-15.85.

1.2 Fundamental questions:

For producers of official figures within an institution which has chosen to publish its figures in current year prices and chain-linked prices sometimes we are faced with situations which require some nonstandard solutions. Here I state three different situations which are somewhat related and ordered according to complexity.

A. When no observation of the price of a product (micro-) transaction is available, which value of the corresponding product's price should be considered and perhaps chosen when computing the aggregate price index? Consider the cases of prices not collected, when transactions are in fact taking place.

B. When the quantity of a product (transacted in a given period) is zero, how is the (chain-linked) price and quantity index for this and the following periods constructed? Consider (the prices of) rarely traded products. E.g. hand-built instruments, investment projects of buildings or infrastructure.

C. When a variable is defined as the sum of positive and negative valued elements. Does this have implications for the formulae for (chain-linked) price index and (chain-linked) quantity index? Consider changes in inventory.[5]

2 A. Which value of the price variable when price observations are unavailable?

If price observations are missing what do we do ? In the literature a division is made between temporarily missing and permanently missing observations. The CPI2004[5] gives recommendations

[5] Diewert (2009)[2] names this case as a future challenge (chapter 16, section 4.6): "These inventory accounting problems seem to carry over to the national accounts in that for virtually all OECD countries, there are time periods where the real change in inventories has the opposite sign to the corresponding nominal change in inventories. This seems logically inconsistent. This is another area for future research."

on what to do in both cases.[6]

> CPI2004: §9.48 The treatment of temporarily missing prices. In the case of temporarily missing observations for non- seasonal items, one of four actions may be taken:
> – omit the item for which the price is missing so that a matched sample is maintained (like is compared with like) even though the sample is depleted;
> – carry forward the last observed price;
> – impute the missing price by the average price change for the prices that are available in the elementary aggregate;
> – impute the missing price by the price change for a particular comparable item from another similar outlet.

Broadly summarising either we may *delete*, *impute* or *substitute* our way out of this. Within the quality adjustment literature Triplett (2004)[10] takes on the issue with even more sophistication.

There seems to be an underlying issue on which we need to know our own position. How could you construct a price index when the products are not traded, commissioned nor built ? Apparently an exotic question which seems to require both a theoretical discussion and many practical examples. In practice the situation may not be quite as rare, cf. the example in appendix 1 from the Danish National Accounts' commodity flow system.

Regarding the theoretical discussion of the meaningfulness of deriving the index values in the case of a quantity passing a zero nothing more than making a presumption clear is made in this paper.

A theoretical clarification of the individual's presumption about the objects we are measuring seems necessary in order to determine how to approach the questions raised.

Briefly relevant presumptions may be classified into at least two main groups:

2.1 Presumption A

We only know the prices when they are observed. Unobserved prices have a default value, either "missing", or a fixed numerical value, e.g. Zero. This assumption leads to what may be called an "*atomistic* price theory", i.e. only the prices observed are deemed to "exist". This position seems associated with the group of national accounts intellectuals who insist on the national accounts should restrict itself to the empirical core, rather than including constructs such as FISIM, etc.

One way to represent this point of view is to consider the expected value of the price variable,

[6] See chapter 7, and §§9.47-9.63, pp.160-162.

given the observations {say $E[p(.,.,.)|observations]$}, is a *discontinuous function* dominated by a "missing" surface with specific observations as isolated points. Represented in this way *the average of the observations is not very close to the "common" surface*, which is dominated by the missing values. Even with a huge amount of observations available we don't allow ourselves to "guess" at the prices of potential transactions for which we have no observation, even if we have some observations resembling the transaction quite closely (according to some metric).

2.2 Presumption B

We allow extrapolation from actual observations of the values of prices, to the values of prices which we do not have observations of. This assumption leads to what may be called "price *field* theory", i.e. prices are apriori considered to "exist" for a continuum of possible trades, even if not observed. This position seems to be associated with the group of national accounts intellectuals who insist that important products should be reconstructed even by indirect methods, as long as the products are in principle observable. This group would usually applaud the inclusion of FISIM into the national accounts framework, despite increasing the volume of necessary imputations.

Looking at the same mathematical representation for this point of view, the expected value of the price variable, given the observations {$E[p(.,.,.)|observations]$}, is a *continuous smooth surface* with observations scattered around as isolated points. Here *the average of the observations is not very far from the surface (on average)* – as this is how we permit ourselves to construct the surface (and its location).

2.3 Which presumption ?

In this paper I shall make presumption B and demonstrate that solutions to the three questions raised above will be available.

For readers inclined to make presumption A at present I can offer no solution to these questions.

2.4 Price data should never only be implicitly available

We also need to assume that the price variable is available at every transformation of a set of valuations of a variable into another set of valuations of a variable. Because allowing quantity variables to be zero makes it impossible to reconstruct the price information from the set of only variables in current prices and previous year prices.

But the cost of retrieving the price information by direct observation may in certain cases be so prohibitively high that indirect methods have to be relied on. A back-up method is important to have at hand.

Of the *constructive indirect methods* to reconstruct the price of a product when no observation is available, the production side method used in *input-output practice* of dividing the market price of a product into separately estimated parts: i) intermediate consumption in market prices[7], ii) wages and salaries, iii) other production taxes and subsidies, and iv) operating surplus and mixed income, seems to be the method we may always rely on when other more direct methods are not available. Depending on the resources available for the reconstruction the results may come closer to the "true" price (of the product).[8]

3 B. Which quantity index values when the quantity passes a zero?

"When the quantity transacted is zero, how is the index for this and the following periods constructed?"

Accounting and economics provide two general principles: the shadow pricing principle, and the concept of opportunity cost.

Both may be used for estimating the figures of variables for accounting entries in a balance sheet. In economic history this is also a well-known problem with well-known first choices in practical situations.

In principle we may deal in different ways with a variable whether it measures a stock, a flow or a higher derivative (wrt. time) of a stock.

Each observation may be time specific (and have a geographic dimension and other dimensions) or may be (some kind of) average over one or more dimensions.

This paper is primarily concerned with time series variables. So one or more observations of the variable may be missing either in a regular or irregular pattern.

Let us focus on two of these classes of problems:

- (a) a few observations are missing
- (b) a few observations are available.

In case (a) the standard methods from CPI2004[5] could be considered, e.g. some kind of interpolation between closely surrounding observations seems appropriate. Perhaps interpolation of

[7] Perhaps also further broken down into intermediate consumption in basic prices, product taxes and subsidies, trade margins, and VAT, as in the Danish commodity flow system.

[8] This is a simple problem within the class of what may broadly be called *reconstructing counterfactual prices*.

percentage change of price level of last observation prior to the missing observation, to the level of the first observation after the missing observation.

In case (b), if sufficient background data are available on the relevant production function, e.g. from input-output tables in the national accounts (NA), the costs of producing the extra supply could be calculated from the prices of differing factors involved in the production necessary to provide the extra products, and add the going profit mark-up. This way you have a price reconstruction principle from which to derive the values for the period before, the period with and the period after the period with the missing price observation. Critically reviewed the result of this process continued iteratively may result in figures which have sufficient reliability.

Further we may rewrite the chain price index link,

$$pY_{t-1,t} = \sum s_{i,t} \cdot pY_{i,t}/pY_{i,t-1}, \tag{1}$$

where

$$s_{i,t} = fY_{i,t} \cdot pY_{i,t-1} / \left(\sum fY_{i,t} \cdot pY_{i,t-1} \right) \tag{2}$$

with sums over i=1,...,n. Cf. CPI2004[5], §15.24. These (hybrid) expenditure shares, $s_{i,t}$, are continually updated from the figures in previous year prices.

From the right hand side expression of (2) it is evident that if we use this equivalent form it is only when all components of an aggregate variable (in previous year prices) are simultaneously zero that we really have a problem. Previously the user could derive his own aggregates by just having access to the values of his aggregates' components in current year and previous year values. Taking into account the case of one or more zeros, *the user requires access to both the components' individual price indices and their corresponding expenditure shares as well.*

If we develop the form a bit further we see that

$$pY_{t-1,t} = 1 + \sum s_{i,t} \cdot (pY_{i,t} - pY_{i,t-1}) / pY_{i,t-1} \tag{3}$$

Each link in the chain price index is thus a weighted average of the relative price increases of the components. [9]

[9] If we did not observe the price of one component having a zero quantity at time t and would like to determine the price increase which would be consistent with the aggregate then it should be equal to the weighted average of the remaining ones with nonzero quantities at time t.

3.1 Practical examples:

The first example we look at is the simplest possible. The aggregate covers only one component. This simplifies but still brings out the main points in the case of an aggregate's components all passing a zero simultaneously.

The full tables are available in tables A.1-A.3 in this paper's appendix A.

From table A.1 we should notice that the factor of relative price index values from t-1 to t (column 5) become nonsensical the same year that the quantity measured in previous year prices is zero. Turning to the chain-linked price index in column 6 notice that this takes on similarly nonsensical values from the zero value year and all years away from this, as seen from the base year (2005). Thus when the zero value event occurs it has serious consequences.

Also notice that the price (and quantity) index where well-defined is actually exact.

Table A.2 explores the consequences of one simple remedy consisting in adding a small delta to both the value in current year prices and in previous year prices. First notice that the problem with nonsensical values completely disappears from the table. Second notice that for the factors in column 5 the consequence is that the implicit price increase is zero (for the years with the zero quantity), compared to the 5%, which is in the actual prices. The consequences of this "error" in reconstructing the value of the factor of relative price index growth are perpetuated al through the time series for the same years as we had nonsensical values in table A.1.

Table A.3 presents the results of an insistence on a reasonable reconstruction of the value of the price index using statistical methods when necessary. Notice that by guestimating the missing value of the price index we are able to reconstruct the price index and the quantity index variables rather precisely. This result of course depends on the regularity of the year-to-year price increases. The result also points to the importance of the user not have to reconstruct the associated price index values, but have the NSIs provide them.

Conclusion: *first* it is important for NSIs to provide the price index values directly to users, *second* when users define their own aggregates they need to reconstruct the price index variable *before* deriving the quantity variable.

4 C. Consequences when applied to a difference?

"When the quantity is a sum of positive and negative valued components. Does this have implications for the formulae for price index and quantity index ?"

Typical examples of this may be the change in (the stock of) inventories, and other kinds of net capital formation.

If components of an aggregate variable have values of different signs at time t the weights may become negative, which violates an intuitive requirement that the value of the price increase of the individual link of the (aggregate) chain price index is a convex combination of the price increases of the components with nonzero quantities. It is exactly in these cases (of the requirement being violated) in which a variable may have a value in current year prices of the opposite sign of its values in previous year prices, which Diewert(2009) mentions “seems logically inconsistent”.

When all quantities of components are zero then extending the period which the component (flow) variables “cover” may solve the problem. E.g. using *annual* quantity figures for weights in a *monthly* price index.

4.1 Example

In the remaining part of the paper we look at a situation with two products, *product 1* increasing 3 pct. p.a. in price, and *product 2* increasing 7 pct. p.a.. First we look at the case of both changing sign over the period. Later we look at the case of the two quantities consistently have opposite signs. In both cases in 2002 the quantities of both products are zero simultaneously, and in one year, 2004, the quantities have opposite signs and quantities which in current year prices cancel. In previous year prices they do not cancel. Year 2005 is the base year of all price indices in these examples. The first subcase is meant to simulate the change in inventory, and the second subcase, the net exports.

The relative annual price increase is usually calculated as $pX_t \cdot fX_t/(pX_{t-1} \cdot fX_t)$ i.e. the current year value divided by the value in previous year prices.

The proposed method of reconstructing the price index first is in the simple setup easy as the relative price increases of the two products are so uniform over time that 4.5 pct.p.a. is a natural choice. But of course this is an estimate subject to measurement error.

From table A.4 and A.7 the results of the standard method are given, in column 6, resulting in price index change which is inconsistent with being an average of 3% and 7% p.a.. Even turning negative in later years.

The values resulting from equation (1) and (2) leaves most cells left uncalculable for the chain price indexes in columns 8-10 which are the Paasches, Laspeyres respectively Fisher.

Clearly this situation is very unsatisfactory. The next section presents an alternative, which has merits.

4.2 Proposal

One way to ensure the nonnegativity of the individual weights, $s_{i,t} = fY_{i,t} \cdot pY_{i,t-1} / \left(\sum fY_{i,t} \cdot pY_{i,t-1}\right)$, would be to have all quantities, $fY_{i,t}$, enter only numerically. Then no seeming logical inconsistency would arise.

$$s^{\star}_{i,t} = |fY_{i,t}| \cdot pY_{i,t-1} / \left(\sum |fY_{i,t}| \cdot pY_{i,t-1}\right) \tag{4}$$

This would solve the cases when the aggregate's zero value is due to non-zero canceling components, but we need to explore the consequences of this change in definition of the weights in more detail, which is done in the next section of this paper.

By coinciding with the usual expressions for quantities which are all positive or all negative, this definition is a natural extension to situations where the quantities have different signs.

4.3 Consequences of proposal

The illustrative examples in table A.6 and A.9 seem encouraging. In the case of table A.9 covering the net exports subcase, the weights used when the quantities were simultaneously zero were taken fra the year before, both for the chain Paasches index and the chain Laspeyres index. This has been indicated by marking the figures by a star in column 8, year 2002 respectively column 9, year 2003.

Let us think a bit about what the requirements are about the data in order to be able to apply these new expressions more generally.

When the data are broken down over a product dimension[10] and a supply-use dimension this implies a number of restrictions over (at least) two dimensions. For simplicity of reference we call each intersection of all the dimensions a "cell", like in a matrix. In order to be able to respect these restrictions for each cell we require more data than just the values in current year prices and previous year prices. We need the values in previous year prices to have access to the relevant expenditure shares, but we also need the price index for each "cell" in order to be able to derive the price index for aggregates over these dimensions. We need the values in current year prices in addition to the derived price index in order to derive the value of the implicit quantity index. When we only have access to data at an intermediate level of disaggregation. Consistency of conceptual definitions of different aggregates seems to require that we have access to the data at the lowest level of disaggregation. This would clearly be impractical. One smaller sufficient set of data would

[10] And possibly also over other dimensions, like geographic space.

be to have the data broken down in cells at the intermediate level of aggregation, but keeping the expenditure data subdivided into the sum of positive components and the sum of negative components. I.e. for each cell it seems enough to require two subcells, and for each subcell, its value in current year prices, its value in previous year prices and its price index value. This would ensure that from this database we can derive *consistent* figures for all aggregates at intermediate or higher levels of aggregations.

The logic of this line of thinking makes it simple to determine the solution of a number of challenges which have been ad hoc previously, like net exports. For net exports we keep imports and exports separately and calculate the price index of net exports as the weighted average of the price index of imports (exports) using the share of imports (exports) from imports *plus* eksports as its weight. Similarly for changes in inventories. For each cell we keep the additions to the inventory separate from the subtractions from inventories, in current year prices, in previous year prices and their price index values.

4.4 Conclusion on proposal

The attempt to extend the scope of price indexes from dealing only with sets of products with non-negative quantities to dealing with set of products with non-negative as well as non-positive quantities has achieved the objective of yielding indexes which have characteristics of price indexes, and make it simple to handle some of the cases which have been difficult with previous price index theory and in practice only solved by *ad hoc* suggestions.

From the simple analysis in subsection 4.3 the proposed use of *numerical* weights, rather than weights from signed quantities, for our expenditure shares requires us to keep our basic data in a particular form which is more complicated than previously necessary. A matrix has been defined which ensures that aggregations on the basis of a set of maximally disaggregate data can be made in one or more steps and yet reach the same aggregate values.

5 Conclusions

In this paper we have studied some problems of how to handle statistics involving a variable passing a zero when we need to decompose it into the corresponding chain-linked quantity and price index variables.

The main conclusion is that in all cases we need to pay attention to the values of the chain price index to make sure that its values are sensible *before* we use it to deflate the values in current prices.

Two default methods are recommended to reconstruct the price index variable for a chain-linked variable: either relying on input-output matrix information to compute indicator time series, or simpler imputational methods from CPI2004[5].

For the cases of a chained Paasche price index of an aggregate variable which is a sum of component variables with values of different signs a proposal is made to take the numerical value of the quantity variables when the weights are computed. A similar form has been used for the chain Laspeyres price index and chain Fisher price index. The properties of indices with such weights need to be explored in much greater detail than was possible for this conference.

Taken as a whole the paper has presented a method to cope with chain indexes for variables which pass a zero.

A description of how to reconstruct such price indices under more generalized circumstances is a topic of future research.

One practical conclusion from this paper is that, *for variables passing a zero* over a period, in order to be able to handle the associated chain-linked quantity and price index variables *we require more price information than is currently available* where National Statistical Institutes follow the practice of releasing IO-tables in current and previous year prices only.

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A Appendix: Tables

Description of table layout for A.1-A.3:

We assume full knowledge of the values of the price and quantity variable over the period 2000-2012.

- The price variable is increasing by 5% each year. Column 1.
- The quantity transacted (column 2) takes positive, zero and negative values over this period.
- First we compute the corresponding values in current year prices (column 3) and in previous year prices (column 4).
- The factor of the relative growth in the price index is then defined as the value in current year prices divided by the value in previous year prices (column 5).
- Multiplying these factors up from a base year, when the price index takes the value one, to the current year amounts to calculating a chained Paasches price index (column 6).
- The corresponding quantity index is derived by dividing the price index into the values in current year prices (column 7).
- For comparison the directly computed price index and the corresponding quantity index are printed in columns 8 respectively 9.
- In the last two columns the differences are given, between the price (quantity) index values derived through access only to the values in current year prices and previous year prices and the directly calculated one.

A.1 Table 1: Standard method

Time	Price	Quantity	p(t)q(t)	p(t-1)q(t)	p(t)q(t)/p(t-1)q(t)	P(t)	Q(t)	P^*(t)	Q^*(t)	P(t)-P^*(t)	Q(t)-Q^*(t)
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
2000	10,000	20	200,000			#DIV/0!	#DIV/0!	0,78353	255,256	#DIV/0!	#DIV/0!
2001	10,500	10	105,000	100,000	1,050	#DIV/0!	#DIV/0!	0,82270	127,628	#DIV/0!	#DIV/0!
2002	11,025	0	0,000	0,000	#DIV/0!	0,864	0,000	0,86384	0,000	0	0
2003	11,576	-10	-115,763	-110,250	1,050	0,907	-127,628	0,90703	-127,628	0	0
2004	12,155	-20	-243,101	-231,525	1,050	0,952	-255,256	0,95238	-255,256	0	0
2005	12,763	-10	-127,628	-121,551	1,050	1,000	-127,628	1,00000	-127,628	0	0
2006	13,401	0	0,000	0,000	#DIV/0!	#DIV/0!	#DIV/0!	1,05000	0,000	#DIV/0!	#DIV/0!
2007	14,071	10	140,710	134,010	1,050	#DIV/0!	#DIV/0!	1,10250	127,628	#DIV/0!	#DIV/0!
2008	14,775	20	295,491	281,420	1,050	#DIV/0!	#DIV/0!	1,15763	255,256	#DIV/0!	#DIV/0!
2009	15,513	30	465,398	443,237	1,050	#DIV/0!	#DIV/0!	1,21551	382,884	#DIV/0!	#DIV/0!
2010	16,289	40	651,558	620,531	1,050	#DIV/0!	#DIV/0!	1,27628	510,513	#DIV/0!	#DIV/0!
2011	17,103	30	513,102	488,668	1,050	#DIV/0!	#DIV/0!	1,34010	382,884	#DIV/0!	#DIV/0!
2012	17,959	20	359,171	342,068	1,050	#DIV/0!	#DIV/0!	1,40710	255,256	#DIV/0!	#DIV/0!

A.2 Table 2: Standard method with fixing by adding delta

Time	Price	Quantity	p(t)q(t)	p(t-1)q(t)	p(t)q(t)/p(t-1)q(t)	P(t)	Q(t)	P^*(t)	Q^*(t)	P(t)-P^*(t)	Q(t)-Q^*(t)
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
2000	10,000	20	200,000	0,000	0,000	0,823	243,101	0,78353	255,256	0,03918	-12,15506
2001	10,500	10	105,000	100,000	1,050	0,864	121,551	0,82270	127,628	0,04114	-6,07753
2002	11,025	0	0,001	0,001	1,000	0,864	0,001	0,86384	0,001	0,00000	0,00000
2003	11,576	-10	-115,763	-110,250	1,050	0,907	-127,628	0,90703	-127,628	0,00000	0,00000
2004	12,155	-20	-243,101	-231,525	1,050	0,952	-255,256	0,95238	-255,256	0,00000	0,00000
2005	12,763	-10	-127,628	-121,551	1,050	1,000	-127,628	1,00000	-127,628	0,00000	0,00000
2006	13,401	0	0,001	0,001	1,000	1,000	0,001	1,05000	0,001	-0,05000	0,00005
2007	14,071	10	140,710	134,010	1,050	1,050	134,010	1,10250	127,628	-0,05250	6,38141
2008	14,775	20	295,491	281,420	1,050	1,103	268,019	1,15763	255,256	-0,05513	12,76282
2009	15,513	30	465,398	443,237	1,050	1,158	402,029	1,21551	382,884	-0,05788	19,14422
2010	16,289	40	651,558	620,531	1,050	1,216	536,038	1,27628	510,513	-0,06078	25,52563
2011	17,103	30	513,102	488,668	1,050	1,276	402,029	1,34010	382,884	-0,06381	19,14422
2012	17,959	20	359,171	342,068	1,050	1,340	268,019	1,40710	255,256	-0,06700	12,76282

A.3 Table 3: Standard method with adding imputed price index value

Time	Price	Quantity	p(t)q(t)	p(t-1)q(t)	p(t)q(t)/p(t-1)q(t)	P(t)	Q(t)	P^*(t)	Q^*(t)	P(t)-P^*(t)	Q(t)-Q^*(t)
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
2000	10,000	20	200,000	0,000	0,000	0,784	255,256	0,78353	255,256	0,00000	0,00000
2001	10,500	10	105,000	100,000	1,050	0,823	127,628	0,82270	127,628	0,00000	0,00000
2002	11,025	0	0,000	0,000	1,050	0,864	0,000	0,86384	0,000	0,00000	0,00000
2003	11,576	-10	-115,763	-110,250	1,050	0,907	-127,628	0,90703	-127,628	0,00000	0,00000
2004	12,155	-20	-243,101	-231,525	1,050	0,952	-255,256	0,95238	-255,256	0,00000	0,00000
2005	12,763	-10	-127,628	-121,551	1,050	1,000	-127,628	1,00000	-127,628	0,00000	0,00000
2006	13,401	0	0,000	0,000	1,050	1,050	0,000	1,05000	0,000	0,00000	0,00000
2007	14,071	10	140,710	134,010	1,050	1,103	127,628	1,10250	127,628	0,00000	0,00000
2008	14,775	20	295,491	281,420	1,050	1,158	255,256	1,15763	255,256	0,00000	0,00000
2009	15,513	30	465,398	443,237	1,050	1,216	382,884	1,21551	382,884	0,00000	0,00000
2010	16,289	40	651,558	620,531	1,050	1,276	510,513	1,27628	510,513	0,00000	0,00000
2011	17,103	30	513,102	488,668	1,050	1,340	382,884	1,34010	382,884	0,00000	0,00000
2012	17,959	20	359,171	342,068	1,050	1,407	255,256	1,40710	255,256	0,00000	0,00000

Description of table layout for A.4-A.9:

We assume full knowledge of the values of the price and quantity variable over the period 2000-2012.

- The price variable for product 1 (2) is increasing by 3% (7%) each year. Column 1.
- The quantity transacted (column 2) takes positive, zero and negative values over this period.
- First we compute the corresponding values in current year prices (column 3) and in previous year prices (column 4).
- The factor of the relative growth in the price index is then defined as the value in current year prices divided by the value in previous year prices (column 5).
- Multiplying these factors up from a base year, when the price index takes the value one, to the current year amounts to calculating a chained Paasches price index (column 6).
- The corresponding quantity index is derived by dividing the price index into the values in current year prices (column 7).
- Column 8-10 present the values of the chained Paasches, Laspeyres respectively Fisher price index. When marked by *, the figures are for the version with *numerical* weights.
- In the last column the year-to-year price growth factor of the Fisher price index.

A.4 Table 4: Standard method

Time	Price 1	Price 2	Quantity 1	Quantity 2	p(t)q(t)	p(t-1)q(t)	p(t)q(t)/p(t-1)q(t)	P(t)	Q(t)	P(P,t)	P(L,t)	P(F,t)	P(F,t-1,t)
	(1a)	(1b)	(2a)	(2b)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
2000	10,000	5,000	20	20	300,000			#DIV/0!	#DIV/0!	#DIV/0!	#DIV/0!	#DIV/0!	#DIV/0!
2001	10,300	5,350	10	10	156,500	150,000	1,043	#DIV/0!	#DIV/0!	#DIV/0!	#DIV/0!	#DIV/0!	#DIV/0!
2002	10,609	5,725	0	0	0,000	0,000	#DIV/0!	#DIV/0!	#DIV/0!	#DIV/0!	#DIV/0!	#DIV/0!	#DIV/0!
2003	10,927	6,125	-10	-10	-170,525	-163,335	1,044	#DIV/0!	#DIV/0!	#DIV/0!	#DIV/0!	#DIV/0!	#DIV/0!
2004	11,255	6,554	5	-8,586	0,000	2,042	0,000	0,957	0,000	0,95719	#DIV/0!	#DIV/0!	#DIV/0!
2005	11,593	7,013	-10	-10	-186,055	-178,091	1,045	1,000	-186,055	1,00000	1,00000	1,00000	#DIV/0!
2006	11,941	7,504	0	5	37,518	35,064	1,070	1,070	35,064	1,07000	1,04508	1,05746	1,05746
2007	12,299	8,029	10	10	203,276	194,442	1,045	1,119	181,721	1,11862	1,11823	1,11842	1,05765
2008	12,668	8,591	20	-30	-4,374	5,108	-0,856	-0,958	4,566	1,16985	1,16945	1,16965	1,04580
2009	13,048	9,192	30	30	667,201	637,759	1,046	-1,002	-665,761	1,22385	1,22343	1,22364	1,04616
2010	13,439	9,836	40	40	930,997	889,601	1,047	-1,049	-887,681	1,28080	1,28036	1,28058	1,04653
2011	13,842	10,524	30	30	730,998	698,248	1,047	-1,098	-665,761	1,34088	1,34042	1,34065	1,04690
2012	14,258	11,261	20	20	510,371	487,332	1,047	-1,150	-443,840	1,40427	1,40379	1,40403	1,04728

A.5 Table 5: Standard method with fixing by adding delta

Time	Price 1	Price 2	Quantity 1	Quantity 2	p(t)q(t)	p(t-1)q(t)	p(t)q(t)/p(t-1)q(t)	P(t)	Q(t)	P(P,t)	P(L,t)	P(F,t)	P(F,t-1,t)
	(1a)	(1b)	(2a)	(2b)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
2000	10,000	5,000	20	20	300,000			1795,721	0,167				
2001	10,300	5,350	10	10	156,500	150,000	1,043	1873,535	0,084				
2002	10,609	5,725	0	0	0,001	0,001	1,000	1873,535	0,000				
2003	10,927	6,125	-10	-10	-170,525	-163,335	1,044	1956,007	-0,087				
2004	11,255	6,554	5	-8,586	0,001	2,043	0,000	0,957	0,001				
2005	11,593	7,013	-10	-10	-186,055	-178,091	1,045	1,000	-186,055				
2006	11,941	7,504	0	5	37,518	35,064	1,070	1,070	35,064				
2007	12,299	8,029	10	10	203,276	194,442	1,045	1,119	181,721				
2008	12,668	8,591	20	-30	-4,374	5,108	-0,856	-0,958	4,566				
2009	13,048	9,192	30	30	667,201	637,759	1,046	-1,002	-665,761				
2010	13,439	9,836	40	40	930,997	889,601	1,047	-1,049	-887,681				
2011	13,842	10,524	30	30	730,998	698,248	1,047	-1,098	-665,761				
2012	14,258	11,261	20	20	510,371	487,332	1,047	-1,150	-443,840				

A.6 Table 6: Standard method with adding imputed price index value

Time	Price 1	Price 2	Quantity 1	Quantity 2	p(t)q(t)	p(t-1)q(t)	p(t)q(t)/p(t-1)q(t)	P(t)	Q(t)	P*(P,t)	P*(L,t)	P*(F,t)	P*(F,t-1,t)
	(1a)	(1b)	(2a)	(2b)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
2000	10,000	5,000	20	20	300,000			0,806	372,095	0,80244	0,80242	0,80243	#DIV/0!
2001	10,300	5,350	10	10	156,500	150,000	1,043	0,841	186,047	0,83721	0,83720	0,83721	1,04333
2002	10,609	5,725	0	0	0,000	0,000	1,044	0,878	0,000	0,87349	0,87376	0,87363	1,04350
2003	10,927	6,125	-10	-10	-170,525	-163,335	1,044	0,916	-186,079	0,91194	0,91192	0,91193	1,04385
2004	11,255	6,554	5	-8,586	0,000	2,042	1,045	0,957	0,000	0,95719	0,95238	0,95478	1,04699
2005	11,593	7,013	-10	-10	-186,055	-178,091	1,045	1,000	-186,055	1,00000	1,00000	1,00000	1,04736
2006	11,941	7,504	0	5	37,518	35,064	1,070	1,070	35,064	1,07000	1,04508	1,05746	1,05746
2007	12,299	8,029	10	10	203,276	194,442	1,045	1,119	181,721	1,11862	1,11823	1,11842	1,05765
2008	12,668	8,591	20	-30	-4,374	5,108	1,046	1,170	-3,740	1,16985	1,16945	1,16965	1,04580
2009	13,048	9,192	30	30	667,201	637,759	1,046	1,224	545,320	1,22385	1,22343	1,22364	1,04616
2010	13,439	9,836	40	40	930,997	889,601	1,047	1,280	727,093	1,28080	1,28036	1,28058	1,04653
2011	13,842	10,524	30	30	730,998	698,248	1,047	1,340	545,320	1,34088	1,34042	1,34065	1,04690
2012	14,258	11,261	20	20	510,371	487,332	1,047	1,404	363,546	1,40427	1,40379	1,40403	1,04728

A.7 Table 7: Standard method for case of aggregate of opposite signed variables

Time	Price 1	Price 2	Quantity 1	Quantity 2	p(t)q(t)	p(t-1)q(t)	p(t)q(t)/p(t-1)q(t)	P(t)	Q(t)	P(P,t)	P(L,t)	P(F,t)	P(F,t-1,t)
	(1a)	(1b)	(2a)	(2b)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
2000	10,000	5,000	20	-20	100,000			1,092	91,552	#DIV/0!	#DIV/0!	#DIV/0!	#DIV/0!
2001	10,300	5,350	10	-10	49,500	50,000	0,990	1,081	45,776	#DIV/0!	#DIV/0!	#DIV/0!	#DIV/0!
2002	10,609	5,725	0	0	0,000	0,000	0,987	1,067	0,000	#DIV/0!	#DIV/0!	#DIV/0!	#DIV/0!
2003	10,927	6,125	10	-10	48,021	48,845	0,983	1,049	45,786	#DIV/0!	#DIV/0!	#DIV/0!	#DIV/0!
2004	11,255	6,554	5	-8,586	0,000	2,042	0,979	1,026	0,000	1,02645	#DIV/0!	#DIV/0!	#DIV/0!
2005	11,593	7,013	10	-10	45,800	47,011	0,974	1,000	45,800	1,00000	1,00000	1,00000	#DIV/0!
2006	11,941	7,504	0	-5	-37,518	-35,064	1,070	1,070	-35,064	1,07000	0,96875	1,01812	1,01812
2007	12,299	8,029	10	-10	42,698	44,369	0,962	1,030	41,466	1,02972	1,03657	1,03314	1,01475
2008	12,668	8,591	20	-30	-4,374	5,108	1,072	1,104	-3,961	-0,88181	0,98970	nonsensical	nonsensical
2009	13,048	9,192	30	-60	-160,106	-135,425	1,182	1,305	-122,649	-1,04252	3,35206	nonsensical	nonsensical
2010	13,439	9,836	40	-40	144,136	154,217	0,935	1,220	118,138	-0,97437	3,91451	nonsensical	nonsensical
2011	13,842	10,524	30	-30	99,542	108,102	0,921	1,123	88,604	-0,89722	3,60455	nonsensical	nonsensical
2012	14,258	11,261	20	-20	59,933	66,362	0,903	1,015	59,069	-0,81030	3,25537	nonsensical	nonsensical

A.8 Table 8: Standard method with fixing by adding delta (for case of aggregate of opposite signed variables)

Time	Price 1	Price 2	Quantity 1	Quantity 2	p(t)q(t)	p(t-1)q(t)	p(t)q(t)/p(t-1)q(t)	P(t)	Q(t)	P(P,t)	P(L,t)	P(F,t)	P(F,t-1)
	(1a)	(1b)	(2a)	(2b)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
2000	10,000	5,000	20	-20	100,000			1,092	91,552				
2001	10,300	5,350	10	-10	49,500	50,000	0,990	1,081	45,776				
2002	10,609	5,725	0	0	0,000	0,000	0,987	1,067	0,000				
2003	10,927	6,125	10	-10	48,021	48,845	0,983	1,049	45,786				
2004	11,255	6,554	5	-8,586	0,000	2,042	0,979	1,026	0,000				
2005	11,593	7,013	10	-10	45,800	47,011	0,974	1,000	45,800				
2006	11,941	7,504	0	-5	-37,518	-35,064	1,070	1,070	-35,064				
2007	12,299	8,029	10	-10	42,698	44,369	0,962	1,030	41,466				
2008	12,668	8,591	20	-30	-4,374	5,108	1,072	1,104	-3,961				
2009	13,048	9,192	30	-60	-160,106	-135,425	1,182	1,305	-122,649				
2010	13,439	9,836	40	-40	144,136	154,217	0,935	1,220	118,138				
2011	13,842	10,524	30	-30	99,542	108,102	0,921	1,123	88,604				
2012	14,258	11,261	20	-20	59,933	66,362	0,903	1,015	59,069				

A.9 Table 9: Standard method with adding imputed price index value (for case of aggregate of opposite signed variables)

Time	Price 1	Price 2	Quantity 1	Quantity 2	p(t)q(t)	p(t-1)q(t)	p(t)q(t)/p(t-1)q(t)	P(t)	Q(t)	P*(P,t)	P*(L,t)	P*(F,t)	P*(F,t-1,t)
	(1a)	(1b)	(2a)	(2b)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
2000	10,000	5,000	20	-20	100,000			1,092	91,552	0,80244	0,80242	0,80243	#DIV/0!
2001	10,300	5,350	10	-10	49,500	50,000	0,990	1,081	45,776	**0,83721***	0,83720	0,83721	1,04333
2002	10,609	5,725	0	0	0,000	0,000	0,987	1,067	0,000	0,87349	**0,87376***	0,87363	1,04350
2003	10,927	6,125	10	-10	48,021	48,845	0,983	1,049	45,786	0,91194	0,91192	0,91193	1,04385
2004	11,255	6,554	5	-8,586	0,000	2,042	0,979	1,026	0,000	0,95719	0,95238	0,95478	1,04699
2005	11,593	7,013	10	-10	45,800	47,011	0,974	1,000	45,800	1,00000	1,00000	1,00000	1,04736
2006	11,941	7,504	0	-5	-37,518	-35,064	1,070	1,070	-35,064	1,07000	1,04508	1,05746	1,05746
2007	12,299	8,029	10	-10	42,698	44,369	0,962	1,030	41,466	1,11862	1,11823	1,11842	1,05765
2008	12,668	8,591	20	-30	-4,374	5,108	1,072	1,104	-3,961	1,17431	1,16945	1,17188	1,04779
2009	13,048	9,192	30	-60	-160,106	-135,425	1,182	1,305	-122,649	1,23658	1,22812	1,23234	1,05160
2010	13,439	9,836	40	-40	144,136	154,217	0,935	1,220	118,138	1,29412	1,29369	1,29391	1,04996
2011	13,842	10,524	30	-30	99,542	108,102	0,921	1,123	88,604	1,35482	1,35437	1,35460	1,04690
2012	14,258	11,261	20	-20	59,933	66,362	0,903	1,015	59,069	1,41887	1,41840	1,41864	1,04728

B Appendix: Expression for extended chain Laspeyres price index

Extended chain Laspeyres price index

$$p^L Y_{t-1,t} = \sum s^{\star L}_{i,t-1} \cdot pY_{i,t}/pY_{i,t-1}, \tag{5}$$

where

$$s^{\star L}_{i,t-1} = |fY_{i,t-1}| \cdot pY_{i,t-1} / \left(\sum |fY_{i,t-1}| \cdot pY_{i,t-1}\right) \tag{6}$$

with sums over i=1,...,n.

The corresponding extended chain Fisher price index

$$p^F Y_{t-1,t} = (p^L Y_{t-1,t} \cdot p^P Y_{t-1,t})^{1/2}, \tag{7}$$