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Inequality of Opportunity and Inequality of Effort: A Canonical Growth Model

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Inequality of Opportunity and Inequality of Effort: a Canonical Growth Model

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Abstract

Theoretical and empirical studies exploring the effects of income inequality upon growth reach a disappointing inconclusive result. Some recent empirical papers have emphasized that one reason for this ambiguity could be that income inequality is actually a composite measure of inequality of opportunity (IO) and inequality of effort (IE). These types of inequality would affect growth through opposite channels, so the relationship between inequality and growth would depend on which component is larger. Based on this preliminary empirical result, we build an intergenerational model with human capital of inequality and development. The existence of a trap in the process of human capital accumulation generates multiplicity of equilibria and permits the inclusion of social mobility in the analysis.

The model is able to explain how IO and IE affect human capital accumulation and hence ongoing long-run growth. The existence of social mobility in society makes the relationship between income inequality and growth to be non-linear, and the final sign of the influence of inequality on growth to be dependent on the degree of development

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and overall inequality of the economy. We find that IE is generally beneficial to human capital accumulation and, therefore, to ongoing growth, while IO positively affects human capital (income) only for less developed economies.

Keywords: inequality of opportunity; social mobility; human capital; economic growth.

1 Introduction

On February 9, 2012, a fellow of the Center on Children and Families Brookings Institution, Scott Winship, declared for the Senate Budget Committee that the problem with most discussions of income inequality is that they do not distinguish between good and bad inequality. In a world of perfect equality, there would be no rewards for hard work or risk, which would cripple economic growth. At the same time, as pointed out by the Chairman of the Council of Economic Advisers to the US President, Alan Krueger, in his talk on "The rise and consequences of inequality in the United States" at the Center for American Progress, higher inequality might hurt US economic growth, among other things, by reducing intergenerational mobility.[1] As with cholesterol, there would be two kinds of inequality, inequality that enhances growth (good inequality) and inequality that harms growth (bad inequality). This is in a nutshell the 'inequality as cholesterol' hypothesis.[2] In this paper we try to disentangle the effect of both kinds of inequality on growth by presenting an intergenerational model with human capital of inequality and development, where the effects of effort (good source of inequality) and lack of opportunity (bad source of inequality) are explicitly distinguished.

A surge of literature on the effect of income inequality on development and growth has emerged over the last two decades, leading to controversial conclusions.[3] On one hand, the theoretical literature has suggested many channels through which inequality could (positively or negatively) affect economic growth. Unfortunately, the vast empirical literature has not found a channel with a predominant influence. As a result, the empirical relationship between income inequality and growth is also ambiguous.[4] As pointed out by Partridge (1997 and 2005), Barro (2000), Bleaney and Nishizama (2004) and Voitchovsky (2005), this ambiguity could be due to the fact that inequality may affect growth through distinct avenues that offset to each other. In this respect, recent contributions by the World Bank (2006), Bourguignon et al. (2007a) and Marrero and Rodríguez (2010) have noted

[1] For this author rising inequality in the United States has increased the intergeneration earnings elasticity, i.e., the capacity of parent's earnings for predicting their child's future earnings (he called this relationship the "Great Gatsby Curve").

[2] For more on this issue read the blog: http://blogs.worldbank.org/impactevaluations/rising-inequality-in-the-united-states-lessons-from-developing-countries.

[3] Surveys on this issue can be found in Bénabou (1996), Bourguignon (1996), Aghion et al. (1999), Bertola et al. (2005) and Ehrhart (2009).

[4] See Banerjee and Duflo (2003) on the inconclusiveness of the cross-country empirical literature on economic inequality and growth.

that inequality of opportunities could exert a different effect (i.e., negative) on growth (or development) than inequality of (pure) effort, whose impact would be positive. Accordingly, the reason for the ambiguous relationship between overall inequality and development would be that income inequality is actually a composite measure of inequality of opportunity and inequality of (pure) effort. A complementary explanation proposed by Galor and Moav (2004) shows that the relationship between inequality and growth changes with the degree of development of the economy: positive at the former stages of development, when physical capital is more important than human capital in the process of development; and negative in a second stage, when human capital takes the lead as the most important factor favoring growth and development.

In a parallel way, a vast growth literature has emphasized the role of human capital in development and growth (Nelson and Phelps, 1966; Lucas, 1988; Barro, 2000). Indeed, there is growing consensus that human capital is the main immediate factor of development and growth in economies. For example, Glaeser et al. (2004) have recently argued that human capital is even a more basic source of growth than institutions.[5] Taking human capital as the main engine of development and growth, the main goal of this paper is to characterize the relationship between different sources of inequality (opportunities and pure effort) and the average human capital in the economy. To this goal, we build an overlapping generation economy with heterogenous agents and populated by a continuum of dynasties, with human capital formation and wage determination (Lucas, 1988; Glomm and Ravikumar, 1992; Galor and Tsidon, 1997; Boldrin and Montes, 2005; Galor and Moav, 2004). The model attempts to combine the basic principles of the inequality of opportunity literature, with that of wage determination and human capital accumulation. Individuals of the dynasty accumulate human capital by an individual effort decision, although the final amount of human capital accumulated depends also on a set of factors that are beyond individual's control, which are referred as circumstances. Individual circumstances are related to parental status and socioeconomic background, race, health endowments, nationality, etc..[6] For example, high parental human capital

[5]Following the works of Lipset (1960), Przeworski (2004a and 2004b) and Barro (1997), Glaeser et al. (2004) show that countries that emerge from poverty but have accumulated human and physical capital under dictatorships, are increasingly likely to improve their institutions.

[6]The important role of circumstances has been largely emphasized in the literature of inequality of opportunity. For example, Roemer (1998 and 2000) and Bowles et al. (2005), among others, have shown that even if individuals have high inborn talent, the

creates a better environment for the accumulation of human capital (i.e., local home environmental externality, Galor and Tsidon, 1997), and favors the bequest devoted to the offspring in the form of the quality of schooling (Glomm and Ravikumar, 1992). The ultimate sources of heterogeneity in the model come from individual aversion to effort, individual purely exogenous circumstances (such as race or health endowments) and initial level of parental human capital.

Assuming that individual human capital accumulation follows a non-convex process, we find first explicit expressions of the dynamics of human capital accumulation for the dynasty. The non-convexity of the model leads to two interesting properties in our model. First, there is multiple steady-state equilibria (Galor and Zeira, 1993). Thus, each dynasty faces up two potential equilibria: a 'low' equilibrium, common to all dynasties, and a 'high', dynasty specific, equilibrium. The bad equilibrium can be interpreted as a human-capital trap for the dynasty that shows up because the amount of effort required to scape from the trap might be too high what may disincentive the individual to exerting positive effort. This could be the case even for a dynasty with large preference for effort if it has strong unfavorable circumstances. Clearly, identifying and correcting for these situations could be crucial in the design of a policy targeted to improve opportunities and economic growth at the same time. A second important implication of the non-convexity is the existence of social mobility. This mobility could be 'upward' or 'downward'. The 'upward' occurs when a dynasty starts accumulating human capital at the 'low' equilibrium level, but future generations of the same dynasty manage to scape from the trap. The 'downward' mobility occurs when a dynasty starts accumulating human capital above its low level, but future generations end up converging towards the low equilibrium. Understanding the effect of individual circumstances and effort on this kind of social mobility is, therefore, another crucial target that policy makers should pursue.

Once we characterize multiplicity, social mobility and the particular human capital dynamics of each type of dynasties, we analize the average human capital of the aggregate economy, its dynamics and its relationship with the different sources of inequality at the initial period, along the transition and in the long-run. We study the different routes through which the

likelihood of their being able to realize the benefits of that talent (for example, in terms of admission to university or access to employment) will be affected by social conditions. This reasoning can be also applied to individuals with strong preferences for effort. See also, Arrow et al. (2000), Hertz et al. (2008), Blume and Durlauf (2001), Durlauf (2003) and Loury (1989 and 1999).

components of inequality can affect the average level of human capital: the initial percentage of dynasties accumulating human capital above its minimum (above the tramp); the average human capital accumulated by these dynasties; and, the upward and downward social mobility. Combining initial percentages and results on mobility, we obtain the long-run percentage of dynasties converging towards the low and high equilibria.

In addition, the relationship between aggregate human capital and the different sources of inequality, at the very short-run and at the steady-state equilibrium is characterized. We show that, the different sources of inequality affect the average human capital of the economy through different routes (World Bank, 2006; Bourguignon et al., 2007a; Marrero and Rodríguez, 2010; Galor and Moav, 2004). These effects depend on the degree of development and aggregate inequality (Galor and Moav, 2004; Acemoglu, 2010). In general, the impact of inequality of (pure) effort is positive, though under extreme situations, it might have a negative impact on growth. For inequality of opportunity, the effect on aggregate human capital is ambiguous: positive for less developed economies; negative for economies sufficiently developed. We observe therefore that the relationship between inequality of opportunity and growth is non-linear and it depends on the degree of development, while the impact of inequality of effort is in general positive.

Another important contribution of the paper is that the classical decomposition of total income inequality into the inequality-of-opportunity and inequality-of-pure-effort components is reproduced (Peragine, 2004; Chechi and Peragine, 2010; Ferreira and Gignoux, 2011). By using this decomposition for the simplest case (all dynasties end up converging towards their associated high long-run equilibrium), our model may shed some light on the existing controversy about the relationship between inequality and growth. In accordance with previous findings, we show that the effect of overall inequality on aggregate human capital is ambiguous. By using the typical growth-inequality log-linear function, the relationship between development and inequality would depend on the controls included in the equation, and on whether they are related to inequality of opportunity or inequality of effort. For example, overall inequality shows a positive impact on human capital (say development) when an additional term related to inequality of opportunity is included in the regression, while total inequality appears to have a negative effect on development when the controls are more related to inequality of (pure) effort. Despite its difficulty, we extend this result to the general case, with social mobility.

The rest of the paper is organized as follows. The next section presents the overlapping generation economy, solves the problem of the dynasties, comment on the sources of heterogeneity, the definition of circumstances and pure effort, and solves the dynamics of individual human capital for the different kinds of dynasties that inhabit in the economy. Section 3 characterizes the evolution of the average human capital in the aggregate economy and studies the relationship between this average and inequality of opportunities and pure effort in the short-run. Section 4 provides an explanation for the ambiguous result between growth and inequality in the literature and extends this result to inequality of opportunity and inequality of effort. The last section provides some final remarks.

2 The economy

Our framework is a small open overlapping-generations economy with heterogenous agents, populated by a continuum of dynasties - each one indexed by $i \in \Omega \equiv [0, 1]$ - and with perfect competitive markets. Time t is discrete and in every period a single homogenous good is produced using physical capital and efficiency units of labor. Each dynasty i born at t consists of a common individual, who lives for two periods, childhood and adulthood. During the adulthood, each individual gives birth to another so overall population keeps constant over time. At the beginning of their childhood, individuals receive a bequest from their parent, $x_{t-1}(i)$, in the form of resources to be devoted in the quality of his own education (Card and Krueger, 1992; Glomm and Ravikumar, 1992). Next, at the beginning of his adulthood, human capital accumulation, $h_t(i)$, is determined by an individual effort decision, $e_t(i)$, but the final amount accumulated depends also on a set of factors that are beyond individual's control. In the inequality-of-opportunity literature, these factors are referred as circumstances (Roemer, 1993; Van de Gaer, 1993; Bourguignon et al., 2007a; Fleurbaey, 2008), which are related to parental socioeconomic background (family status, social connections, child nourishment, etc.) and to factors such as race, ethnicity, health endowments, gender or region of birth (Bourguignon et al., 2007b; Ferreira and Gignoux, 2011; Li Donni et al., 2012).[7] The individual works during his adulthood and earns the labor income $w_t(i)$ according to his accumulated human capital. Finally, at the end of adulthood, the individual decides his consumption, $c_t(i)$, and the level of bequest to his offspring. By simplicity,

[7] In this paper, we do not consider inborn ability or talent because innate ability is less than perfectly correlated between generations so an explicit modelization of how talent is allocated among dynasties is required (see, among others, Hasler and Rodriguez-Mora, 2000 and Hasler et al. 2007). Another possible source of inequality that is not considered in this paper is luck (see Lefranc et al., 2009).

we assume that consumption during his childhood is included in his parent's consumption (Galor and Zeira, 1993; Benabou, 2000). From now on, time subscript is omitted whenever it is not strictly necessary.

2.1 The aggregate economy

Aggregate output Y is produced according to the following Cobb-Douglas technology:

$$Y = A{\cdot}K^{\lambda}\widetilde{L}^{1-\lambda},\ A > 0,\ \lambda \in (0,1), \tag{1}$$

where K is the aggregate stock of physical capital, $\widetilde{L} = L{\cdot}h$ is the overall effective labor in the economy, with L as raw labor and h the average level of human capital in the economy. Labor is perfectly inelastically supplied and, without loss of generality, normalized to 1. The term A is a technological, Arrow-neutral factor, which is assumed to be constant (i.e., there is no technological progress in this economy). We consider a small open economy with unrestricted international borrowing and lending, thus the real interest rate is exogenous and equal to the world rental rate, which is assumed to be stationary at level $\overline{r}$.[8] Since producers operate in a perfectly competitive environment, $\overline{r}$ determines the aggregate physical to human capital level,

$$\overline{r} = Y'_K = A{\cdot}\lambda{\cdot}\left(\frac{K}{h}\right)^{\lambda-1} \Rightarrow \left(\frac{K}{h}\right) = \left(\frac{A{\cdot}\lambda}{\overline{r}}\right)^{1/(1-\lambda)}, \tag{2}$$

and the wage per unit of human capital (or effective labor) is given by,

$$w = Y'_{\widetilde{L}} = A{\cdot}(1-\lambda){\cdot}\left(\frac{K}{h}\right)^{\lambda} = A^{1/(1-\lambda)}{\cdot}(1-\lambda){\cdot}\left(\frac{\lambda}{\overline{r}}\right)^{\lambda/(1-\lambda)}, \tag{3}$$

which increases with A, decreases with $\overline{r}$ and is constant as long as A and $\overline{r}$ are constant as well.[9]

The average human capital h is determined by

$$h = \int_{\Omega} h(i){\cdot}dG\left[h(i)\right], \tag{4}$$

[8] Following Galor and Tsidon (1997), the choice of a small open economy is basically based on the fact that interest rates do not change significantly in the course of economic growth.

[9] Though beyond the scope of this paper, it could be assumed that the technological factor A depends on the average human capital. In this case, the level of effective labor, w, would encompass the evolution of average human capital as well. In this case, there would be a (global) technological externality coming from the effect of average human capital on aggregate technology and thus on w (Benabou, 1996; Tamura, 1996; Galor and Tsidon, 1997).

where $G[h(i)]$ is the distribution function of human capital, and then K is obtained from (2). Because $\Omega \equiv [0,1]$, h can also be interpreted as the aggregate human capital in the economy.

2.2 The dynasty: preferences and human capital

Formally, preferences of each individual in the dynasty i are represented by:[10]

$$u(i) = \pi(\eta)c(i)^{\eta}x(i)^{1-\eta} - \gamma(i)e(i)^{1+\beta}, \tag{5}$$

where $\eta \in (0,1)$ is a parameter of relative preferences between $c(i)$ and $x(i)$; $\pi(\eta) = \eta^{-\eta}(1-\eta)^{-(1-\eta)}$ is a factor of normalization. Effort is considered as a non-monetary factor that generates desutility, but is needed to accumulate human capital (Agion and Bolton, 1997; Roemer, 1998), with $\beta > 0$ defining the degree of convexity of the effort function; $\gamma(i) > 0$ represents the individual degree of aversion to effort, which is specific to individuals in the same dynasty. Each individual supplies one unit of labor inelastically and faces the following restriction:

$$c(i) + x(i) \leq w(i), \tag{6}$$

which is satisfied with equality because utility is strictly monotonic in c and x. Individual salaries are determined by $h(i)$ and w,

$$w(i) = w \cdot h(i). \tag{7}$$

Human capital accumulation depends on personal decisions (i.e., effort) but also on factors that are beyond individual's control (i.e., circumstances). Initially, the set of circumstances is assumed to be a composite index, $\theta(i) > 0$ (Mejía and St-Pierre, 2008) and individual human capital accumulation is assumed to follow a non-convex process,

$$h(i) = \Pi[e(i), \theta(i)] = \begin{cases} \overline{h} & e(i) < \widetilde{e}(i) \\ \theta(i)^{\psi} \cdot e(i) & e(i) \geq \widetilde{e}(i) \end{cases}, \ \widetilde{e}(i) = \frac{\overline{h}}{\theta(i)^{\psi}}, \ \psi \geq 0, \tag{8}$$

where $\Pi[\cdot]$ is continuous and increasing in $e(i)$ and $\theta(i)$; $\overline{h}$ is some minimum level of human capital that any individual attains even without exerting any effort.[11] The threshold $\widetilde{e}(i)$ denotes the minimum level of effort that

[10] See Acemoglu (2010), Ch. 10.

[11] A similar human capital accumulation function can be found in Acemoglu (2010), Ch. 21.6. We assume that $\overline{h}$ is exogenous and it is common to all dynasties. Instead, the level of $\overline{h}$ could be dynasty specific or related to public provided funds, but these cases are beyond the scope of this paper.

an individual with $\theta(i)$ circumstances needs to run in order to accumulate human capital above $\overline{h}$ (Azariadis and Drazen, 1990; Azariadis, 1996). This threshold depends inversely on the set of individual circumstances, an idea that has been extensively discussed in the inequality-of-opportunity literature (Roemer, 1998; World Bank, 2006; Bourguignon et al., 2007b). The parameter ψ denotes the relative importance of circumstances with respect to effort in determining $h(i)$. Thus, if $\psi = 0$, the accumulation of human capital depends only on individual's effort, i.e., a pure Meritocratic society (Lucas, 1995; Arrow et al., 2000). However, if $\psi > 0$, the accumulation of human capital depends also on initial circumstances (Rawls, 1971; Sen, 1980; Roemer, 1993).

Summing up, for $\psi > 0$, circumstances affect individual's human capital and wage in two different ways. First, $\theta(i)$ affects $\widetilde{e}(i)$ and hence the likely to be *trapped* at $\overline{h}$. Second, for $e(i) > \widetilde{e}(i)$, $\theta(i)$ affects the return-to-effort in terms of accumulated human capital, which is a crucial feature to evaluate the trade-off between the benefit of exerting effort and its desutility.

Finally, we emphasize the implications of the non-convexity of the human capital accumulation process. First, it is a simple way to generate multiple steady-state equilibria.[12] Under certain general conditions (as we will show), each dynasty faces up with two potential equilibria: a low one, given by $\overline{h}$, and a high one, dynasty specific, given by $h^*(i) > \overline{h}$. This multiplicity makes the initial distribution of circumstances and aversion to effort to play an important role on the equilibrium at which the dynasty will end up. Moreover, it may cause the low equilibrium $\overline{h}$ to be a human-capital *trap* for the dynasty. When the amount of effort required to scape from the trap is high in comparison to its return, the individual has no incentives to exert positive effort. This could be the case even for a dynasty with large preference for effort (i.e., low $\gamma(i)$), but strong unfavorable circumstances (i.e., low $\theta(i)$). If this situation does not change, this dynasty will keep trapped in a zero-effort and low-human capital equilibrium, which is harmful not only for the dynasty itself, but also for the economy as a whole because it affects the average human capital in the society.

At the same time, social mobility might also be a consequence of multiple equilibria. In our context, social mobility occurs when either a dynasty

[12] Non-convexities and multiple steady-state equilibria have been traditionally justified in the context of imperfect credit markets (Galor and Zeira, 1993). Instead, we assume the process in (8), which makes explicit the role of circumstances in the accumulation of human capital. Nevertheless, multiple steady-state equilibria are also possible when there are no convexities if credit markets are imperfect and the marginal propensity to save is higher for richer dynasties (Galor and Moav, 2004). See also Galor and Tsidon (1997).

starts accumulating $\overline{h}$ but future generations manage to scape from the trap ('upward mobility'); or when a dynasty starts accumulating human capital above $\overline{h}$ but future generations end up converging towards $\overline{h}$ ('downward mobility').

2.3 Solving the problem of the dynasty

For each dynasty i, generation t maximizes utility (5) subject to (6), (7) and (8). Given the noncovexity in (8), the individual must decide first between exerting effort to accumulate human capital above $\overline{h}$ or, on the contrary, set effort to zero and accumulate $\overline{h}$. For the zero-effort case, the solution is trivial (allocations are denoted with a 0 superscript):

$$\begin{aligned} e^0(i) &= 0, && (9) \\ h^0(i) &= \overline{h}, && (10) \\ w^0(i) &= w \cdot \overline{h}, && (11) \\ c^0(i) &= \eta \cdot w \cdot \overline{h}, && (12) \\ x^0(i) &= (1-\eta) \cdot w \cdot \overline{h}, && (13) \\ V^0(i) &= w \cdot \overline{h}, && (14) \end{aligned}$$

where $V^0(i)$ is the utility attained by dynasty i in this case. If the individual decides a level of effort above $\widetilde{e}(i)$, the problem is solved in two steps (allocations are denoted with a 1 superscript). First, taking $h(i)$ as a given, (5) is maximized subject to (6) and (7), obtaining

$$\begin{aligned} e^1(i) &= \frac{h(i)}{\theta(i)^{\psi}}, && (15) \\ c^1(i) &= \eta \cdot w \cdot h(i), && (16) \\ x^1(i) &= (1-\eta) \cdot w \cdot h(i). && (17) \end{aligned}$$

Substituting these expressions into (5), the following indirect utility function, V^1, in terms of $h(i)$ is obtained:

$$V^1 = w \cdot h(i) - \gamma(i) \cdot \left[\frac{h(i)}{\theta(i)^{\psi}} \right]^{1+\beta}. \tag{18}$$

In the second step, (18) is maximized with respect to $h(i)$, and the solution is substituted into (7) and (15),

$$h^1(i) = \left[\frac{w}{\gamma(i)\cdot(1+\beta)}\right]^{\frac{1}{\beta}} \theta(i)^{\frac{(1+\beta)\cdot\psi}{\beta}}, \tag{19}$$

$$w^1(i) = \left[\frac{w^{1+\beta}}{\gamma(i)\cdot(1+\beta)}\right]^{\frac{1}{\beta}} \theta(i)^{\frac{(1+\beta)\cdot\psi}{\beta}}, \tag{20}$$

$$e^1(i) = \left[\frac{w}{\gamma(i)\cdot(1+\beta)}\right]^{\frac{1}{\beta}} \theta(i)^{\frac{\psi}{\beta}}. \tag{21}$$

Finally, $c^1(i)$ and $x^1(i)$ are obtained plugging (19) into (16) and (17), respectively. We elaborate on some features of these expressions below.

To close with the problem of the dynasty, we need to characterize the incentive-to-effort condition, i.e., whether $V^1(i) - V^0(i) \geq 0$. In Lemma 1, we establish this condition in terms of the minimum level of circumstances required, $\widetilde{\theta}(i)$, which depends directly on $\gamma(i)$ and inversely on w (aspects of the aggregate economy).

Lemma 1 *For any dynasty $i \in \Omega$ and generation $t \geq 0$, the incentive-to-effort condition $V^1(i) - V^0(i) \geq 0$ is fulfilled if and only if $\theta(i) \geq \widetilde{\theta}(i)$,*

$$\widetilde{\theta}(i) = \overline{h}^{\frac{\beta}{(1+\beta)\cdot\psi}} \cdot \left(\frac{1+\beta}{\beta}\right)^{\frac{\beta}{(1+\beta)\cdot\psi}} \cdot \left[\frac{\gamma(i)\cdot(1+\beta)}{w}\right]^{\frac{1}{(1+\beta)\cdot\psi}}. \tag{22}$$

Moreover, condition $\theta(i) \geq \widetilde{\theta}(i)$ implies that $h^1(i) \geq \overline{h}$ and $e^1(i) \geq \widetilde{e}(i)$.
Proof. *See Appendix A1* ■

2.4 Circumstances, effort and heterogeneity

From (8) and (7), it is clear that $h(i)$ and $w(i)$ are increasing functions of individual's effort and circumstances. With respect to circumstances, some of them can be seen as endogenous (i.e., those related to parental behavior, such as $h_{t-1}(i)$ and $x_{t-1}(i)$),[13] and others are purely exogenous, such as race,

[13] Macroeconomists have extensily shown that parental education and resourses devoted to the offspring's education have significant effects on the individual's human capital, while, for example, school characteristics have relatively little importance in determining individual achievements (Coleman et al., 1966; Becker and Tomes, 1986; Hanushek, 1996; Ginther et al., 2000). In our framework, it can be interpreted that h_{t-1} would create a better environment for the accumulation of human capital (i.e., a local home environmental externality, Galor and Tsidon, 1997), while x_{t-1} favors the bequest devoted to the offspring

health endowments, nationality, ethnicity or gender, which, by simplicity, we group them in a composite index $a(i)$. Bearing in mind this distinction, $\theta(i)$ can be assumed to be the Cartesian product of $h_{t-1}(i)$, $x_{t-1}(i)$ and $a(i)$ (Roemer, 1993; Van de Gaer, 1993; Rodríguez, 2008; Ferreira and Gignoux, 2011),

$$\theta(i) = a(i)x_{t-1}(i)^{\alpha}h_{t-1}(i)^{\varphi};\ \alpha, \varphi \in (0, 1). \tag{23}$$

Substituting the expression $x_{t-1}(i) = (1-\eta)wh_{t-1}(i)$ into (23), $\theta(i)$ reduces to

$$\theta(i) = (1-\eta)^{\alpha}w^{\alpha}a(i)h_{t-1}(i)^{\alpha+\varphi}. \tag{24}$$

We assume that $\alpha + \varphi \leq 1$ to avoid increasing returns to scale in $\theta(i)$.

From (9) and (21), it is clear that effort and circumstances are related.[14] In fact, when $\theta(i) < \widetilde{\theta}(i)$, $e(i) = 0$. This result accords with the inequality-of-opportunity literature: individual exerted effort is a function of personal circumstances (beyond the individual's control) and of individual preferences (independent to circumstances), which is commonly referred as 'pure effort', a factor that is under the individual own responsibility and it is represented by $1/\gamma(i)$ in our case (Roemer 1998; Björklund et al., 2012).

Consequently, each dynasty is ultimately characterized by

$\Gamma(i) = \{a(i), \gamma(i), h_{-1}(i)\}$, which is revealed at $t = 0$.[15] Hence, heterogeneity in the economy comes from differences in $\Gamma(i)$. We assume that a, γ and h_{-1} follow mean invariant log normal, independent distributions (see Benabou, 1996, among others):

$$\ln a \sim N\left(\ln \hat{a} - \frac{\Delta_a^2}{2}, \Delta_a^2\right), \tag{25}$$

$$\ln \gamma \sim N\left(\ln \widehat{\gamma} - \frac{\Delta_\gamma^2}{2}, \Delta_\gamma^2\right), \tag{26}$$

$$\ln h_{-1} \sim N\left(\ln \widehat{h} - \frac{\Delta_h^2}{2}, \Delta_h^2\right). \tag{27}$$

In this manner, a, γ and h have constant means equal to $\hat{a}$, $\widehat{\gamma}$ and $\widehat{h}$, independent to the corresponding variances. The variance of a lognormal

in the form of quality of schooling (Glomm and Ravikumar, 1992). See also Boldrin and Montes (2005) for a general discussion of this issue.

[14] For example, Roemer observes that "Asian children generally work hard in school and thereby do well because parents press them to do so. The familial pressure is clearly an aspect of their environment outside their control." (Roemer, 1998, p.22).

[15] For $t = 0$, $x_{-1}(i) = (1-\eta)\cdot w \cdot h_{-1}(i)$, given $h_{-1}(i)$.

distribution is closely related to any inequality index consistent with the Lorenz curve (the class of S-convex inequality indices), such as the Gini, the Kolm-Atkinson index, the Mean Logarithmic Deviation (MLD) or the Theil index (Cowell, 2009). For example, the MLD index, $T_0(x)$, and the Theil index, $T_1(x)$, are just half the variance of a log normal: $T_0(j) = T_1(j) = \Delta_j^2/2$, $j = h$, a and γ.[16] Notice that $1/\gamma$ (pure effort as defined above) and γ have (in logarithms) the same variance, Δ_γ^2.

2.5 The human capital dynamics within a dynasty

In order to characterize the human capital dynamics of dynasty i, we rewrite (8) in terms of $h_{t-1}(i)$. First, the incentive-to-effort condition:

Proposition 2 *For any dynasty $i \in \Omega$ and generation $t \geq 0$, the incentive-to-effort condition $V^1(i) - V^0(i) \geq 0$ is fulfilled if and only if the parental level of human capital, $h_{t-1}(i)$, is higher than $\widetilde{h}(i)$; i.e.,$h_{t-1}(i) \geq \widetilde{h}(i)$,*

$$\begin{aligned} \widetilde{h}(i) &= \left[\left(\frac{\overline{h}\,(1+\beta)}{\beta}\right)^{\beta}\frac{(1+\beta)\gamma(i)}{w}\right]^{\frac{1}{\vartheta}}\left[(1-\eta)^{\alpha}w^{\alpha}a(i)\right]^{\frac{-1}{\alpha+\varphi}}, \qquad (28) \\ \vartheta &= (\alpha+\varphi)(1+\beta)\psi. \end{aligned}$$

Proof. *It comes directly from plugging (24) into (22), and solving the resultant inequality in h_{t-1}* ■

Condition (28) highlights the role of parental human capital in the return-to-effort of their offsprings and hence in the incentive to accumulate human capital above $\overline{h}$. Parental human capital needs to be high enough to compensate the requirements established by $\widetilde{h}(i)$, which depends on $a(i)$ and $\gamma(i)$ as well as on the characteristics of the aggregate economy (w and $\overline{h}$); otherwise, the dynasty converges towards $\overline{h}$. The following properties of $\widetilde{h}(i)$ are worth noting:

i. $\widetilde{h}(i) > 0$ for all $\gamma(i)$, $a(i) > 0$, and it is independent to $h_{-1}(i)$;

ii. $\widetilde{h}(i)$ can be greater, equal or lower than $\overline{h}$;

iii. The relationship between $\widetilde{h}(i)$ and all other relevant factors is as expected: positive with $\gamma(i)$ and negative with $a(i)$ and w;

[16] The MLD index has a path-independent additive decomposition (Foster and Shneyerov, 2000). For this reason, this inequality index is the most used in the empirical literature on inequality of opportunity (Checchi and Peragine, 2010; Ferreira and Gignoux, 2011; Marrero and Rodríguez, 2011 and 2012b).

iv. $\widetilde{h}(i)$ follows a log normal distribution; i.e., $\ln \widetilde{h} \sim N\left(\widetilde{\mu}, \widetilde{\sigma}^2\right)$, with[17]

$$\begin{aligned}
\widetilde{\mu} &= J - \frac{1}{\alpha+\varphi}\left(\ln \widehat{a} - \frac{\Delta_a^2}{2}\right) + \frac{1}{\vartheta}\left(\ln \widehat{\gamma} - \frac{\Delta_\gamma^2}{2}\right), \qquad (29) \\
\widetilde{\sigma}^2 &= \frac{1}{(\alpha+\varphi)^2}\Delta_a^2 + \frac{1}{\vartheta^2}\Delta_\gamma^2, \\
J &= \frac{1}{\vartheta}\ln\left[\frac{1+\beta}{w}\left(\frac{\overline{h}\,(1+\beta)}{\beta}\right)^{\beta}\right] - \frac{1}{\alpha+\varphi}\ln\left[(1-\eta)^{\alpha} w^{\alpha}\right].
\end{aligned}$$

We next rewrite the dynasty human capital accumulation as a function of $h_{t-1}(i)$ by plugging (24) and (21) into (8),

$$\begin{aligned}
h_t(i) &= \Omega\left[h_{t-1}(i)\right] = \begin{cases} \overline{h} & h_{t-1}(i) < \widetilde{h}(i) \\ \zeta[h_{t-1}(i)] & h_{t-1}(i) \geq \widetilde{h}(i) \end{cases}, \qquad (30) \\
\zeta[h_{t-1}(i)] &= \left[\frac{w}{(1+\beta)\gamma(i)}\right]^{\frac{1}{\beta}} \left[(1-\eta)^{\alpha} w^{\alpha} a(i) h_{t-1}(i)^{\alpha+\varphi}\right]^{\frac{(1+\beta)\cdot\psi}{\beta}}.
\end{aligned}$$

The following properties of $\Omega\left[h_{t-1}(i)\right]$ are worth noting:
i. $\Omega\left[0\right] = \overline{h} > 0$;
ii. $\Omega\left[\widetilde{h}(i)\right] = \frac{\overline{h}\cdot(1+\beta)}{\beta} > \overline{h}$;
iii. $\Omega\left[h_{t-1}(i)\right]$ is increasing in $h_{t-1}(i)$;[18]
iv. If $\beta - \vartheta > 0$, the function $\Omega\left[\cdot\right]$ is concave and $\zeta[h_{t-1}(i)]$ is strictly concave with $\lim_{h_{t-1}(i)\to 0^+} \zeta'[h_{t-1}(i)] = \infty$ (i.e., the function $\zeta[h_{t-1}(i)]$ starts above the main diagonal, given that $\zeta[0] = 0$). This result comes immediately from the second derivative of $\zeta[h_{t-1}(i)]$ and implies that individual human capital presents decreasing returns with respect to parental human capital.

Proposition 3 *A dynasty $i \in \Omega$ with $\Gamma(i)$ faces, a priory, two potential steady-state equilibria: a 'low' equilibrium, given by $\overline{h}$ and common to all dynasties; and a 'high', dynasty specific, equilibrium, given by $h^*(i) = \zeta[h^*(i)]$,*

$$h^*(i) = \left\{\left[(1-\eta)^{\alpha} w^{\alpha} a(i)\right]^{(1+\beta)\cdot\psi}\left[\frac{w}{(1+\beta)\gamma(i)}\right]\right\}^{\frac{1}{\beta-\vartheta}}. \qquad (31)$$

[17] For this latter result, take logs in (28) and use the expressions in (25)-(27).

[18] For $h_{t-1}(i) < \widetilde{h}(i)$, $\Omega\left[\cdot\right] = \overline{h}$. For $h_{t-1}(i) = \widetilde{h}(i)$, we know from property *ii)* that $\Omega[\widetilde{h}(i)] = \frac{\overline{h}\cdot(1+\beta)}{\beta}$ which is greater than $\overline{h}$. Finally, for $h_{t-1}(i) > \widetilde{h}(i)$, $\partial\zeta[h_{t-1}(i)]/\partial h_{t-1}(i) = \frac{\vartheta}{\beta}\cdot B\cdot C\cdot h_{t-1}(i)^{\frac{\vartheta-\beta}{\beta}} > 0$, with B and C positive constants.

For each dynasty i, $\widetilde{h}(i) > \overline{h}$ is a necessary and sufficient condition for existence of the 'low' equilibrium, which is, at least, locally stable. With respect to the 'high' equilibrium, its existence requires that $\widetilde{h}(i) \leq h^*(i)$, while the condition $\beta - \vartheta > 0$ guarantees its local stability.
Proof. *The characterization of $h^*(i)$ comes immediately from properties i)-iv) of $\Omega\left[h_{t-1}(i)\right]$ and from solving the fixed point $h^*(i) = \zeta[h^*(i)]$. Existence and unicity come directly from the properties of $\Omega\left[h_{t-1}(i)\right]$* ■

As for $\widetilde{h}(i)$, the following properties of $h^*(i)$ are highlighted:

i. $h^*(i) > 0$ for all $\gamma(i)$, $a(i) > 0$, and it is independent to $h_{-1}(i)$;

ii. The relationship between $h^*(i)$ and $\gamma(i)$, $a(i)$ and w is the following: negative with respect to $\gamma(i)$ and positive with respect to $a(i)$ and w;

iii. $h^*(i)$ follows a log normal distribution; i.e., $\ln h^* \sim N\left[\mu^*, (\sigma^*)^2\right]$, with[19]

$$\begin{aligned}
\mu^* &= \frac{G}{\beta-\vartheta} + \frac{(1+\beta)\psi}{\beta-\vartheta}\left(\ln\widehat{a} - \frac{\Delta_a^2}{2}\right) - \frac{1}{\beta-\vartheta}\left(\ln\widehat{\gamma} - \frac{\Delta_\gamma^2}{2}\right), \quad (32)\\
(\sigma^*)^2 &= \left(\frac{(1+\beta)\psi}{\beta-\vartheta}\right)^2\Delta_a^2 + \left(\frac{1}{\beta-\vartheta}\right)^2\Delta_\gamma^2,\\
G &= \ln\left(\frac{w}{1+\beta}\right) + (1+\beta)\psi\ln\left[(1-\eta)^\alpha w^\alpha\right].
\end{aligned}$$

iv. $\ln h^*(i)$ *is negatively correlated with* $\ln\widetilde{h}(i)$, *with* $cov(\ln h^*, \ln\widetilde{h}) = -\left(\frac{\Delta_\gamma^2}{(\beta-\vartheta)\vartheta} + \frac{(1+\beta)\psi\Delta_a^2}{(\beta-\vartheta)(\alpha+\varphi)}\right)$.[20]

If all dynasties would converge to their high equilibrium, μ^* would represent the steady-state human capital average in the economy. From (29) and (32), it is easy to show that the effects of $\widehat{\gamma}$, $\widehat{a}$, w, Δ_a^2 and Δ_γ^2 on $\widetilde{\mu}$ and μ^* are of opposite signs, while their impacts on the corresponding variances have the same sign. Thus, an increase of Δ_a^2 simultaneously raises $\widetilde{\mu}$ (i.e., it makes more difficult to accumulate human capital above $\overline{h}$) and reduces μ^*. The opposite relationship is found for Δ_γ^2, while Δ_h^2 does not affect either $\widetilde{\mu}$ or μ^*. The impacts through $\widetilde{\mu}$ and μ^* can be seen as the two simplest channels through which inequality might affect human capital accumulation. Looking only at these channels, inequality of opportunity would have

[19] For this latter property, take logs in (31) and use the expressions in (25)-(27).

[20] Taking logs in the definitions of h^* and $\widetilde{h}$, we compute $cov(\ln h^*, \ln\widetilde{h}) = E(\ln h^*, \ln\widetilde{h}) - \mu^*\widetilde{\mu}$. After tedious manipulations, we obtain that $cov(\ln h^*, \ln\widetilde{h}) = \frac{2\cdot cov(a,\gamma)}{(\beta-\vartheta)(\alpha+\varphi)} - \frac{\Delta_\gamma^2}{(\beta-\vartheta)\vartheta} - \frac{(1+\beta)\psi\Delta_a^2}{(\beta-\vartheta)(\alpha+\varphi)}$, which can be simplified because $cov(a,\gamma) = 0$.

a negative impact on average human capital, while the effect of inequality of pure effort would be positive. However, as we show in the next section, there exist additional routes that could make ambiguous the overall impact of opportunity and, less likely of pure effort on human capital.

After characterizing the two potential steady-state equilibria in the economy, we study the possible groups of dynasties. Depending on the relative magnitudes of $\widetilde{h}(i)$, $h^*(i)$ and $\overline{h}$, dynasties can be classified in 3 groups. Figures 1, 2 and 3 illustrate the three feasible cases, while Figure 4 shows an unfeasible situation. Depending on the group, initial parental human capital would play a leading or a minor role in determining whether the dynasty ends up converging towards $\overline{h}$ or $h^*(i)$.

Proposition 4 *Assuming $\beta - \vartheta > 0$, a dynasty $i \in \Omega$ belongs to any of the following cases:*

Case 1. *$\overline{h} > \widetilde{h}(i)$: the dynasty always converges to $h^*(i)$, regardless the initial value of $h_{-1}(i)$. Thus, $h^*(i)$ is globally stable in this case.*

Case 2. *$\overline{h} \leq \widetilde{h}(i) \leq h^*(i)$ (at least one with inequality): depending on whether $h_{-1}(i)$ is below or above $\widetilde{h}(i)$, the dynasty converges to either $\overline{h}$ or $h^*(i)$, respectively. Both equilibria are locally stable in this case.*

Case 3. *$h^*(i) < \widetilde{h}(i)$: the dynasty always converge to $\overline{h}$, regardless the initial value of $h_{-1}(i)$. Thus, $\overline{h}$ is globally stable in this case.*

Proof. *From Figures 1 and 3, it follows that Case 1 is characterized by $h^*(i) > \overline{h} > \widetilde{h}(i)$, while Case 3 is characterized by $\overline{h} < h^*(i) < \widetilde{h}(i)$. However, $\overline{h} > \widetilde{h}(i)$ and $h^*(i) < \widetilde{h}(i)$ (Figure 4) cannot be satisfied at the same time, because that would imply that $\Omega\left[\widetilde{h}(i)\right] < \overline{h}$, which contradicts the second property of $\Omega\left[\cdot\right]$. Hence, Case 1 and Case 3 are fully characterized by $\overline{h} > \widetilde{h}(i)$ and $h^*(i) < \widetilde{h}(i)$, respectively. Condition for Case 2 follows immediately from Figure 2* ■

For Case 1 (Figure 1), initial levels of $h(i)$ depends on whether $h_{-1}(i)$ is higher or lower than $\widetilde{h}(i)$, but the dynasty always converges to $h^*(i)$ (i.e., $\overline{h}$ is not a steady-state under Case 1). Hence, dynasties belonging to Case 1 and starting with $h_{-1}(i) < \widetilde{h}(i)$ initially accumulate $\overline{h}$, but they eventually scape from it because they show good enough exogenous circumstances and/or low aversion to effort (i.e., $\widetilde{h}(i)$ is sufficiently small). This situation can be interpreted as an example of 'upward' social mobility. On the other hand, dynasties belonging to Case 3 (Figure 3) and starting with $h_{-1}(i) > \widetilde{h}(i)$ accumulate human capital above $\overline{h}$ during a finite number of periods, but they eventually converge to $\overline{h}$, which can be seen as an example of 'downward' mobility. Finally, regarding Case 2 (Figure 2), there is no mobility and the dynamics of the dynasty depends crucially on its initial levels of parental education: if $h_{-1}(i) \leq \widetilde{h}(i)$, the dynasty stays accumulating $\overline{h}$ from $t = 0$ onwards, while $h(i) > \overline{h}$ for all t and converges to its own $h^*(i)$ otherwise.

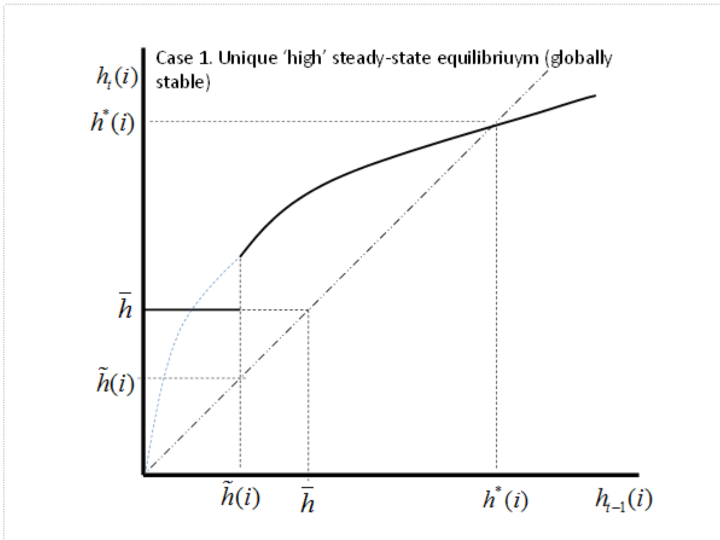

Fig. 1. Human capital dynamics of dynasties. Case 1: unique steady-state, high-equilibrium

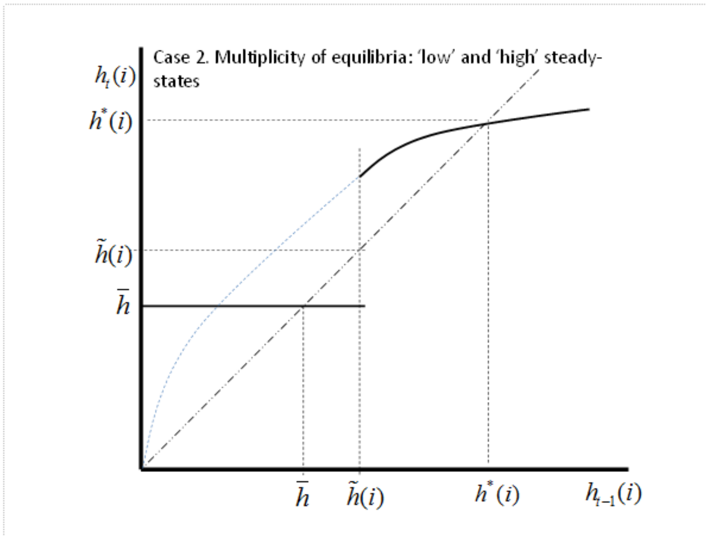

Fig. 2. Human capital dynamics of dynasties. Case 2: multiplicity of equilibria

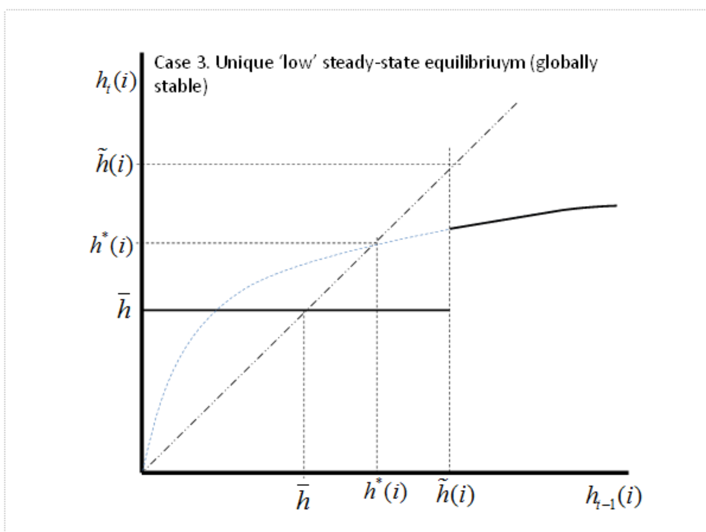

Fig. 3. Human capital dynamics of dynasties. Case 3: unique steady-state, low-equilibrium

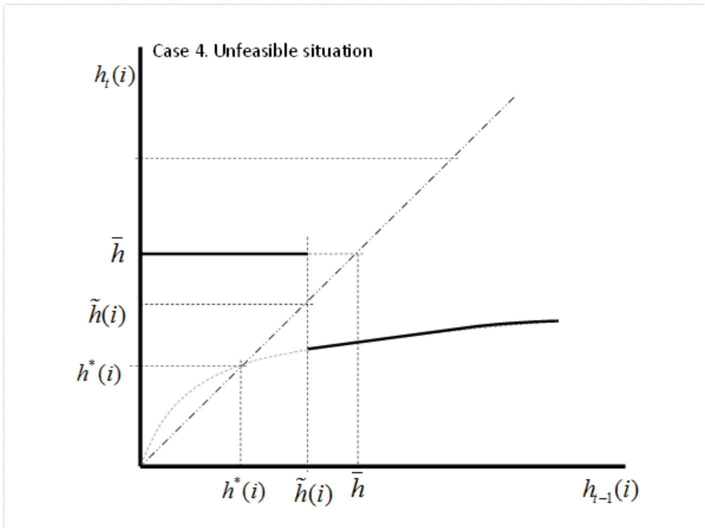

Fig. 4. Human capital dynamics of dynasties. Case 4: unfeasible case

To end with this section, we characterize the probability that a dynasty belongs to a particular case. Denote the probability that a dynasty belongs to Case 1, 2 and 3 by p_{C1}, p_{C2} and p_{C3}, respectively. From Proposition 3, we know that $p_{C1} = \Pr\left[\ln \widetilde{h} < \ln \overline{h}\right]$ and $p_{C3} = \Pr\left[\ln h^* < \ln \widetilde{h}\right]$. Therefore, we have

Lemma 5 *The probabilities that a dynasty belongs to either Case 1, 2 or 3 are given by*

$$p_{C1} = \Phi\left(\frac{\ln \overline{h} - \widetilde{\mu}}{\widetilde{\sigma}}\right), \tag{33}$$

$$p_{C3} = \Phi\left(\frac{\beta - \vartheta}{\beta}\frac{\widetilde{\mu} - \mu^*}{\widetilde{\sigma}}\right), \tag{34}$$

$$p_{C2} = 1 - p_{C1} - p_{C3} \tag{35}$$

where Φ is the $N(0,1)$ cumulative distribution function.
Proof. *See Appendix A2* ■

3 Human capital average and inequality

The purpose of this section is twofold. First, to characterize the evolution of the average human capital in the aggregate economy (4). Second, to study the relationship between this average and the ultimate sources of inequality (individual circumstances and pure effort). We assume throughout this section that $\beta - \vartheta > 0$; i.e., $\zeta[h_{t-1}(i)]$ is strictly concave and dynasties belong to either Case 1, 2 or 3. For illustrative purposes, we divide this section in two parts: the initial period, $t = 0$; the long-run, steady-state equilibrium.

3.1 The initial period ($t = 0$)

The probability that a particular dynasty i accumulates human capital above $\overline{h}$ at $t = 0$ is $p_0 = \Pr\left[\ln h_{-1} > \ln \widetilde{h}\right]$. To simplify notation, we define $X = \ln h_{-1} - \ln \widetilde{h}$, which follows a $N\left(\mu_x, \sigma_x^2\right)$ distribution function. Hence, $p_0 = \Pr[X > 0] = \Phi\left(\frac{\mu_x}{\sigma_x}\right)$, where Φ is a $N(0,1)$ cumulative distribution function. Since h_{-1} and $\widetilde{h}$ are uncorrelated, μ_x and σ_x^2 are easily obtained from (25)-(27) and (29),

$$\mu_x = \left(\ln \widehat{h} - \frac{\Delta_h^2}{2}\right) - J - \frac{1}{\vartheta}\left(\ln \widehat{\gamma} - \frac{\Delta_\gamma^2}{2}\right) + \frac{1}{(\alpha+\varphi)}\left(\ln \widehat{a} - \frac{\Delta_a^2}{2}\right) \quad (36)$$

$$\sigma_x^2 = \Delta_h^2 + \widetilde{\sigma}^2 = \Delta_h^2 + \frac{1}{(\alpha+\varphi)^2}\Delta_a^2 + \frac{1}{\vartheta^2}\Delta_\gamma^2. \quad (37)$$

We observe that the magnitude of μ_x depends on: the initial characteristics of the aggregate economy (technology and preferences), represented by the term J (recall from 29); the average characteristics of the dynasties (levels of $\widehat{a}$, $\widehat{h}_{-1}$ and $\widehat{\gamma}$); and, the different sources of inequality Δ_a^2, Δ_h^2 or Δ_γ^2. Thus, economies with high μ_x would be related to economies with initial high levels of w (i.e., according to (3), large productivity, A, and/or low interest rate, $\overline{r}$), initial favorable circumstances and pure effort on average (i.e., high $\widehat{a}$, $\widehat{h}_{-1}$ and $1/\widehat{\gamma}$), low inequality of circumstances (Δ_a^2 and Δ_h^2) and high inequality of pure effort (Δ_γ^2). Hence, in our economy, we could think of μ_x to be positively related to the degree of development. When the economy develops, it typically shows better circumstances and pure effort on average, lower inequality of opportunity and higher inequality of pure effort.[21] On the other hand, σ_x^2 is a weighted sum of the three sources of inequality Δ_a^2, Δ_h^2 and Δ_γ^2. Hence, we could think of σ_x^2 to be closely related

[21] Ferreira and Gignoux (2011) found that between one fifth and one third of all income

to the overall economic inequality. Obviously, if $\sigma_x^2 = 0$, all dynasties are equal and initial inequality is zero.

To characterize the expression for $E[\ln h_0]$, we notice that the average human capital (in logs) accumulated by those dynasties with $X > 0$ is given by $E[\ln\zeta(h_{-1})/X > 0]$, which is the first moment of an incidentally truncated normal distribution. From (30) and (28), it is clear that $\ln\zeta(h_{-1})$ and X are positively correlated, so that the truncated mean is pushed to the right, i.e., $E[\ln\zeta(h_{-1})/X > 0] \geq E[\ln\zeta(h_{-1})]$. Otherwise, those dynasties with $X \leq 0$ accumulate $\ln\overline{h}$ on average. Hence,

$$E[\ln h_0] = (1 - p_0)\ln\overline{h} + p_0 E[\ln\zeta(h_{-1})/X > 0], \tag{38}$$

where p_0 and $E[\ln\zeta(h_{-1})/X > 0]$ affect positively to $E[\ln h_0]$.[22] The following Lemma gives a detailed expression for $E[\ln\zeta(h_{-1})/X > 0]$.

Lemma 6 *Assuming that $(\ln\zeta(h_{-1}), X)$ follows a bivariate normal distribution, the average human capital for those dynasties with $h_{-1}(i) > \widetilde{h}(i)$ is:*

$$E[\ln\zeta(h_{-1})/X > 0] = E[\ln\zeta(h_{-1})] + \frac{\vartheta}{\beta}\frac{\sigma_x}{p_0}\phi\left(\frac{-\mu_x}{\sigma_x}\right), \tag{39}$$

where $\phi(\cdot)$ is the standard normal density function and the unconditional average human capital, $E[\ln\zeta(h_{-1})]$, is

$$E[\ln\zeta(h_{-1})] = \frac{G}{\beta} + \frac{\vartheta\left(\ln\widehat{a} - \frac{\Delta_a^2}{2}\right)}{(\alpha+\varphi)\beta} + \frac{\vartheta\left(\ln\widehat{h}_{-1} - \frac{\Delta_h^2}{2}\right)}{\beta} - \frac{\left(\ln\widehat{\gamma} - \frac{\Delta_\gamma^2}{2}\right)}{\beta}. \tag{40}$$

Proof. *See Appendix A3* ■

Using (38) and (39), $E[\ln h_0]$ can be rewritten as follows:

$$E[\ln h_0] = \ln\overline{h} + p_0\cdot\left[E\ln\zeta(h_{-1}) - \ln\overline{h}\right] + M(\mu_x, \sigma_x), \tag{41}$$

where $M(\mu_x,\sigma_x) = \frac{\vartheta}{\beta}\sigma_x\phi\left(-\frac{\mu_x}{\sigma_x}\right)$. This expression shows the three routes through which the alternative sources of inequality can affect $E[\ln h_0]$: i) the unconditional mean of $\ln\zeta(h_{-1})$ (taking into account all dynasties); ii)

inequality is explained by opportunities in six countries in Latin America. Meanwhile, Marrero and Rodriguez (2012a) found that between 2% and 22% of overall inequality is explained by opportunities in a set of 23 European countries.

[22] Because $E\left[\ln\zeta(h_{-1})\middle/\ln h_{-1} > \ln\widetilde{h}\right] > \ln\overline{h}$ by definition, the larger p_0, the higher $E[\ln h_0]$.

the probability p_0; iii) the term $M(\mu_x, \sigma_x)$, a term related to those dynasties with better conditions, i.e., those with $h_{-1}(i) > \widetilde{h}(i)$.

From (40), we observe that the effect through $E[\ln \zeta(h_{-1})]$ is strictly positive for Δ_γ^2, but negative for Δ_h^2 and Δ_a^2. This channel was described in the previous section when commenting on the effects of inequality on μ^*. A more equal distribution of circumstances would increase the unconditional mean of $\ln \zeta(h_{-1})$ because the marginal returns to human capital accumulation are higher for those individuals who have lower circumstances.[23] On the contrary, a more equal distribution of 'pure effort' would decrease $E[\ln \zeta(h_{-1})]$ because the marginal returns to human capital accumulation are higher for those individuals who have more aversion to effort. While these former effects show a well defined sign, the relationship through the other two channels are ambiguous and, in general, depend on the relative magnitude of μ_x and σ_x^2 (see the Proof of the following Proposition). As a result, the overall impact of the different sources of inequality on $E[\ln h_0]$ might be ambiguous and would depend, in general, on the initial degree of development.

Proposition 7 *The effect of the different sources of inequality, Δ_j^2, $j = h,a,\gamma$, on $E[\ln h_0]$ is characterized by the following condition:*

$$\frac{\partial E[\ln h_0]}{\partial \Delta_j^2} > 0 \text{ iff } \Pi_j(\mu_x, \sigma_x) < 0, \tag{42}$$

$$\Pi_j(\mu_x, \sigma_x) = \frac{\mu_x}{\sigma_x^2} + \varepsilon_j - \frac{\vartheta}{\beta} \frac{1 - \varepsilon_j \sigma_x \lambda(-\frac{\mu_x}{\sigma_x})}{\ln\left(\frac{1+\beta}{\beta}\right)}, \tag{43}$$

where $\varepsilon_h = 1$, $\varepsilon_a = (\alpha + \varphi)$, $\varepsilon_\gamma = -\vartheta$; $\lambda\left(-\frac{\mu_x}{\sigma_x}\right) = \Phi\left(\frac{\mu_x}{\sigma_x}\right) / \phi\left(-\frac{\mu_x}{\sigma_x}\right)$ is the Mill's ratio at $\left(-\frac{\mu_x}{\sigma_x}\right)$; and $\Pi_j(\mu_x, \sigma_x)$ is an implicit function on (μ_x, σ_x) with the following properties:

i) $\Pi_j(\mu_x, \sigma_x)$ is C^2 on the $(-\infty, +\infty) \times [0, +\infty)$ space for $j = h,a,\gamma$;

ii) $\lim_{\mu_x \to -\infty} \Pi_j(\mu_x, \sigma_x) = -\infty$ for $j = h, a, \gamma$;

iii) $\lim_{\mu_x \to +\infty} \Pi_j(\mu_x; \sigma_x) = +\infty$ for $j = a, h$; $\lim_{\mu_x \to +\infty} \Pi_j(\mu_x; \sigma_x) = -\infty$ for $j = \gamma$;

iv) $\Pi_j(\mu_x; \sigma_x)$ is strictly monotone (increasing) in μ_x for $j = a, h$, and no monotone for $j = \gamma$;

v) $\Pi_j(\mu_x; \sigma_x)$ is strictly convex in μ_x for $j = a, h$; it is strictly concave for $j = \gamma$, and it shows a global maximum, $\mu_x^{\gamma\max}$, in this case.

[23] See the dicussion in Deaton (2003) applied to health inequality.

Proof. *See Appendix A4* ■

In principle, condition (42) implies that the impact of any source of inequality on $E[\ln h_0]$ is ambiguous. The following Corollary elaborates on the characteristics of these ambiguities, which might be very different depending on the source of inequality. Correspondingly, the Corollary distinguishes between the properties of $\Pi_j(\mu_x, \sigma_x)$ for inequality of opportunity (Part A) and inequality of pure effort (Part B).

Corollary 8 ***Part A. Inequality of opportunity***

For $j = a, h$, the effect of Δ_j^2 on $E[\ln h_0]$ is always ambiguous. Given σ_x, the function $\Pi_j(\cdot)$ shows a unique root, $\widehat{\mu}_x^j$, such that $\partial E[\ln h_0] / \partial \Delta_j^2 > 0$ iff $\mu_x < \widehat{\mu}_x^j$ and $\partial E[\ln h_0] / \partial \Delta_j^2 \leq 0$ otherwise. Hence, the smaller $\widehat{\mu}_x^j$, the more likely is that inequality of opportunity affects negatively to $E[\ln h_0]$. Two properties of the roots $\widehat{\mu}_x^a$ and $\widehat{\mu}_x^h$ are worth noting:

i) $\widehat{\mu}_x^h \leq \widehat{\mu}_x^a$, hence the range of μ_x under which Δ_h^2 harmfully affects $E[\ln h_0]$ is larger than for Δ_a^2.

ii) $\widehat{\mu}_x^h, \widehat{\mu}_x^a < 0$ iff $\sigma_x > \frac{\sqrt{2/\pi}}{(\alpha+\varphi)} \left[1 - \frac{\beta \ln\left(\frac{1+\beta}{\beta}\right)}{(1+\beta)} \frac{1}{\psi} \right]$, which is always satisfied if the society is sufficiently meritocratic, i.e., $\psi < \frac{\beta \ln\left(\frac{1+\beta}{\beta}\right)}{(1+\beta)}$.

Part B. Inequality of pure effort

For $j = \gamma$, the effect of Δ_γ^2 on $E[\ln h_0]$ depends on the sign of $\Pi_\gamma^{\max} = \Pi_\gamma(\mu_x^{\gamma \max})$ as follows:

i) if $\Pi_\gamma^{\max} < 0$, the function $\Pi_\gamma(\cdot)$ is never positive and hence $\partial E[\ln h_0] / \partial \Delta_\gamma^2 > 0$. This is always the case if $\mu_x^{\gamma \max} < 0$ (sufficient condition), which is equivalent to $\sigma_x^2 > \frac{\beta}{\vartheta^2} \ln\left(\frac{1+\beta}{\beta}\right)$;

ii) if $\Pi_\gamma^{\max} \geq 0$, the effect is ambiguous. Condition $\mu_x^{\gamma \max} \geq 0$, which is equivalent to $\sigma_x^2 \leq \frac{\beta}{\vartheta^2} \ln\left(\frac{1+\beta}{\beta}\right)$, is now necessary but not sufficient. In this case, there exists two positive roots, $\underline{\mu_x^\gamma} > \overline{\mu}_x^\gamma > 0$, that divide the real line in three zones: $\mu_x < \underline{\mu_x^\gamma}$; $\mu_x \in \left[\underline{\mu_x^\gamma}, \overline{\mu}_x^\gamma\right]$ and $\mu_x > \overline{\mu}_x^\gamma$. For the 1st and 3rd zones, the effect of Δ_γ^2 on $E[\ln h_0]$ is positive, while it is negative for the 2nd zone.

Proof. *See Appendix A5* ■

We illustrate the results of Proposition 4 and Corollary 1 in Figures 5a and 5b, where the functions $\Pi_j(\cdot)$ $j = a, h, \gamma$ are represented for two alternative situations. On one hand, Figure 5a illustrates an economy with sufficiently high initial levels of inequality, for example with $\sigma_x^2 > \frac{\beta}{\vartheta^2} \ln\left(\frac{1+\beta}{\beta}\right)$.

Under this situation, $\Pi_\gamma(\cdot)$ is always negative, hence inequality of pure effort always benefits $E[\ln h_0]$. Moreover, because σ_x^2 is high, $\widehat{\mu}_x^a$ and $\widehat{\mu}_x^h$ are more likely to be lower than zero, and the range of economies showing opposite impacts of inequality of effort and opportunity on $E[\ln h_0]$ (positive and negative, respectively) increases.

On the other hand, Figure 5b assumes that $\sigma_x^2 \leq \frac{\beta}{\vartheta^2} \ln\left(\frac{1+\beta}{\beta}\right)$, which illustrates the case of multiple roots in $\Pi_\gamma(\mu_x, \sigma_x)$. In this case, inequality of pure effort can be harmful for $E[\ln h_0]$ only if $\mu_x \in \left[\underline{\mu}_x^\gamma, \overline{\mu}_x^\gamma\right]$. Despite this latter possibility is feasible, using simulations (some of these simulations are shown below), we can conclude that this is an unlikely situation; moreover, whether those cases may exist, the range $\left[\underline{\mu}_x^\gamma, \overline{\mu}_x^\gamma\right]$ is quite small.

Another general result is the following: inequality of opportunity is always harmful for $E[\ln h_0]$ if μ_x is sufficiently large, i.e., $\mu_x > \widehat{\mu}_x^a$. In this respect, it is worth noting that the average of human capital is more likely shortened by inequality of opportunity in meritocratic economies. From above we know that less developed economies are characterized by small μ_x, high inequality of circumstances (Δ_a^2 and Δ_h^2) and low inequality of pure effort (Δ_γ^2). In this situation (for sufficiently small μ_x), if the economy as a whole shows a sufficient high initial level of inequality (σ_x^2), raising any source of inequality (of opportunity and/or pure effort) might benefit $E[\ln h_0]$. The intuition of this result is the following. A high percentage of dynasties are trapped at $\overline{h}$ in less developed economies. Hence, raising any source of inequality would favor dynasties with better circumstances to scape from the trap and accumulate human capital above $\overline{h}$. By using simulations, we show below that for an economy of this type, the effect of raising inequality of pure effort on $E[\ln h_0]$ is much higher than raising inequality of opportunity.

When the economy develops, μ_x increases and the effect of inequality of opportunity on $E[\ln h_0]$ turns negative, mainly because of its negative impact on $E[\ln \zeta(h_{-1})]$. In this situation, a relevant fraction of dynasties are now above the trap, and the aforementioned route through $E[\ln \zeta(h_{-1})]$ prevails over the other two alternatives. The impact of inequality of pure effort on $E[\ln h_0]$ remains positive unless σ_x^2 is below $\frac{\beta}{\vartheta^2} \ln\left(\frac{1+\beta}{\beta}\right)$ and $\mu_x \in \left[\underline{\mu}_x^\gamma, \overline{\mu}_x^\gamma\right]$.

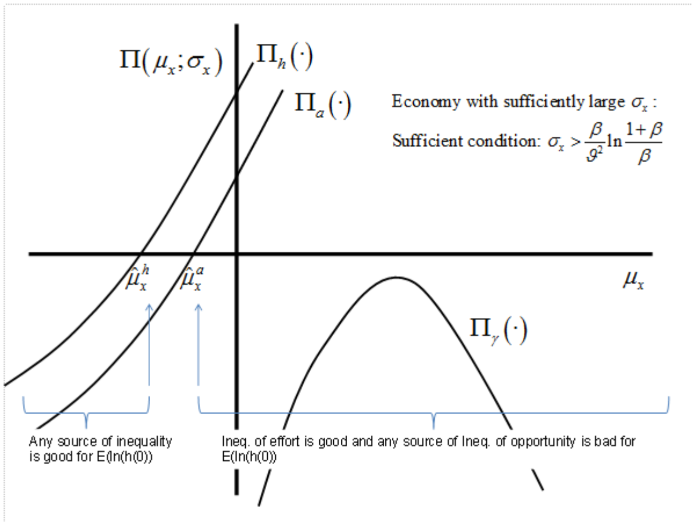

Fig. 5a. Inequality (opportunity and pure effort) and initial human capital (case 1)

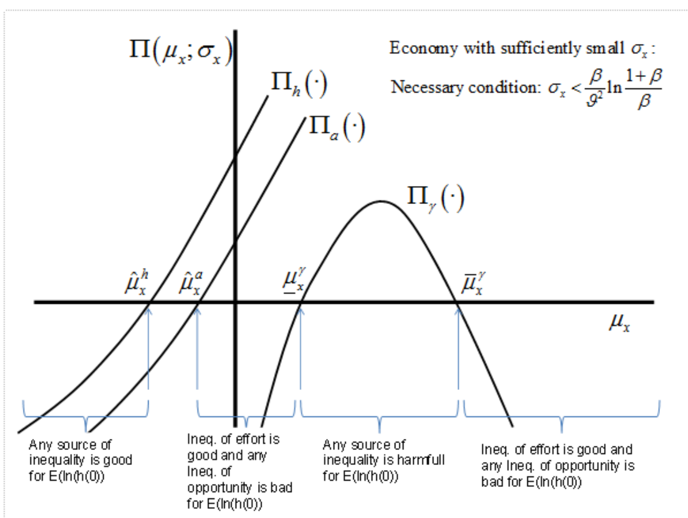

Fig. 5b. Inequality (opportunity and pure effort) and initial human capital (case 2)

3.1.1 Simulation exercise for the initial period

We illustrate our model for the initial period by simulating an economy at $t = 0$. To begin with, we fix the required parameters of the model according to the values in Table 1. Without loss of generality, we normalize to 1 the values of $\overline{h}$ and A and assume that the real interest rate, $\overline{r}$, is 0.05. Following the related literature, we assume also that the parameter of the Cobb-Douglas technology λ is 0.4 and the parameter η of relative preferences between $c(\cdot)$ and $x(\cdot)$ is 0.7. In addition, we suppose that the parameter ψ representing the level of meritocracy in the economy and the degree of convexity of the effort function β are both 0.5. Finally, to guarantee that conditions $\alpha + \varphi \leq 1$ and $\beta - \vartheta > 0$ are fulfilled, we assume that the parameters of the composite index $\theta(\cdot)$, α and φ, are equal to 0.30.

Table 1. The parameters of the model

A	$\overline{h}$	$\overline{r}$	λ	η	ψ	β	α	φ
1	1	0.05	0.4	0.7	0.5	0.5	0.30	0.30

Next, we compute the impact of each source of inequality ($T_0(j) = \Delta_j^2/2$, $j = h, a, \gamma$) on $E[\ln h_0]$ for different values of $\widehat{a}$, $\widehat{h}_{-1}$ and $\widehat{\gamma}$. In particular, we consider these average levels equal to 0.25, 0.50, 1.0 (the benchmark), 1.5 and 2.0 (see Figures 6a, 6b and 6c). Thus, in Figure 6a, we represent the relationship between $E[\ln h_0]$ and $T_0(a)$ for different values of $\widehat{a}$. When $\widehat{a}$ is small (which causes μ_x to be also small), the relationship is positive, i.e., higher inequality of exogenous circumstances favors $E[\ln h_0]$. On the contrary, when $\widehat{a}$ is large enough (which causes μ_x to be also sufficiently large), the relationship becomes unambiguously negative, so that a higher $T_0(a)$ reduces initial human capital on average. Results are similar for the case of $T_0(h)$ (see Figure 6b). However, the incidence of $T_0(\gamma)$ on $E[\ln h_0]$ is in general positive (Figure 6c). Only for a small range of values of $\widehat{\gamma}$ and Δ_γ^2 (not shown in the Figure), we could find a negative relationship between $T_0(\gamma)$ and $E[\ln h_0]$.[24] Therefore, in general, inequality of pure effort helps to increase initial human capital on average.

Finally, by comparing slopes in Figures 6a, 6b and 6c, it is observed that economies with unfavorable (average) initial circumstances and pure effort (low $\widehat{a}$, $\widehat{h}_{-1}$ and $1/\widehat{\gamma}$), the positive impact on $E[\ln h_0]$ of raising inequality of

[24] Though these extreme cases are not shown, they can be obtained from the authors upon request.

pure effort is higher than raising inequality of opportunity. For instance, the slope of $E[\ln h_0]$ for low levels of $\widehat{a}$ and $\widehat{h}_{-1}$ and low inequality of opportunity is positive, but small, while it tends to infinite for low levels of $\widehat{\gamma}$ and low inequality of pure effort.

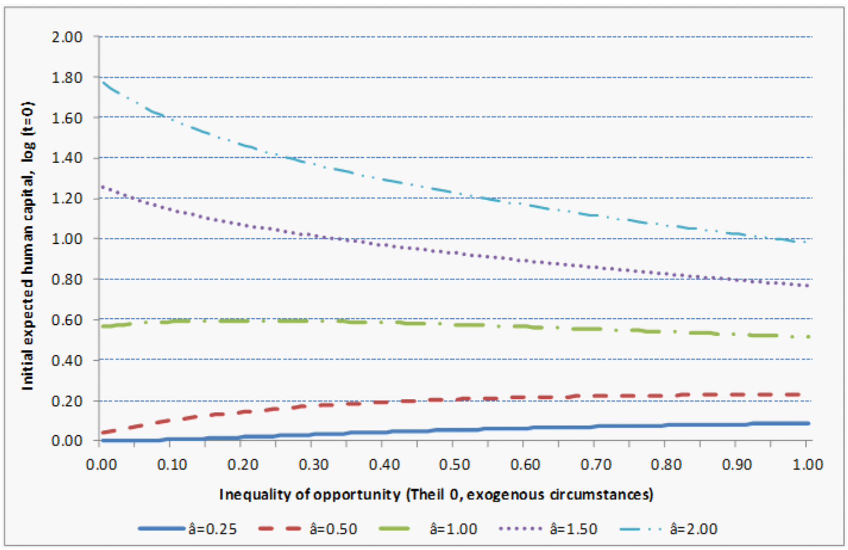

Fig 6a. Initial human capital and inequality of opportunity (exog. circumstances)

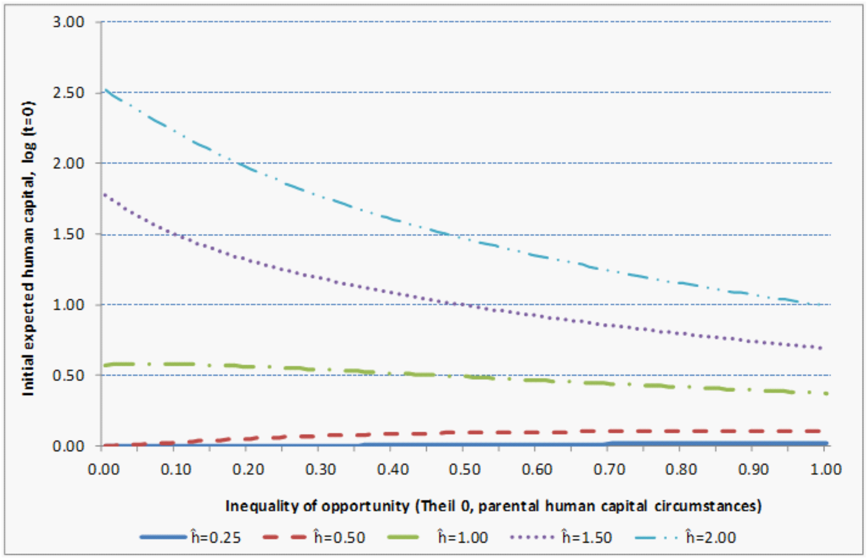

Fig 6b. Initial human capital and inequality of opportunity (parental human capital)

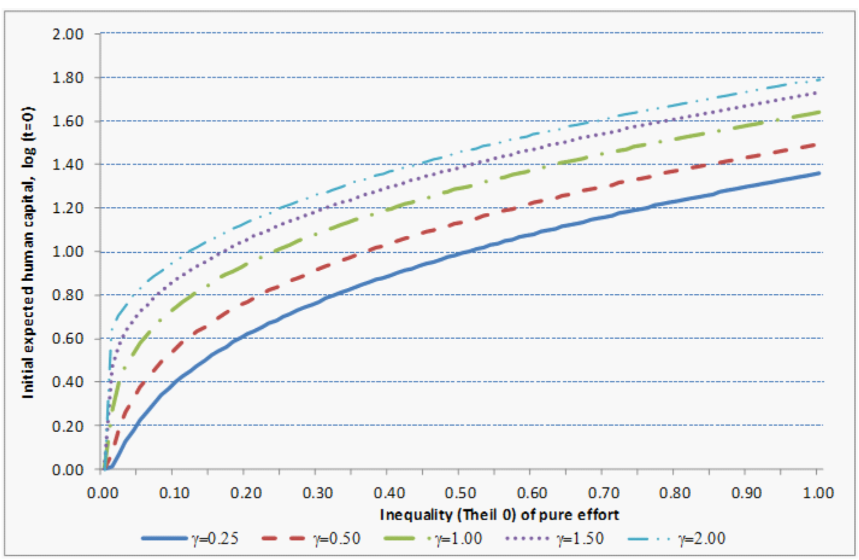

Fig 6c. Initial human capital and inequality of pure effort

3.2 The long-run ($t = \infty$)

After $t = 0$, the evolution of the average human capital for the aggregate economy depends on p_0 and on social mobility. Social mobility occurs when a dynasty accumulates initially $\overline{h}$ and then covergences towards h^* (upward mobility) or, the other way around, when initially accumulates above $\overline{h}$ and then converges to $\overline{h}$ (downward mobility). As a result, the probability that a dynasty accumulates human capital above $\overline{h}$ may change after $t = 0$.[25]

Let us denote the proportion of individuals converging to their high steady-state in the long run by p_∞. This proportion would include all dynasties belonging to Case 1 and those dynasties belonging to Case 2 with $h_{-1}(i) > \widetilde{h}(i)$. Alternatively, p_∞ can be seen as the proportion of all dynasties with $h_{-1}(i) > \widetilde{h}(i)$ (given by the probability p_0) plus the dynasties belonging to Case 1 with $h_{-1}(i) \leq \widetilde{h}(i)$ (conditioning on upward mobility) and minus the dynasties belonging to Case 3 with $h_{-1}(i) > \widetilde{h}(i)$ (conditioning to downward mobility). Defining the proportion of dynasties moving upward by $p^u = \Pr\left[B^{up}\right]$, where $B^{up} \equiv \left\{\ln \overline{h} - \ln \widetilde{h} > 0\right\} \cap \left\{\ln h_{-1} - \ln \widetilde{h} \leq 0\right\}$, and the proportion of dynasties moving downward by $p^d = \Pr\left[B^{down}\right]$, where $B^{down} \equiv \left\{\ln h^* - \ln \widetilde{h} < 0\right\} \cap \left\{\ln h_{-1} - \ln \widetilde{h} > 0\right\}$, we can define p_∞ as follows:

$$p_\infty = p_0 + p^u - p^d. \tag{44}$$

This expression shows clearly that p_∞ can be lower, higher or equal to p_0 depending on the net mobility $p^u - p^d$.

Likewise, the expected level of human capital in the long run would be given by:

$$\begin{aligned} E\left[\ln h_\infty\right] &= (1 - p_\infty) \cdot \ln \overline{h} + p_0 E\left[\ln h^* / X > 0\right] + \\ &\quad + p^u E\left[\ln h^* / B^{up}\right] - p^d E\left[\ln h^* \Big/ B^{down}\right]. \end{aligned} \tag{45}$$

Using the definition of p_∞, we can rewritte $E\left[\ln h_\infty\right]$ as follows:

$$\begin{aligned} E\left[\ln h_\infty\right] &= (1 - p_0) \ln \overline{h} + p_0 E\left[\ln h^* / X > 0\right] + \\ &\quad + p^u \left[E\left(\ln h^* / B^{up}\right) - \ln \overline{h}\right] \\ &\quad - p^d \left[E\left(\ln h^* \Big/ B^{down}\right) - \ln \overline{h}\right]. \end{aligned} \tag{46}$$

[25] For simplicity, we have assumed that $a(i)$ and $\gamma(i)$ are given by the initial generation. Otherwise, a dynasty might move randomly from one status to another, depending on the realizations of $a(i)$ and $\gamma(i)$, and the resultant $\widetilde{h}(i)$, at any period t.

Note that all terms in parenthesis are positive because $h^*(i) > \overline{h}$ for all i by construction. Moreover, it is expected that $E\left[\ln h^* / B^{up}\right] > E\left[\ln h^* / X > 0\right] > E\left[\ln h^* / B^{down}\right]$ because h^* and $\widetilde{h}$ are negative correlated. If this is the case, a powerful intuition emerges. Given the value of p_0, if social mobility of both kinds decreases by the same amount, i.e., $\nabla p^u = \nabla p^d$, the lost in $E\left[\ln h_\infty\right]$ will be higher than the gain. Consequently, reducing social mobility would be harmfull for $E\left[\ln h_\infty\right]$. A lower upward mobility harms dynasties with better conditions (low aversion to effort and/or good circumstances) and higher levels of $h^*(i)$, while lower downward mobility permits those dynasties with worse conditions and lower levels of $h^*(i)$ to keep accumulating human capital above $\overline{h}$.

For illustrative purposes, we analyze first the average human capital for the aggregate economy in the long-run when there is no social mobility.

3.2.1 The no social mobility case

Assuming no social mobility (NM) implies that $p^u = p^d = 0$, hence $p_\infty = p_0$. In this situation, we can rewrite the expression for $E\left[\ln h_\infty\right]$ in (46) as follows:

Lemma 9 *The average human capital in the long-run when there is no social mobility is:*

$$E\left[\ln h_\infty^{NM}\right] = (1 - p_0)\cdot \ln \overline{h} + p_0 \cdot \mu^* + \frac{\vartheta}{\beta - \vartheta} \frac{\sigma_x^2 - \Delta_h^2}{\sigma_x} \phi\left(-\frac{\mu_x}{\sigma_x}\right). \qquad (47)$$

Proof. *See Appendix A6* ■

We can easily perceive the similarity between this expression and equation (41) in the previous subsection. Following the strategy adopted for $t = 0$, we obtain for the case of no social mobility at $t = \infty$ also the following result,

Proposition 10 *The effect of the different sources of inequality,* Δ_j^2*,* $j = a,\gamma$*, on* $E\left[\ln h_0\right]$ *is characterized by the following condition:*

$$\frac{\partial E\left[\ln h_\infty^{NM}\right]}{\partial \Delta_j^2} > 0 \text{ iff } \Pi_{j,\infty} < 0, \qquad (48)$$

$$\Pi_{j,\infty} = \frac{\mu_x}{\sigma_x^2} + \varepsilon_j - \frac{1 - \varepsilon_j \sigma_x \lambda\left(-\frac{\mu_x}{\sigma_x}\right) + \frac{\Delta_h^2}{\sigma_x^2}}{\left[\frac{\beta}{\vartheta}\ln\left(\frac{1+\beta}{\beta}\right) + \left(\ln \overline{h} - \ln \widehat{h}_{-1} + \frac{\Delta_h^2}{2}\right) + \frac{\mu_x}{\sigma_x^2}\Delta_h^2\right]},$$

where $\varepsilon_a = (\alpha + \varphi)$, $\varepsilon_\gamma = -\vartheta$; $\lambda\left(-\frac{\mu_x}{\sigma_x}\right) = \Phi\left(\frac{\mu_x}{\sigma_x}\right)/\phi\left(-\frac{\mu_x}{\sigma_x}\right)$ *is the Mill's ratio at* $\left(-\frac{\mu_x}{\sigma_x}\right)$; *and* $\Pi_{j,\infty}$ *is an implicit function on* (μ_x, σ_x).
Proof. *See Appendix A7* ■

The no mobility case is a very particular one and, for this reason, we do not go any further here. However, it is worth noting the great similarity between the results in Propositions 5 and 4.

3.2.2 The social mobility case

When considering a society with social mobility, it is no longer true that $p_\infty = p_0$. From above we have that $p^u = \Pr\left[\left\{\ln \overline{h} > \ln \widetilde{h}\right\} \cap \left\{\ln h_{-1} \leq \ln \widetilde{h}\right\}\right]$ and $p^d = \Pr\left[\left\{\ln \widetilde{h} > \ln h^*\right\} \cap \left\{\ln h_{-1} > \ln \widetilde{h}\right\}\right]$, which can be rewritten as $p^u = \Pr\left[\ln h_{-1} \leq \ln \widetilde{h} < \ln \overline{h}\right]$ and $p^d = \Pr\left[\ln h^* < \ln \widetilde{h} < \ln h_{-1}\right]$, respectively. From Lemma 2, we know also that $p_{C1} = \Pr\left[\ln \widetilde{h} < \ln \overline{h}\right]$, $p_{C3} = \Pr\left[\ln h^* < \ln \widetilde{h}\right]$ and $p_0 = \Pr\left[\ln h_{-1} > \ln \widetilde{h}\right]$ hence, in this case, it is true that

$$
\begin{aligned}
p^u &= p_{C1} - p_0, & (49)\\
p^d &= p_{C3} - (1 - p_0), & (50)\\
p_\infty &= (1 - p_0) + (p_{C1} - p_{C3}). & (51)
\end{aligned}
$$

Considering these probabilities and recalling the value of $E\left[\ln h_\infty^{NM}\right]$ in (47), the average human capital for the aggregate economy in the long-run in this general case would be given by the following expression:

$$
\begin{aligned}
E\left[\ln h_\infty\right] &= (p_0 - p_{C1} + p_{C3}) \cdot \ln \overline{h} + p_0 \cdot \mu^* + \frac{\vartheta}{\beta - \vartheta} \frac{\sigma_x^2 - \Delta_h^2}{\sigma_x} \phi\left(-\frac{\mu_x}{\sigma_x}\right) + & (52)\\
&\quad + (p_{C1} - p_0) E\left(\ln h^* / B^{up}\right) - (p_{C3} + p_0 - 1) E\left(\ln h^* \middle/ B^{down}\right) & (53)
\end{aligned}
$$

Notice the complexity of the last expression given the fact that the averages $E\left(\ln h^* / B^{up}\right)$ and $E\left(\ln h^* / B^{down}\right)$ refer to the first moment of a truncated trivariate normal distribution.

(... Unfinished section. To be completed ...)

4 The inequality-development relationship

Over the last two decades the literature on the relationship between income inequality and development has arrived to an unconclusive result. According to our findings in the previous section, the reason for this ambiguity is that income inequality is actually a composite measure of inequality of opportunity and inequality of pure effort. This result has been empirically tested by Marrero and Rodríguez (2010) and Ferreira et al. (2012). However, these authors do not provide a formal model to justify such a result. Under the model presented in the previous sections, we can explain the opposite impact on growth of inequality of opportunity and inequality of pure effort. But we can go further if we assume that there is not trap in the economy. By assuming this, we avoid explicitly all the complexities raised by the presence of social mobility and, as a result, we are able to replicate some significant results found in the microeconomic literature on inequality of opportunity and in the macroeconomic literature on the relationship between growth and income inequality.

First, we prove that wage inequality (our proxy of income inequality) is actually a weighted aggregation of inequality of opportunity and inequality of pure effort. This result reproduces the classical decomposition of total income inequality into the inequality-of-opportunity and inequality-of-pure-effort components in the inequality-of-opportunity literature (Chechi and Peragine, 2010; Ferreira and Gignoux, 2011). From (20), the long-run expected level of wages, $E(\ln w_\infty)$, can be rewritten as follows:

$$E(\ln w_\infty) = E(\ln h_\infty) + \ln w. \tag{54}$$

Among all inequality indices that are consistent with the Lorenz curve, following the inequality-of-opportunity literature, we adopt the MLD index, which is defined for continuous distributions as follows:

$$T_0(w_\infty) = \int \ln\left(\frac{E(w_\infty)}{w_\infty}\right) dF(w_\infty), \tag{55}$$

where $F(w_\infty)$ is the distribution function of wages in the long-run. After several simple operations, it can be rewritten as follows:

$$T_0(w_\infty) = \ln E(w_\infty) - E(\ln w_\infty). \tag{56}$$

Note that $T_0(w_\infty)$ is always non-negative because the logarithmic function is strictly concave.

While $E(\ln w_\infty)$ is given by (54), $\ln E(w_\infty)$ comes directly from taking logs in

$$E(w_\infty) = \left[\left[\frac{w^{1+\beta-\vartheta}}{(1+\beta)}\right]^{\frac{1}{\beta-\vartheta}} \cdot [(1-\eta)^{\alpha} \cdot w^{\alpha}]^{\frac{(1+\beta)\cdot\psi}{(\beta-\vartheta)}}\right] \cdot E\left[a^{\frac{(1+\beta)\psi}{\beta-\vartheta}}\right] \cdot E\left[\gamma^{-\frac{1}{(\beta-\vartheta)}}\right]. \tag{57}$$

Bearing in mind that a and γ are independent, the last term is obtained by substituting (24) into (20), then substituting (31) into the resultant expression of wages and, finally, taking expected values. Hence, overall inequality in this simple case without social mobility reduces to:

$$T_0(w_\infty) = \ln E\left[a^{\frac{(1+\beta)\psi}{\beta-\vartheta}}\right] + \ln E\left[\gamma^{-\frac{1}{(\beta-\vartheta)}}\right] - \frac{(1+\beta)\psi}{\beta-\vartheta} \cdot E(\ln a) + \frac{1}{\beta-\vartheta} \cdot E(\ln\gamma), \tag{58}$$

because the constant terms cancel each other. Manipulating this expression we obtain

$$T_0(w_\infty) = \ln E\left[a^{\frac{(1+\beta)\psi}{\beta-\vartheta}}\right] - E\left(\ln a^{\frac{(1+\beta)\psi}{\beta-\vartheta}}\right) + \ln E\left[\gamma^{-\frac{1}{(\beta-\vartheta)}}\right] - E\left(\ln \gamma^{-\frac{1}{\beta-\vartheta}}\right) \tag{59}$$

$$= T_0\left[a^{\frac{(1+\beta)\psi}{\beta-\vartheta}}\right] + T_0\left[\gamma^{-\frac{1}{(\beta-\vartheta)}}\right]. \tag{60}$$

Because individual circumstances and pure effort follow log normal distributions, we have the following

$$\ln a^{\frac{(1+\beta)\psi}{\beta-\vartheta}} \sim N\left(\frac{(1+\beta)\psi}{\beta-\vartheta}\left(\ln\hat{a} - \frac{\Delta_a^2}{2}\right), \frac{(1+\beta)^2\psi^2}{(\beta-\vartheta)^2}\Delta_a^2\right) \tag{61}$$

$$\ln \gamma^{-\frac{1}{(\beta-\vartheta)}} \sim N\left(-\frac{1}{(\beta-\vartheta)}\left(\ln\widehat{\gamma} - \frac{\Delta_\gamma^2}{2}\right), \frac{1}{(\beta-\vartheta)^2}\Delta_\gamma^2\right). \tag{62}$$

Bearing in mind that half the variance of any log normal variable is equal to the MLD index of such variable, we obtain our first important result: wage inequality in the steady-state equilibrium is the weighted aggregation of inequality of opportunity and inequality of pure effort.

$$T_0(w_\infty) = \frac{(1+\beta)^2\psi^2}{(\beta-\vartheta)^2}T_0(a) + \frac{1}{(\beta-\vartheta)^2}T_0(\gamma). \tag{63}$$

Note that this result does not depend on parental education because all dynasties converge to their corresponding high equilibrium, which is independent from h_{-1}.

Once we have obtained the above decomposition of total wage inequality in the long-run, we show that the effect of total inequality of wages on the stationary average of human capital is ambiguous. To see this result, we first consider expression (63) and find the value of $T_0(a)$:

$$T_0(a) = \frac{(\beta-\vartheta)^2}{(1+\beta)^2\psi^2} T_0(w_\infty) - \frac{1}{(1+\beta)^2\psi^2} T_0(\gamma). \tag{64}$$

This expression is then plugged into equation (32) to obtain the following:

$$E\left[\ln h_\infty\right] = \mu^* = \pi_0 + \pi_1 \cdot T_0(\gamma) - \pi_2 \cdot T_0(w_\infty), \tag{65}$$

where $\pi_0 = \frac{1}{\beta-\vartheta}\left[G + (1+\beta)\cdot\psi\cdot \ln\widehat{a} - \ln\widehat{\gamma}\right]$, $\pi_1 = \frac{(1+\beta)\cdot\psi-1}{(\beta-\vartheta)\cdot(1+\beta)\cdot\psi}$ and $\pi_2 = \frac{1}{(1+\beta)\cdot\psi}$. In this case, it is clear that $T_0(w_\infty)$ has a negative impact on the average human capital in the long-run, which is consistent with results of the previous section. However, if alternatively, we consider again expression (63) but now we consider the pure effort inequality term, $T_0(\gamma)$, we have:

$$T_0(\gamma) = (\beta-\vartheta)^2 T_0(w_\infty) - (1+\beta)^2\psi^2 T_0(a). \tag{66}$$

Substituting this expression into equation (32), we obtain,

$$E\left[\ln h_\infty\right] = \mu^* = \delta_0 - \delta_1 \cdot T_0(a_0) + \delta_2 \cdot T_0(w_\infty) \tag{67}$$

where $\delta_0 = \pi_0$, $\delta_1 = \frac{(1+\beta)\psi+(1+\beta)^2\psi^2}{\beta-\vartheta}$ and $\delta_2 = \frac{(\beta-\vartheta)^2}{\beta-\vartheta}$. Now, we get the opposite result, i.e., $T_0(w_\infty)$ has a positive impact on the stationary average of human capital. Moreover, if we aggregate expressions (65) and (67) and divide the sum by two, we obtain an expression including total inequality, inequality of opportunity and inequality of pure effort:

$$E\left[\ln h_\infty\right] = \pi_0 + \frac{\pi_1}{2} T(\gamma) - \frac{\delta_1}{2} T(a_0) + \frac{\delta_2 - \pi_2}{2} T(w_\infty). \tag{68}$$

This expression is even more appealing to illustrate the existing controversy in the inequality-growth relationship: the relationship between $E\left[\ln h_\infty\right]$ and $T(w_\infty)$ depends on the sign of $(\delta_2 - \pi_2)$, which is undefined.

Summing up, we have found that total inequality of wages negatively affects average human capital when we control for the inequality of pure effort, while total inequality of wages positively affects average human capital when we control for the inequality of circumstances. This finding might be the reason that explains why empirical studies exploring the effects of income inequality upon growth reach an inconclusive result. When individual circumstances are specified in the empirical model, the impact of income inequality on growth follows closely the effect of pure effort on human capital which is positive. On the contrary, when pure effort is considered in empirical estimations, the impact of income inequality on growth becomes negative as it follows closely the effect of individual circumstances on human capital which is negative. Therefore, it becomes apparent that total inequality of wages can increase or decrease growth depending on the kind of inequality, opportunity or pure effort, that prevails.

5 Conclusions

There are crucial factors complementary to human capital accumulation (race, genes, family background, parental education, social connections, installation of preferences and aspirations in children, health endowments, etc.) that are beyond the individual's control and are non-purchasable: there is no market for individual circumstances. For these factors the functioning of credit markets is irrelevant and it causes that those with bad complementary factors end up either investing little in human capital or not investing. In addition, there are decreasing returns to the accumulation of human capital. Hence, a more equal distribution of circumstances would increase growth (in terms of human capital) given that the marginal returns to human capital accumulation are higher for those individuals who have more unfavorable circumstances. On the contrary, a more equal distribution of 'pure effort' would decrease growth (in terms of human capital) given that the marginal returns to human capital accumulation are higher for those individuals who have more aversion to effort.

These are some of the results that we have obtained with an intergenerational model of inequality and development as proposed here. However, this is by no means the whole story. We have also shown the importance of considering social mobility. After taking into account this consubstantial element of any society, the picture of the relationship between income inequality and growth becomes less evident. The existence of social mobility in the economy causes a non-linear relationship between income inequality and growth, and makes the final sign of the influence of inequality on growth to be dependent on the degree of development of the economy. The model proposed here has pointed out that the way income inequality affects growth is more complex and difficult than what the literature has commonly assumed. Despite this difficulty, our results reveal that two general conclusions can be outlined: inequality of pure effort generally benefits human capital accumulation, and hence ongoing growth, while more likely inequality of opportunity positively affects growth only in less developed economies.

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6 Appendix

A1. Proof of Lemma 1

The proof of the first part of the Proposition comes directly from plugging *(19) into* $V^1(i)$ *in (18), using (14) for* $V^0(i)$ *and solving the inequality* $V^1(i) - V^0(i) \geq 0$ *for* $\theta(i)$. For the second part of the proposition comes from using (14) and rewriting (18) in terms of $e^1(i)$. Hence, the incentive-to-effort condition can be rewritten as follows:

$$V^1(i) - V^0(i) \geq 0 \Leftrightarrow w\cdot \left[h^1(i) - \overline{h}\right] - \gamma(i)\cdot e^1(i)^{1+\beta} \geq 0. \tag{69}$$

From this expression, it is clear that the incentive-to-effort condition implies that $h^1(i) \geq \overline{h}$. Plugging (19) and (21) into (69), condition (69) can be rewritten as

$$V^1(i) - V^0(i) \geq 0 \Leftrightarrow \Phi\left[\theta(i), \gamma(i), w\right] \geq \overline{h}, \tag{70}$$

where

$$\Phi\left[\theta(i), \gamma(i), w\right] = \left(\frac{\beta}{1+\beta}\right) \cdot \left[\frac{w}{\gamma(i)\cdot(1+\beta)}\right]^{\frac{1}{\beta}} \cdot \theta(i)^{\frac{(1+\beta)\cdot\psi}{\beta}}. \tag{71}$$

Finally, using the expressions for $\widetilde{e}(i)$ and $e^1(i)$ in (8) and (21), respectively, and the definition of $\Phi\left[\cdot\right]$ in (71), we have

$$e^1(i) \geq \widetilde{e}(i) \Leftrightarrow \Phi\left[\theta(i), \gamma(i), w\right] \geq \overline{h}\left(\frac{\beta}{1+\beta}\right). \tag{72}$$

Because $\beta \geq 0$, condition (70) implies (72), which proves that the incentive-to-effort condition implies $e^1(i) \geq \widetilde{e}(i)$.

A2. Proof of Lemma 2

Recall that $\ln\widetilde{h} \sim N\left(\widetilde{\mu}, \widetilde{\sigma}^2\right)$ and $\ln h^* \sim N\left[\mu^*, (\sigma^*)^2\right]$. From Proposition 3, we can express the probability p_{C1} in logarithms as $p_{C1} = \Pr\left[\ln\widetilde{h} < \ln\overline{h}\right]$ and easily conclude that $p_{C1} = \Phi\left(\frac{\ln\overline{h} - \widetilde{\mu}}{\widetilde{\sigma}}\right)$. Defining $R = \ln h^* - \ln\widetilde{h}$ and following a similar reasoning, $p_{C3} = \Pr\left[\ln h^* < \ln\widetilde{h}\right] = \Pr\left[R < 0\right] = \Phi\left(\frac{-\mu_R}{\sigma_R}\right)$, where $R \sim N(\mu_R, \sigma_R^2)$, with $\mu_R = \mu^* - \widetilde{\mu}$ and $\sigma_R^2 = (\sigma^*)^2 + \widetilde{\sigma}^2 - 2\cdot cov(\ln h^*, \ln\widetilde{h})$. The expression of μ_R can be obtained from (32) and (29). For σ_R^2, we know from property *iv)* in page 15 that $cov(\ln h^*, \ln\widetilde{h}) = -\frac{\Delta_\gamma^2}{(\beta-\vartheta)\vartheta} - \frac{(1+\beta)\psi\Delta_a^2}{(\beta-\vartheta)(\alpha+\varphi)}$. Using the expressions of $(\sigma^*)^2$, $\widetilde{\sigma}^2$ and $cov(\ln h^*, \ln\widetilde{h})$, we obtain $\sigma_R^2 = \frac{\beta^2}{\vartheta^2 \cdot (\beta-\vartheta)^2}\Delta_\gamma^2 + \frac{\beta^2}{(\alpha+\varphi)^2 \cdot (\beta-\vartheta)^2}\Delta_a^2$, which is indeed equal to $\frac{\beta^2}{(\beta-\vartheta)^2}\widetilde{\sigma}^2$. Finally, from Proposition 3, we have that $p_{C2} = \Pr\left[\ln \overline{h} \leq \ln \widetilde{h}(i) \leq \ln h^*(i)\right]$. Accordingly, $p_{C2} = \Pr\left[\ln \widetilde{h}(i) \leq \ln h^*(i)\right] - \Pr\left[\ln \widetilde{h}(i) \leq \ln \overline{h}\right] = 1 - \Pr\left[\ln \widetilde{h}(i) > \ln h^*(i)\right] - \Pr\left[\ln \widetilde{h}(i) \leq \ln \overline{h}\right] = 1 - p_{C1} - p_{C2}$.

A3. Proof of Lemma 3

Assuming that $\ln \zeta(h_{-1})$ and X follow a bivariate normal distribution, we can apply the following result for truncated bivariate normal distributions (see Green, 2008, pp. 883):

$$E\left[\ln \zeta(h_{-1})/X > 0\right] = E\left[\ln \zeta(h_{-1})\right] + \frac{Cov(\ln \zeta(h_{-1}), X)}{\sigma_x}\kappa(\alpha_x), \tag{73}$$

where

$$Cov(\ln \zeta(h_{-1}), X) = \frac{\vartheta}{\beta}\Delta_h^2 + \frac{\vartheta}{\beta(\alpha+\varphi)^2}\Delta_a^2 + \frac{1}{\beta\vartheta}\Delta_\gamma^2 = \frac{\vartheta}{\beta}\sigma_x^2 \tag{74}$$

$$\kappa(\alpha_x) = \frac{\phi\left(\frac{-\mu_x}{\sigma_x}\right)}{1 - \Phi\left(\frac{-\mu_x}{\sigma_x}\right)} = \frac{\phi\left(\frac{-\mu_x}{\sigma_x}\right)}{p_0}. \tag{75}$$

Thus, the result in (39) is straighforward.

A4. Proof of Proposition 4

Deriving the expression in (41), we have $\frac{\partial E[\ln h_0]}{\partial \Delta_j^2} = \frac{\partial p_0}{\partial \Delta_j^2}\left[E \ln \zeta(h_{-1}) - \ln \overline{h}\right] + p_0 \frac{\partial E[\ln \zeta(h_{-1})]}{\partial \Delta_j^2} + \frac{\partial M(\mu_x, \sigma_x)}{\partial \Delta_j^2}$. From the condition in (40) and the definitions of G and J, we obtain $E \ln \zeta(h_{-1}) = \ln \overline{h} + \ln\left(\frac{1+\beta}{\beta}\right) + \frac{\vartheta}{\beta}\mu_x$. Then, substituting the latter expression into the former, we find that:

$$\frac{\partial E\left[\ln h_0\right]}{\partial \Delta_j^2} = \Phi'\left(\frac{\mu_x}{\sigma_x}\right)\frac{\partial\left(\frac{\mu_x}{\sigma_x}\right)}{\partial \Delta_j^2}\left[\ln\left(\frac{1+\beta}{\beta}\right) + \frac{\vartheta}{\beta}\mu_x\right] + \tag{76}$$

$$+\Phi\left(\frac{\mu_x}{\sigma_x}\right)\frac{\partial E\left[\ln \zeta(h_{-1})\right]}{\partial \Delta_j^2} + \frac{\partial M(\mu_x; \sigma_x)}{\partial \Delta_j^2} \tag{77}$$

for $j = a, h$ and γ. From (36) and (37) it can be shown that $\frac{\partial\left(\frac{\mu_x}{\sigma_x}\right)}{\partial \Delta_j^2} = -\frac{1}{2\varepsilon_j^2 \sigma_x}\left[\varepsilon_j + \frac{\mu_x}{\sigma_x^2}\right]$. Moreover, we know that $\Phi'\left(\frac{\mu_x}{\sigma_x}\right) = \phi\left(\frac{\mu_x}{\sigma_x}\right) = \phi\left(-\frac{\mu_x}{\sigma_x}\right)$.

As a result, we can divide both sides of (76) by $\phi\left(-\frac{\mu_x}{\sigma_x}\right)$ and obtain $\frac{1}{\phi\left(-\frac{\mu_x}{\sigma_x}\right)}\frac{\partial E[\ln h_0]}{\partial \Delta_j^2} = \frac{-1}{2\varepsilon_j^2\sigma_x}\left[\varepsilon_j + \frac{\mu_x}{\sigma_x^2}\right]\left[\ln\left(\frac{1+\beta}{\beta}\right) + \frac{\vartheta\mu_x}{\beta}\right] - \frac{\vartheta\cdot\lambda\left(-\frac{\mu_x}{\sigma_x}\right)}{2\beta\varepsilon_j} + \frac{\vartheta}{2\beta\varepsilon_j^2\sigma_x}\left[1+\varepsilon_j\mu_x + \frac{\mu_x^2}{\sigma_x^2}\right]$ where $\lambda(\cdot)$ is the Mill's ratio (Greene, 2008). Hence, $\frac{\partial E[\ln h_0]}{\partial \Delta_j^2} > 0$ iff $-\left[\varepsilon_j + \frac{\mu_x}{\sigma_x^2}\right]\left[\ln\left(\frac{1+\beta}{\beta}\right) + \frac{\vartheta\mu_x}{\beta}\right] - \frac{\vartheta\cdot\lambda\left(-\frac{\mu_x}{\sigma_x}\right)\varepsilon_j\sigma_x}{\beta} + \frac{\vartheta}{\beta}\left[1+\varepsilon_j\mu_x+\frac{\mu_x^2}{\sigma_x^2}\right] > 0$. Rearranging and simplifying terms, this condition can be rewritten as $\frac{\beta}{\vartheta}\left[\varepsilon_j + \frac{\mu_x}{\sigma_x^2}\right]\ln\left(\frac{1+\beta}{\beta}\right) + \lambda\left(-\frac{\mu_x}{\sigma_x}\right)\varepsilon_j\sigma_x - 1 < 0$. Thus, $\frac{\partial E[\ln h_0]}{\partial \Delta_j^2} > 0$ iff $\Pi_j(\mu_x,\sigma_x) = \frac{\mu_x}{\sigma_x^2} + \varepsilon_j - \frac{\vartheta}{\beta}\frac{1-\varepsilon_j\sigma_x\lambda\left(-\frac{\mu_x}{\sigma_x}\right)}{\ln\left(\frac{1+\beta}{\beta}\right)} < 0$. The intuition behind this ambiguity is the following. From above, it can be shown that $\partial E\left[\ln\zeta\left(h_{-1}\right)\right]/\partial\Delta_j^2 = -\frac{\vartheta}{2\beta\varepsilon_j}$ so the effect of Δ_j^2 on $E\left[\ln\zeta\left(h_{-1}\right)\right]$ is strictly positive for Δ_γ^2 and strictly negative for Δ_h^2 and Δ_a^2. However, we find that $\partial p_0/\partial\Delta_j^2 > 0$ iff $\frac{\mu_x}{\sigma_x^2} + \varepsilon_j < 0$ and $\partial M(\mu_x,\sigma_x)/\partial\Delta_j^2 > 0$ iff $\frac{\mu_x}{\sigma_x^2} + \varepsilon_j - \frac{1}{\mu_x} < 0$ for $j = h$, a and γ. Therefore, we see that the last two effects are ambiguous (they depend on the relative magnitude of μ_x and σ_x^2) and, consequently, the overall impact of Δ_j^2 ($j = h$, a and γ) on $E[\ln h_0]$ is ambiguous in general.

Regarding the properties of $\Pi_j(\mu_x,\sigma_x)$, we use the following well known characteristics of the Mill's ratio (Baricz, 2008): the Mill's ratio $\lambda(z)$ is C^2 on the $(-\infty,+\infty)$ space; it is strictly monotone (decreasing), $\lambda'(z) < 0$, and convex, $\lambda''(z) \geq 0$; and its limits are $\lim_{z\to-\infty}\lambda(z) = +\infty$ and $\lim_{z\to+\infty}\lambda(z) = 0$. From the first property the proof of *i)* is trivial. Moreover, we have that $\lim_{\mu_x\to-\infty}\Pi_j(\mu_x,\sigma_x) = -\infty$ for all $j = h$, a and γ, because $\lim_{\mu_x\to-\infty}\lambda(-\frac{\mu_x}{\sigma_x}) = 0$. On the other hand, $\lim_{\mu_x\to+\infty}\Pi_j(\mu_x,\sigma_x) = +\infty$ for $j = a$ and h, because in these cases $\varepsilon_j > 0$; however, $\varepsilon_j < 0$ for $j = \gamma$ so $\lim_{\mu_x\to+\infty}\Pi_j(\mu_x,\sigma_x) = \infty - \infty$. After applying the L'Hopital rule, we find that $\lim_{\mu_x\to+\infty}\Pi_j(\mu_x,\sigma_x) = -\infty$ for $j = \gamma$ (note that the Mill's ratio is convex while μ_x/σ_x is linear, therefore, the former converges faster to $-\infty$ than the latter to $+\infty$). To prove condition *iv)*, we calculate the derivative $\frac{\partial\Pi(\mu_x,\sigma_x)}{\partial\mu_x} = \frac{1}{\sigma_x^2} - \frac{\vartheta}{\beta}\frac{\varepsilon_j\lambda'\left(-\frac{\mu_x}{\sigma_x}\right)}{\ln\left(\frac{1+\beta}{\beta}\right)}$, which is positive for $j = a$ and h because their associated ε_a and ε_h are positive and $\lambda'\left(-\frac{\mu_x}{\sigma_x}\right) < 0$. On the contrary, $\frac{\partial\Pi(\mu_x,\sigma_x)}{\partial\mu_x}$ can be positive or negative for $j = \gamma$, because $\varepsilon_\gamma = -\vartheta$ so that the final result depends on the levels of μ_x and σ_x. Finally, for condition *v)*, the second derivative of

$\Pi_j(\cdot)$ is equal to $\frac{\vartheta}{\beta}\frac{\varepsilon_j}{\sigma_x}\frac{\lambda''\left(-\frac{\mu_x}{\sigma_x}\right)}{\ln\left(\frac{1+\beta}{\beta}\right)}$ for all j, which is strictly positive for $j = a$ and h, but strictly negative for $j = \gamma$. Hence, for $j = \gamma$, the solution of $\frac{\partial\Pi(\mu_x,\sigma_x)}{\partial\mu_x} = 0$ defines a global maximum.

A5. Proof of Corollary 1

Part A.

Condition *i)* is straighforward from the definition of $\Pi_j(\mu_x, \sigma_x)$ ($j = a$ and h) in Proposition 4 and the assumption that $\alpha + \varphi \leq 1$. To prove condition *ii)*, notice that $\Pi_j(0, \sigma_x) = \varepsilon_j \left[1 + \frac{\vartheta}{\beta\ln\left(\frac{1+\beta}{\beta}\right)}\left(\sqrt{\frac{\pi}{2}}\sigma_x - \frac{1}{\varepsilon_j}\right)\right]$ because $\lambda(0) = \sqrt{\pi/2}$ (Baricz, 2008). Moreover, there is a negative root, $\widehat{\mu}_x^j < 0$, if and only if $\Pi_j(0, \sigma_x) > 0$ because $\Pi_j(\cdot)$ is monotone increasing for $j = a$ and h. From condition *i)* we know that $\widehat{\mu}_x^h \leq \widehat{\mu}_x^a$ so that the condition for $j = a$ is sufficient to guarantee that both thresholds are negative. Setting $\varepsilon_a = \alpha + \varphi$, it is straighforward to show that $\Pi_a(0, \sigma_x) > 0$ iff $\sigma_x > \frac{\sqrt{2/\pi}}{(\alpha+\varphi)}\left[1 - \frac{\beta}{(1+\beta)}\ln\left(\frac{1+\beta}{\beta}\right)\frac{1}{\psi}\right]$. This condition is always satisfied if the term in parenthesis is negative; i.e., $\psi < \frac{\ln\left(\frac{1+\beta}{\beta}\right)}{(1+\beta)/\beta} < 1$.

Part B.

The existance of cases *i)* (no ambiguity) and *ii)* (ambiguity) is trivial from the properties of $\Pi_\gamma(\mu_x, \sigma_x)$ (see Proposition 4). With respect to the first case, the sufficient condition is obtained as follows. First, we set $\varepsilon_\gamma = -\vartheta$ and compute $\Pi_\gamma(\mu_x^{\gamma\max}, \sigma_x) = \frac{\mu_x^{\gamma\max}}{\sigma_x^2} - \vartheta - \frac{\vartheta}{\beta}\frac{1+\vartheta\sigma_x\lambda\left(-\frac{\mu_x^{\gamma\max}}{\sigma_x}\right)}{\ln\left(\frac{1+\beta}{\beta}\right)}$. From this result it is clear that $\mu_x^{\gamma\max} < 0$ guarantees that $\Pi_\gamma(\mu_x^{\gamma\max}, \sigma_x) < 0$. Next, we ellaborate on $\frac{\partial\Pi_\gamma(\mu_x,\sigma_x)}{\partial\mu_x} = \frac{1}{\sigma_x^2} + \frac{\vartheta^2}{\beta}\frac{\lambda'\left(-\frac{\mu_x}{\sigma_x}\right)}{\ln\left(\frac{1+\beta}{\beta}\right)} = 0$ to obtain the equivalent condition for $\mu_x^{\gamma\max}$. By using $\lambda'\left(-\frac{\mu_x}{\sigma_x}\right) = -\frac{\mu_x}{\sigma_x}\lambda\left(-\frac{\mu_x}{\sigma_x}\right) - 1$, we have $\mu_x^{\gamma\max} = \frac{\sigma_x}{\lambda\left(-\frac{\mu_x^{\gamma\max}}{\sigma_x}\right)}\left[\frac{1}{\sigma_x^2}\frac{\beta}{\vartheta^2}\ln\left(\frac{1+\beta}{\beta}\right) - 1\right]$. Hence, $\mu_x^{\gamma\max} < 0$ is equivalent to $\sigma_x^2 > \frac{\beta}{\vartheta^2}\ln\left(\frac{1+\beta}{\beta}\right)$. With respect to the second case, given the previous result, it is easy to see that $\mu_x^{\gamma\max} \geq 0$ is a necessary (but not sufficient) condition for this case. In addition, it is true that $\Pi_\gamma(0, \sigma_x) < 0$ for $\varepsilon_\gamma = -\vartheta$ so that whenever the roots of $\Pi_\gamma(\mu_x, \sigma_x)$ exist, they are always positive.

A6. Proof of Lemma 4

When there is no social mobility in the economy, we know that $E\left[\ln h_\infty^{NM}\right] = (1-p_0)\ln \overline{h} + p_0 E\left[\ln h^* / X > 0\right]$. Making use of the strategy in Lemma 3 for $t = 0$, we obtain

$$E\left[\ln h_\infty^{NM}\right] = (1-p_0)\ln \overline{h} + p_0 \cdot \mu^* + \frac{Cov(\ln h^*, X)}{\sigma_x}\phi\left(\frac{-\mu_x}{\sigma_x}\right), \qquad (78)$$

where $Cov(\ln h^*, X) = -\frac{1}{\beta-\vartheta}\frac{1}{\vartheta}\Delta_\gamma^2 - \frac{1}{\alpha+\varphi}\frac{(1+\beta)\cdot\psi}{\beta-\vartheta}\Delta_a^2$. Then, recalling from (37) that $\sigma_x^2 = \Delta_h^2 + \frac{1}{(\alpha+\varphi)^2}\Delta_a^2 + \frac{1}{\vartheta^2}\Delta_\gamma^2$ the result in (47) is straightforward.

A7. Proof of Proposition 5

We know by construction that $\mu^* = \frac{1}{\beta-\vartheta}(G+\vartheta J) - \frac{\vartheta}{\beta-\vartheta}E\ln h_{-1} + \frac{\vartheta}{\beta-\vartheta}\mu_x$, where $G+\vartheta J = \beta\cdot\ln\overline{h} + \ln\left(\frac{1+\beta}{\beta}\right)^\beta$ and $E\ln h_{-1} = \left(\ln\widehat{h} - \frac{\Delta_h^2}{2}\right)$. As a result, $\mu^* - \ln\overline{h} = \frac{1}{\beta-\vartheta}\left[\vartheta\left(\ln\overline{h} - E\ln h_{-1}\right) + \beta\ln\left(\frac{1+\beta}{\beta}\right) + \vartheta\mu_x\right]$ so

$$\begin{aligned} E\left[\ln h_\infty^{NM}\right] &= \ln\overline{h} + \Phi\left(\frac{\mu_x}{\sigma_x}\right)\frac{1}{\beta-\vartheta}\left[\beta\ln\left(\frac{1+\beta}{\beta}\right) + \vartheta\mu_x\right] + \frac{\vartheta}{\beta-\vartheta}\sigma_x\phi\left(-\frac{\mu_x}{\sigma x}\right) \qquad (79)\\ &\quad + \frac{\vartheta}{\beta-\vartheta}\Delta_h^2\left(\frac{\Phi\left(\frac{\mu_x}{\sigma_x}\right)}{2} - \frac{\phi\left(-\frac{\mu_x}{\sigma_x}\right)}{\sigma_x}\right) + \Phi\left(\frac{\mu_x}{\sigma_x}\right)\frac{\vartheta}{\beta-\vartheta}\left(\ln\overline{h} - \ln\widehat{h}_{-1}\right)\end{aligned}$$

Recalling that $E\left[\ln h_0\right] = \ln\overline{h} + \Phi\left(\frac{\mu_x}{\sigma_x}\right)\left[\ln\left(\frac{1+\beta}{\beta}\right) + \frac{\vartheta}{\beta}\mu_x\right] + \frac{\vartheta}{\beta}\sigma_x\phi\left(-\frac{\mu_x}{\sigma_x}\right)$, it is straightforward to see the paralelism between $E\left[\ln h_\infty^{NM}\right]$ and $E\left[\ln h_0\right]$. Let us divide $E\left[\ln h_\infty^{NM}\right]$ in two parts: $E\left[\ln h_\infty^{NM}\right]^{(part1)}$ that includes the first three terms in $E\left[\ln h_\infty^{NM}\right]$; and, $E\left[\ln h_\infty^{NM}\right]^{(part2)}$ that includes the last two terms in (79). Now, we derive these parts with respect to Δ_j^2, $j = a, h, \gamma$. After several manipulations, we obtain for the first part the following:

$$\Pi_j^{(1)} = 2\sigma_x\frac{(\beta-\vartheta)\varepsilon_j^2}{\vartheta\phi\left(-\frac{\mu_x}{\sigma_x}\right)}\frac{\partial E\left[\ln h_\infty^{NM}\right]^{(part1)}}{\partial\Delta_j^2} = -\left[\varepsilon_j + \frac{\mu_x}{\sigma_x^2}\right]\frac{\beta}{\vartheta}\ln\left(\frac{1+\beta}{\beta}\right) - \varepsilon_j\sigma_x\lambda\left(-\frac{\mu_x}{\sigma_x}\right) + 1. \qquad (80)$$

From the expression obtained for $\frac{\partial E[\ln h_0]}{\partial\Delta_j^2}$ in the proof of Proposition 4 we know that the last term is ambiguous. Again, after several manipulations, we have for the second part the following:

$$\Pi_j^{(2)} = 2\sigma_x \frac{(\beta - \vartheta)\,\varepsilon_j^2}{\vartheta\phi\left(\frac{\mu_x}{\sigma_x}\right)} \frac{\partial E\left[\ln h_\infty^{NM}\right]^{(part2)}}{\partial \Delta_{a,\gamma}^2} = \frac{\Delta_h^2}{\sigma_x^2} - \left(\varepsilon_j + \frac{\mu_x}{\sigma_x^2}\right)\left[\left(\ln \overline{h} - \ln \widehat{h}_{-1} + \frac{\Delta_h^2}{2}\right) + \frac{\mu_x}{\sigma_x^2}\Delta_h^2\right]. \tag{81}$$

As a result, $2\sigma_x \frac{(\beta-\vartheta)\varepsilon_j^2}{\vartheta\phi\left(\frac{\mu_x}{\sigma_x}\right)} \frac{\partial E\left[\ln h_\infty^{NM}\right]}{\partial \Delta_j^2} = \Pi_j^{(1)} + \Pi_j^{(2)}$ is positive if and only if $\left[\frac{\mu_x}{\sigma_x^2} + \varepsilon_j\right]\left[\frac{\beta}{\vartheta}\ln\left(\frac{1+\beta}{\beta}\right) + \left(\ln \overline{h} - \ln \widehat{h}_{-1} + \frac{\Delta_h^2}{2}\right) + \frac{\mu_x}{\sigma_x^2}\Delta_h^2\right] + \varepsilon_j \sigma_x \lambda\left(-\frac{\mu_x}{\sigma_x}\right) - 1 - \frac{\Delta_h^2}{\sigma_x^2} < 0$, i.e.,

$$\Pi_{j,\infty} = \frac{\mu_x}{\sigma_x^2} + \varepsilon_j - \frac{1 - \varepsilon_j \sigma_x \lambda\left(-\frac{\mu_x}{\sigma_x}\right) + \frac{\Delta_h^2}{\sigma_x^2}}{\left[\frac{\beta}{\vartheta}\ln\left(\frac{1+\beta}{\beta}\right) + \left(\ln \overline{h} - \ln \widehat{h}_{-1} + \frac{\Delta_h^2}{2}\right) + \frac{\mu_x}{\sigma_x^2}\Delta_h^2\right]} < 0. \tag{82}$$