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The Next Generation of the Penn World Table

Robert C. Feenstra

Robert Inklaar

Marcel Timmer

For additional information please contact:

Name: Robert Inklaar
Affiliation: University of Groningen

Email Address: r.c.inklaar@rug.nl

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by

Robert C. Feenstra
University of California, Davis and NBER

Robert Inklaar
University of Groningen

Marcel Timmer
University of Groningen

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Abstract

We describe the theory and practice of real GDP comparisons across countries and over time. Effective with version 8.0, the Penn World Table (PWT) will be taken over by the University of California, Davis and the University of Groningen, with continued input from Alan Heston at the University of Pennsylvania. Version 8.0 will expand on previous versions of PWT by including a distinction between real GDP on the expenditure side and on the output side, which differ by the terms of trade faced by countries. We also introduce a real GDP measure that allows for simultaneous comparisons of income and output across countries and over time. Preliminary results from PWT version 8.0 are discussed.

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1. Introduction

For over four decades, the Penn World Table (PWT) has been a standard source of data on real GDP across countries. Making use of prices collected across countries in benchmark years by the International Comparisons Program (ICP), and using these prices to construct purchasing-power-parity exchange rates, PWT converted GDP at national prices to a common currency – U.S. dollars – making them comparable across countries. The most recent version PWT 7.0 extends these data to 2009, using the benchmark ICP prices for 2005. Effective with version 8.0, the construction of these data will pass to the University of California, Davis and the University of Groningen, while retaining the PWT initials and with continued input from Alan Heston at the University of Pennsylvania.[1]

In this paper we describe the changes to the measurement of real GDP that will be introduced in this "next generation" of PWT. We begin in section 2 with a brief overview of the theory behind real GDP comparisons, including a new theorem. That discussion is intended to indicate the challenges in making multilateral comparisons of real GDP. Diewert (1999) and Van Veelen (2002) have argued that no multilateral measure of real GDP can satisfy all the axioms we might like, so there are tradeoffs involved with any construction of this concept. Our approach, which is an extension of the so-called Geary (1958)-Khamis (1970,1972) method,[2] has two major benefits: it is a natural extension of what is already done in PWT; and it will allow for an improved construction of real GDP across countries *and over time*, thereby addressing a criticism of PWT growth rates made by Johnson *et al* (2010).

In section 3 we describe the current PWT calculation of real GDP. As argued by Feenstra, Heston, Timmer and Deng (2009), real GDP as currently measured by PWT is closer to

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[1] The data will be available at pwt@ucdavis.edu, pwt@ugroningen.edu, and other websites.
[2] Balk (2008) provides a modern treatment of all the various multilateral comparison methods we employ here.

what is called "command-basis GDP" in the United States, or "real income" in the United Nations System of National Accounts. That is, it is a measure of real GDP that is intended to reflect the purchasing power of the economy rather than the production possibilities. Feenstra *et al* (2009) refer to this concept as "real GDP on the expenditure side," or $RGDP^e$. Countries that have strong terms of trade – meaning a higher than average prices for exports or lower than average prices for imports – will have higher $RGDP^e$ as a result. We contrast this with "real GDP on the output-side", or $RGDP^o$, which is intended to measure the production possibilities of an economy. Both concepts will be reported in PWT 8.0.

In section 4 we move to the measurement of real GDP over time. This is the area where there is perhaps the greatest confusion over concepts. The terms "real" in multilateral comparisons of real GDP refers to the use of a common unit, i.e. U.S. dollars, to compare GDP across countries. When the calculation is made in two different years, the values of real GDP obtained are not necessarily comparable. The term "current-price real GDP" refers to calculations across countries that are *not comparable over time*, because the comparison is based on reference prices of that particular year. In contrast, the term "constant-price real GDP" refers to calculations across countries that are *also comparable over time*, because an appropriate correction has been made for changing reference prices. In this case, we can readily compare GDP levels across countries at different points in time.

To summarize, the next generation of the PWT will have four series for real GDP: real GDP^e and real GDP^o based on current reference prices; and real GDP^e and real GDP^o using constant (or chained) reference prices. All of these series need a normalization, as discussed in section 5, and for that purpose we choose the United States, so that real GDP^o for the U.S. will match that in the national accounts in the reference year of 2005. But for other countries, trends in GDP (using national prices) will differ from trends in relative real GDP levels (using reference prices). A major reason for such differences is the changing consumption pattern as countries grow richer. In section 6 we compare summary statistics for these calculations and describe particular countries where the calculations differ most, such as India. The data discussed there will be made available in PWT 8.0, which we expect to release in 2012. In the concluding section we indicate how PWT 8.1 will improve on these calculations in several respects. The appendix contains the proof of our main theorem along with additional details on the methods used in our calculations.

2. Theory of Real Output Comparisons

Denote the final goods by $i = 1,\ldots,M$, which includes all consumption goods, investment, and government expenditures. These can be thought of as the goods for which prices are collected by the ICP in benchmark years, and we treat them as non-traded in the sense that they are purchased from local retail outlets. In addition, suppose there are $i = M+1,\ldots, M+N$ exported and imported goods: these can be thought of as the categories in the Standard International Trade Classification (SITC) or Harmonized System (HS), for example. A car that is imported and then sold domestically would appear twice, once as an import and again as a non-traded final consumer good, for example.

The free trade price vectors for exports and imports are p_j^x and p_j^m in country j, measured in the local currency. The domestic prices for those traded goods are $p_j^x + s_j$ and $p_j^m + t_j$, where s_j is the vector of export subsidies and t_j is the vector of import tariffs. The column vector of prices is then $P_j = (p_j, p_j^x + s_j, p_j^m + t_j)$, and we let $y_j \equiv (q_j, x_j, -m_j)$ denote the corresponding column vector of outputs and inputs. Then the revenue function for the economy is defined as:

$$r_j(P_j, v_j) \equiv \max_{q_{ij}, x_{ij}, m_{ij} \geq 0,} \left\{ P_j' y_j \;\middle|\; F_j(y_j, v_j) = 1 \right\}, \tag{1}$$

where $F_j(y_j, v_j)$ is a transformation function for each country, which depends on the vector v_j representing primary factor endowments, and also depends on the index of country *j* due to technological differences across countries.

We will distinguish the reference prices π_i for final goods, i =1,…,M, and two sets of reference prices π_i^x, π_i^m for exports and imported intermediate inputs, i =M+1,…,M+N. Denote the M+2N dimensional vector of reference prices by $\Pi = (\pi, \pi^x, \pi^m)$. We suppose that the country is engaged in *free trade* at these reference prices, and evaluate GDP on the output-side using the revenue function:

$$r_j(\Pi, v_j) \equiv \max_{q_{ij}, x_{ij}, m_{ij} \geq 0,} \left\{ \Pi' y_j \;\middle|\; F_j(y_j, v_j) = 1 \right\}. \tag{2}$$

Then real output can be compared across countries using the ratio of revenue functions:

$$\frac{r_j\,(\Pi, v_j)}{r_k\,(\Pi, v_k)}. \tag{3}$$

In practice, we should not expect to estimate the revenue functions across countries, as Neary (2004) does for the expenditure function, for example, because the revenue functions are indexed by the country *j* indicating technological differences between them. Even with a parsimonious specification of such technological differences it would be difficult to estimate revenue functions while pooling across countries. For this reason, researchers rely on indexes that can be used to approximate (3).

The simplest index that could be used to measure real output across countries is the ratio of quantities in the two evaluated at the prices of one country or the other. Gerschenkron (1951) was the first to document that there is a systematic relationship between real output evaluated at each country's prices, which is called the "Gerschenkron effect":

$$\left(\frac{P_j^{'} y_j}{P_j^{'} y_k}\right) < \left(\frac{P_k^{'} y_j}{P_k^{'} y_k}\right). \qquad (4)$$

This inequality states that real GDP is higher when measured with the prices of another country, or "the grass is greener on the other side." Another way to interpret (4) is that the Laspeyres quantity index (on the right) exceeds the Paasche quantity index (on the left). That inequality is familiar from consumer theory, where goods whose prices have fallen the most will have the greatest quantity increase, and since the Laspeyres quantity index uses the last-period prices it overstates the quantity increase. The same holds in the cross-country comparison in (4), despite the fact that this comparison is being made using *production* data rather than consumption data. In production theory, the upward bias of the Laspeyres index is reversed (since those goods whose prices have risen the most will have the greatest quantity increase). Nevertheless, Gerschenkron and many later studies confirm that the "demand-side bias" in (4) holds. One reason for this finding is that prices are determined in general equilibrium, and with similar tastes (demand curves) across countries but different technologies (shifting supply curves), the highest-priced goods in a country will indeed have less quantity, and the upward-bias of the Laspeyres index will follow.

Diewert (1983) refers to the measure of real output in (3) as a Samuelson-Swamy-Sato index. An alternative measure of real output, referred to as the Malmquist index, can be obtained by using the distance between the transformation functions. Specifically, Caves, Christensen and Diewert (1982a,b) define two measures of the difference in real output δ_j and δ_k as follows:

$$F_j(y_k \delta_k, v_j) = 1 \text{ and } F_k(y_j / \delta_j, v_k) = 1. \qquad (5)$$

To interpret the first of these conditions, it states that if we start with the observed output vector for country *k*, and inflate it by δ_k , then we obtain an output vector that is feasible to produce with the technology and endowments of country *j*. Likewise, the second condition states that by deflating the observed output vector in country *j* by δ_j , we obtain an output vector that is feasible to produce in country k.

The question then arises as to how to measure Malmquist distance factors defined by (5) without full knowledge of the transformation function. Caves, Christensen and Diewert (1982a,b) provide a powerful result by showing that if the transformation function is translog, then the geometric mean of δ_j and δ_k is measured by a Törnqvist quantity index of the outputs. We instead provide a result that does not depend on the form of the transformation function, but just relies on having observed output vectors be revenue-maximizing at the observed prices:

Theorem

Suppose that the outputs are revenue-maximizing and the Gerschenkron effect in (4) holds. Then there exists a reference price vector Π in-between P_j and P_k such that:

$$\delta_k \leq \frac{r_j(\Pi, v_j)}{r_k(\Pi, v_k)} = Q_{jk}^F \equiv \left[\left(\frac{P_j' y_j}{P_j' y_k}\right)\left(\frac{P_k' y_j}{P_k' y_k}\right)\right]^{0.5} \leq \delta_j . \tag{6}$$

Proof: See Appendix A.

This result says that computing a Fisher quantity index Q_{jk}^F between the countries will be a valid comparison of real output between them. Remarkably, it does not depend on the functional form of the revenue function but only on optimizing behavior.

In practice, an extension of the Fisher quantity index is often used to measure real output across countries, both by the World Bank in the *World Development Indicators* and by Eurostat and the OECD (2006), as we discuss in the next section. There is, however, an important problem that arises with its use: the reference prices Π used in the comparison are not made explicit. This problem arises in cross-country comparisons, where the implicit reference prices used to compare countries *j* and *k* are not the same as those use to compare countries *k* and *l*, and even more seriously in time-series comparisons: if the reference prices are not known, then it is impossible to correct for changes in these prices over time. Accordingly, in the next section we also discuss the leading alternative to the use of Fisher indexes, which is the Geary-Khamis (GK) method as used by PWT 7.0. This is an attractive alternative as it involves the explicit calculation of reference prices and we use this for PWT 8.0. Even with the reference prices made explicit, however, it is still challenging to adjust for changes in these over time, as we shall discuss in section 4.

3. Measurement of Real GDP in Practice

Notice that in place of the Fisher quantity index defined in (6), we can equivalently deflate the ratio of nominal GDPs by the Fisher price index, obtaining:

$$Q_{jk}^{F} = \left(\frac{P_j^{'} y_j}{P_k^{'} y_k}\right) \Bigg/ \left[\left(\frac{P_j^{'} y_j}{P_k^{'} y_j}\right)\left(\frac{P_j^{'} y_k}{P_k^{'} y_k}\right)\right]^{0.5} .$$

There are two modifications that have been made to the Fisher price index – the denominator of the above ratio – before it is used for multilateral comparisons. First, recall that the output vector $y_j \equiv (q_j, x_j, -m_j)$ consists of final goods, exports and (the negative of) imports. In practice, detailed data on exports and imports have never been incorporated into international comparisons of real GDP (which we shall rectify with PWT 8.0) and only the prices of final goods – as collected by the ICP – have been used. Accordingly, let the Fisher price index between country *j* and *k* computed over final goods be denoted by,

$$P_{jk}^{F} = \left[\left(\frac{p_j^{'} q_j}{p_k^{'} q_j}\right)\left(\frac{p_j^{'} q_k}{p_k^{'} q_k}\right)\right]^{0.5} .$$

If this formula is used to measure the price of final goods in country *j* relative to *k*, and then again in country *k* relative to *l*, and multiply these, we do not get the same numerical result as if we directly measured the price of final goods in country *j* relative to *l*. In other words, the Fisher price or quantity indexes are not transitive. To achieve this, the second modification made to the Fisher price index is to apply a transformation to obtain the so-called EKS system:[3]

$$P_{jk}^{EKS} \equiv \prod_{l=1}^{C} \left(P_{jl}^{F} P_{lk}^{F}\right)^{1/C} . \tag{7}$$

It is apparent that (7) is transitive by construction. Then to obtain real GDP (or *RGDP*) in country *j* relative to *k*, the World Bank and Eurostat-OECD use (7) to deflate nominal GDP in the two countries:

$$\frac{RGDP_j}{RGDP_k} = \left(\frac{GDP_j}{GDP_k}\right) / \, P_{jk}^{EKS} . \tag{8}$$

Notice that the EKS price index is computed using only the prices for final goods, i.e. consumption, investment and government expenditures, but it is used to deflate the ratio of nominal GDPs, which include the trade balance in both countries.

An improvement that we introduce in PWT8.0 is to explicitly account for price differences of exported and imported products. Feenstra *et al* (2009) first proposed to extend the GK system by incorporating prices for *N* export and import goods, p_{ij}^{x} and p_{ij}^{m}, $i = M+1,\ldots,M+N$. As

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[3] Or the GEKS system, after Gini (1931), Eltetö and Köves (1964), and Szulc (1964).

discussed in some detail in Appendix B, we aggregate detailed imports and exports prices to six groups, defined by the Broad Economic Categories (BEC) classification. These prices are used are used to obtain the reference prices as the weighted averages across countries:

$$\pi_i = \sum_{j=1}^{C} (p_{ij} / PPP_j^o) q_{ij} \Big/ \sum_{j=1}^{C} q_{ij} , \qquad i = 1,\ldots,M, \tag{9}$$

$$\pi_i^x = \sum_{j=1}^{C} (p_{ij}^x / PPP_j^o) x_{ij} \Big/ \sum_{j=1}^{C} x_{ij} , \qquad i = M+1,\ldots,M+N, \tag{10}$$

$$\pi_i^m = \sum_{j=1}^{C} (p_{ij}^m / PPP_j^o) m_{ij} \Big/ \sum_{j=1}^{C} m_{ij} , \qquad i = M+1,\ldots,M+N, \tag{11}$$

from which we obtain the PPP exchange rate for each country,

$$PPP_j^o = \frac{\text{GDP}_j}{\sum_{i=1}^{M} \pi_i q_{ij} + \sum_{i=M+1}^{M+N} (\pi_i^x x_{ij} - \pi_i^m m_{ij})}, \; j = 1,\ldots,C. \tag{12}$$

The system of equations in (9)-(12) is identified up to a normalization.

The concept of real GDP^o is obtained by multiplying the reference prices for final outputs π_i , exports π_i^x and imports π_i^m by their respective quantities, obtaining:

$$\begin{aligned} RGDP_j^o &\equiv \sum_{i=1}^{M} \pi_i q_{ij} + \sum_{i=M+1}^{M+N} (\pi_i^x x_{ij} - \pi_i^m m_{ij}) \\ &= \sum_{i=1}^{M} \pi_i q_{ij} + (X_j / PPP_j^x) - (M_j / PPP_j^m), \end{aligned} \tag{13}$$

where the equality follows by defining the PPPs of exports and imports, for $j = 1,\ldots,C$:

$$PPP_j^x = \sum_{i=M+1}^{M+N} p_{ij}^x x_{ij} / \sum_{i=M+1}^{M+N} \pi_i^x x_{ij} \text{ and } PPP_j^m = \sum_{i=M+1}^{M+N} p_{ij}^m m_{ij} / \sum_{i=M+1}^{M+N} \pi_i^m m_{ij} . \tag{14}$$

Thus, it is apparent that nominal exports and imports in (13) are *not* deflated by a PPP computed over final goods, but rather, are deflated by PPP exchange rates that are specific to exports and imports. Feenstra *et al* (2009) argue that this feature makes real GDP^o an appropriate measure of the real output of countries. We can also use the reference prices for final goods, π, to compute real GDP^{e} as a measure of the standard of living across countries:

$$RGDP_j^e \equiv \sum_{i=1}^{M} \pi_i \, q_{ij} + (X_j - M_j)/PPP_j^e \; = GDP_j / PPP_j^e \, , \tag{15}$$

where the equality follows because the numerator in (15) equals nominal absorption, $C_j+I_j+G_j$, and of course, $GDP_j = C_j+I_j+G_j+ X_j - M_j$. Notice that the trade balance $(X_j - M_j)$ in (15) is deflated by the PPP that is computed over final goods, without using any prices for exports or imports, much as we found as in EKS system in (8).

In practice, PWT 7.0 uses a blend of the EKS and the GK system. In particular, the cross-country price indexes for each of consumption, investment, and government expenditures are computed using the EKS method, and then these three categories of final expenditure are aggregated using the GK system (with M=3). This approach neatly combines the transitive property of the EKS indexes with the "adding up" property of the GK system, whereby the outputs of all countries are evaluated at the same reference prices. In PWT 8.0, we shall retain this convenient feature: the price indexes for consumption, investment, and government expenditures will be computed using the EKS method, and in addition, we shall also compute EKS price indexes of exports and imports and build these into the calculation of *real GDP on the output-side*, or real GDP^{o}.[4]

These two measures will differ due to the terms of trade faced by countries as reflected in the PPP exchange rates for exports and imports, or more precisely, in the "price level" for exports and imports, obtained by dividing PPP_j^x and PPP_j^m by the nominal exchange rate.

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[4] In the Appendix B we provide details on how the EKS price indexes for exports and imports are constructed.

Countries with high exports prices or low import prices will tend to have real GDP^e above real GDP^o due to their strong terms of trade. Extending PWT to include these two measures of real GDP is one of the contributions of version 8.0. The other contribution is to improve the calculation of real GDP over time, as discussed next.

4. Real GDP over time

The EKS method defined in (8) provides a comparison of real GDP at a point in time. But there is a difficulty in applying this formula to data over time: when a calculation of real GDP is made in two years, *there is no known method for comparing those EKS results over time because they have been made at different prices.* That is, the "real" in real GDP refers to the comparison across countries, but when these formulas are applied at different points in time, then they are called "current-price" real GDP, because they are made at prices of the current year. In contrast, a "constant price" real GDP comparison would make an appropriate adjustment for *changing reference prices*, and therefore be comparable across space and time. In this section we show how such a "constant price" comparison can be made in theory, and then extend our discussion of the GK system from the previous section to make this comparison in practice.

We now introduce a subscript t on all variables to indicate time. It turns out that we can apply our earlier theorem to obtain a consistent comparison of real GDP over space and time. Specifically, suppose that we start in a situation where we have two reference price vectors at two points in time, $\Pi_\tau = \left(\pi_t, \pi_\tau^x, \pi_\tau^m\right)$, $\tau = t\text{-}1, t$, using the reference prices from the GK system extended to include exports and imports. In order to also compare real output over time, it would be desirable to use a single vector Π and compute the ratios:

$$\frac{r_{jt}(\Pi, v_{jt})}{r_{jt-1}(\Pi, v_{jt-1})}, \qquad j = 1,\ldots,C, \tag{16}$$

for each country. Notice that the endowments in (16) can change over time, as well as the revenue function itself due to technological change.

We can apply the earlier theorem by treating the bilateral comparison as between country *j* using reference prices Π_{t-1} and Π_t in the two periods. The optimal outputs at these prices are denoted by $y^*_{j\tau} \equiv \partial r_{j\tau}(\Pi_\tau, v_{j\tau}) / \partial \Pi_\tau, \tau = t-1, t$. We assume that the time-series analogue of the Gerschenkron effect holds, which states that for country *j*:

$$\left(\frac{\Pi_t ' y^*_{jt}}{\Pi_t ' y^*_{jt-1}} \right) < \left(\frac{\Pi_{t-1} ' y^*_{jt}}{\Pi_{t-1} ' y^*_{jt-1}} \right). \tag{17}$$

Again, we interpret (17) as stating that the Laspeyres quantity index (on the right) exceeds the Paasche quantity index (on the left). This inequality cannot hold for the optimal quantities obtained from a revenue function *in the absence of* changes in endowments or technology, since in that case the goods with rising relative prices will also have rising quantities, and the left side of (17) will exceed the right. But as Gerschenkron (1951) found when comparing countries, the time-series evidence within a country also supports the idea that goods with the greatest increase in quantity (due to changing endowments or technology) have falling relative prices, so that (17) holds. Then an immediate application of the earlier theorem is:

Corollary

Suppose that the outputs are revenue-maximizing and the Gerschenkron effect in (17) holds. Then there will exist reference prices Π lying in-between Π_{t-1} and Π_t such that:

$$\frac{r_{jt}(\Pi,v_{jt})}{r_{jt-1}(\Pi,v_{jt-1})}=\left[\left(\frac{\Pi'_{jt-1}y^*_{jt}}{\Pi'_{jt-1}y^*_{jt-1}}\right)\left(\frac{\Pi'_{jt}y^*_{jt}}{\Pi'_{jt}y^*_{jt-1}}\right)\right]^{0.5}. \tag{18}$$

In words, this result states that we can take the geometric mean of the growth rates obtained at each vector of reference prices to obtain a constant-reference-price growth rate. To see the usefulness of this result, suppose that instead of using the *optimal* quantities in (18) we apply this formula using the *observed* quantities, obtaining:

$$\begin{aligned}\left[\left(\frac{\Pi'_{jt-1}y_{jt}}{\Pi'_{jt-1}y_{jt-1}}\right)\left(\frac{\Pi'_{jt}y_{jt}}{\Pi'_{jt}y_{jt-1}}\right)\right]^{0.5} &\equiv \left(\frac{RGDP^o_{jt}}{RGDP^o_{jt-1}}\right)\\ &=\left[\left(\frac{\sum_i \pi_{it-1}q_{ijt}+\pi^x_{it-1}x_{ijt}-\pi^m_{it-1}m_{ijt}}{\sum_i \pi_{it-1}q_{ijt-1}+\pi^x_{it-1}x_{ijt-1}-\pi^m_{it-1}m_{ijt-1}}\right)\left(\frac{\sum_i \pi_{it}q_{ijt}+\pi^x_{it}x_{ijt}-\pi^m_{it}m_{ijt}}{\sum_i \pi_{it}q_{ijt-1}+\pi^x_{it}x_{ijt-1}-\pi^m_{it}m_{ijt-1}}\right)\right]^{0.5}.\end{aligned} \tag{19}$$

Thus, the quantities of final goods, exports and imports change from *t*–1 to *t* in both ratios, and are evaluated using the reference prices from each period. The next generation of PWT will include a measure of “constant-price” real GDP^{o} using the growth rates defined as in (19).

In addition, the “constant price” growth rates of real GDP^{e} will be included, obtained by considering only the first *M* final consumption goods and using the reference prices π_{t-1} and π_t . Real GDP^{e} defined as in (15) is the current-price and constant-price measure in the final benchmark year *T*, and applied to other (benchmark or interpolated) years gives current-price real GDP^{e}, or $CGDP^e_t$. Then the growth rate of constant-price real GDP^{e} is:

$$\left[\left(\frac{\Pi^{e'}_{jt-1}y_{jt}}{\Pi^{e'}_{jt-1}y_{jt-1}}\right)\left(\frac{\Pi^{e'}_{jt}y_{jt}}{\Pi^{e'}_{jt}y_{jt-1}}\right)\right]^{0.5} \equiv \left(\frac{RGDP^{e}_{jt}}{RGDP^{e}_{jt-1}}\right)$$

$$= \left[\left(\frac{\sum_i \pi_{it-1}q_{ijt} + \frac{X_{jt}}{PPP^{e}_{jt-1}} - \frac{M_{jt}}{PPP^{e}_{jt-1}}}{\sum_i \pi_{it-1}q_{ijt-1} + \frac{X_{jt-1}}{PPP^{e}_{jt-1}} - \frac{M_{jt-1}}{PPP^{e}_{jt-1}}}\right)\left(\frac{\sum_i \pi_{it}q_{ijt} + \frac{X_{jt}}{PPP^{e}_{jt}} - \frac{M_{jt}}{PPP^{e}_{jt}}}{\sum_i \pi_{it}q_{ijt-1} + \frac{X_{jt-1}}{PPP^{e}_{jt}} - \frac{M_{jt-1}}{PPP^{e}_{jt}}}\right)\right]^{0.5}. \quad (20)$$

Notice that in (20) we deflate nominal exports and imports by PPP^{e}_{jt} each year to compute their shares and also their growth rates. We add a superscript *e* to Π to denote that this is only based on reference prices for final goods.

5. Data and Normalizations

Let us now be more explicit about how the calculations in (19) and (20) are made, and also clarify the normalizations that are used. For either real GDP^{e} or real GDP^{o}, the first step is to perform the EKS calculation for household consumption, investment, government consumption, exports and imports. If we denote p^{Z}_{jt} as the prices for the goods in expenditure category $Z = C, I, G, X, M$ in country *j* in benchmark year *t*, then the PPP for each category is computed using the following EKS procedure:

$$PPP^{Z}_{jt} = \prod_{l=1}^{C}\left(\left[\left(\frac{p^{Z}_{j}{}'q^{Z}_{j}}{p^{Z}_{l}{}'q^{Z}_{j}}\right)\left(\frac{p^{Z}_{j}{}'q^{Z}_{l}}{p^{Z}_{l}{}'q^{Z}_{l}}\right)\right]^{0.5}\left[\left(\frac{p^{Z}_{l}{}'q^{Z}_{l}}{p^{Z}_{k}{}'q^{Z}_{l}}\right)\left(\frac{p^{Z}_{l}{}'q^{Z}_{k}}{p^{Z}_{k}{}'q^{Z}_{k}}\right)\right]^{0.5}\right)^{1/C}. \quad (21)$$

Next, the GK calculation as in (9)-(12) is made for each benchmark year, using these PPPs with *M*=3 and *N*=6 and a normalization in each year, obtaining PPP^{o}_{jt}. Denoting the national accounts GDP deflator for the United States by $P^{y}_{us,t}$, we choose the normalization:

$$PPP^{o}_{us,t} = P^{y}_{us,t}, \quad (22)$$

which states that the PPP for GDP in the U.S. equals GDP deflator from national accounts. An implication of this normalization is that all PPPs are in local currencies relative to the U.S. dollar in 2005.

The second step is to interpolate these aggregate prices for years in-between the benchmarks, or extrapolate for years before the first benchmark and after the last benchmark. The interpolation and extrapolation is done using the national-accounts price levels for the components of GDP, P_{jt}^{Z}, for $Z = C, I, G, X, M$. The results are price levels for C, I, G, X and M now for every year t=1,…,T, and every country in the sample.

To illustrate the interpolation and extrapolation procedure, consider the case of India, which was part of the 1970, 1975, 1980, 1985 and 2005 ICP benchmarks, and for illustration purposes focus on the household consumption PPPs. Based on equation (22), we estimate PPPs for each of the benchmark years, normalized to $P_{us,t}^{y}$ in every year. In the years after 2005, we have no further benchmarks, so we extrapolate forward using the trend in consumption prices from the national accounts:

$$PPP_{IND,2006}^{C} = PPP_{IND,2005}^{C} \times \frac{P_{IND,2006}^{C}}{P_{IND,2005}^{C}} \qquad (23)$$

The 2005 consumption PPP for India is 19.1 rupee/2005 US dollar. Between 2005 and 2010, Indian consumption prices rose by 41 percent, so the extrapolated 2010 consumption PPP is 26.8 rupee/US dollar. For years in-between benchmarks, we interpolate using information on consumption prices to inform the trend. So, for example, the consumption PPP for year t between the 1975 and 1980 benchmarks would be calculated as:

$$PPP_{IND,t}^{C} = P_{IND,t}^{C}\left[\left(\frac{1980-t}{1980-1975}\right)\frac{PPP_{IND,1975}^{C}}{P_{IND,1975}^{C}} + \left(\frac{t-1975}{1980-1975}\right)\frac{PPP_{IND,1980}^{C}}{P_{IND,1980}^{C}}\right]. \qquad (24)$$

Between 1975 and 1980, the consumption PPP increased from 1.43 to 2.80 rupee/2005 US dollar, or 96 percent, while over the same period, National Accounts consumption prices increased by 37 percent. However, most of the consumption price increase was concentrated in the last years of this period, with inflation of 12 percent in both 1979 and 1980. Between 1975 and 1977, prices increased by a more modest 7.6 percent, so applying equation (24) shows us that the 1977 consumption PPP is 26 percent higher than the 1975 PPP, compared with the 96 percent overall increase between 1975 and 1980.

This approach to interpolation and extrapolation is similar in spirit to the approach of Rao *et al* (2010), who also discuss a method for estimating PPPs for a full set of years and countries using benchmark PPPs and National Accounts deflators. The key distinction is that we always force the PPP series to be equal to the benchmark estimates, while this is a special case of Rao *et al* (2010); see also Hill (2004).

The third step in the calculation is to apply the extended-GK calculation as in (9)–(12) to the price levels for C, I, G, X and M. See Feenstra and Romalis (2012) for the estimation of quality-adjusted trade prices and Appendix B for further details.

There are several reasons why the growth calculations in (20) and (21) differ from what has been reported in PWT to date. Most obviously, PWT did not report a series for real GDP^o. More importantly for our discussion in this section, PWT has *never interpolated the reference prices for C, I and G between ICP benchmark years, to obtain a time-series consistent with these benchmarks.* Rather, PWT has *extrapolated from each new benchmark year using the national-accounts real growth rates of C, I and G.* Clearly, these two approaches will differ whenever the price trends indicated by the national accounts data differ from the price trends indicated by ICP benchmarks. We will find in the next section that this occurs for some very important countries, and India in particular: the early ICP benchmarks for India indicated that prices are lower, so that real consumption is higher, than would be inferred by just looking at national-accounts inflation of prices over time. One important reason for this, as discussed by Deaton (2012), is the difference in budget shares between poorer and richer countries; see the next section for more details.

6. Results from PWT 8.0

As discussed in the introduction, we will present four series four series of real GDP, all of which will be different from the series found in the National Accounts. For a first overview of the results, Table 1 shows price levels and real GDP per capita for $RGDP^e$ and $RGDP^o$ in 2005 for the G-20 economies. This table mirrors Tables 1 and 2 from Feenstra et al. (2009), but in contrast, the export and import prices shown here are corrected for quality differences. The column labeled '$RGDP^e$' shows the PPP based on equation (15) divided by the U.S. dollar exchange rate, thereby obtaining a "price level"; '$RGDP^o$' shows the PPP based on (13), again divided by the U.S. exchange rate. Also shown is 'Terms of trade', which is the price level for exports divided by the price level for imports, shown in the subsequent two columns. The reference prices for exports and imports are relatively high, so all the price levels are relatively low. There are some notable differences, with particularly low terms of trade (price of exports divided by price of imports) in China and much higher terms of trade in the rich countries. Also noteworthy is that relative import prices are more similar than relative export prices.

The last columns compare real GDP per capita based on $RGDP^e$ price levels and $RGDP^o$ price levels. For this group of countries, these two concepts are fairly close. In other words, 'productive capacity' (measured by $RGDP^o$ per capita) is similar to 'consumptive capacity' (measured by $RGDP^e$ per capita). In Russia and Saudi Arabia these gaps are substantial though.

Table 1:

Price levels and real GDP per capita in 2005 of G-20 economies

Country	**Price level**					**RGDP per capita**		
	$RGDP^e$	$RGDP^o$	Terms of trade	Exports	Imports	$RGDP^e$	$RGDP^o$	Diff (%)
Argentina	0.48	0.48	0.94	0.70	0.75	9961	9821	1.4
Australia	1.05	1.03	0.89	0.67	0.76	35784	36387	-1.7
Brazil	0.64	0.64	1.00	0.74	0.74	7443	7404	0.5
Canada	0.99	0.98	1.01	0.75	0.74	35420	35749	-0.9
China	0.33	0.34	0.94	0.65	0.69	5349	5279	1.3
France	1.17	1.18	1.01	0.77	0.76	29064	28699	1.3
Germany	1.10	1.10	1.03	0.78	0.75	30398	30561	-0.5
India	0.30	0.30	1.00	0.71	0.71	2435	2503	-2.7
Indonesia	0.38	0.39	0.89	0.63	0.71	3386	3302	2.5
Italy	1.08	1.08	1.00	0.75	0.75	28006	27934	0.3
Japan	1.18	1.17	1.01	0.76	0.75	30645	30869	-0.7
Korea	0.66	0.66	1.00	0.73	0.73	11999	12007	-0.1
Mexico	0.75	0.75	1.01	0.73	0.72	24084	24029	0.2
Russia	0.43	0.46	0.90	0.67	0.75	12378	11679	6.0
Saudi Arabia	0.66	0.57	0.73	0.54	0.74	19840	23200	-14.5
South Africa	0.75	0.74	0.96	0.71	0.73	6927	6974	-0.7
Turkey	0.65	0.63	0.94	0.67	0.71	10983	11172	-1.7
UK	1.17	1.20	0.97	0.74	0.76	32314	31540	2.5
US	0.98	1.00	1.01	0.76	0.75	43243	42330	2.2

To provide a broader perspective on these differences, Figure 1 plots GDP^e relative to GDP^o for all 167 countries in PWT8.0. The majority of countries are in the range of 0.8 to 1, with an average (absolute) difference of about 7 percent. Since the countries that are not in Table 1 are generally smaller and more open, it is not surprising to see larger differences. Note that even though the majority of countries is below 1, the sum of GDP^e and GDP^o across all 167 countries is nearly identical.

Figure 1:
GDP^e versus GDP^o and GDP^o per capita in 2005

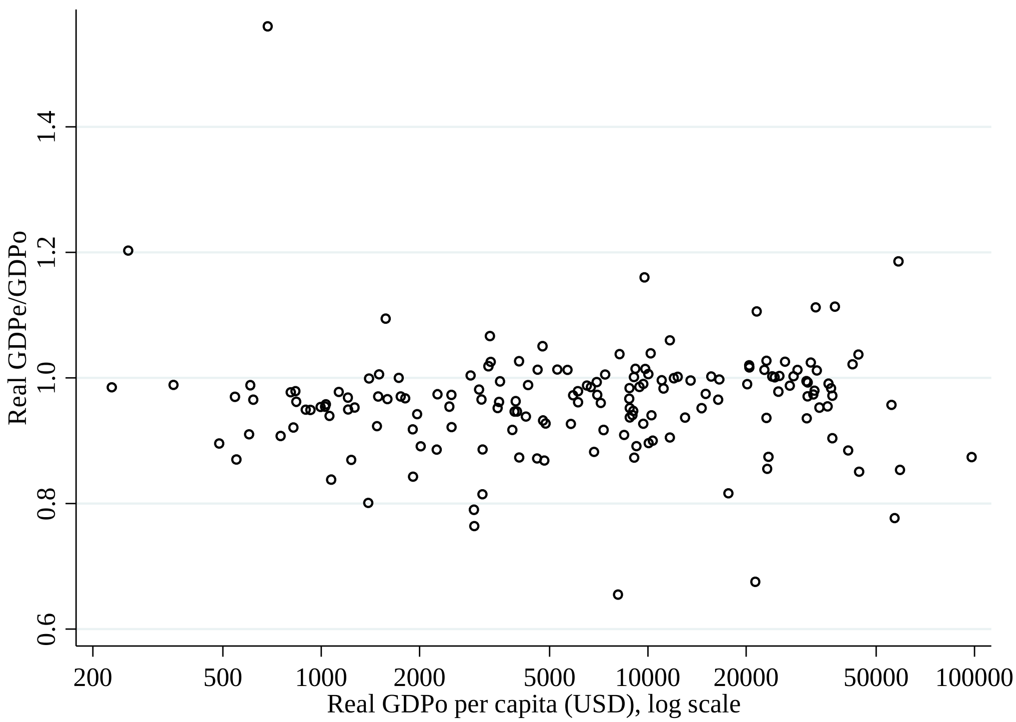

Figure 2 shows the terms of trade against GDP^o per capita for all 167 countries in PWT8.0. This shows how most countries are within a fairly small range between 0.9 and 1. There is no significant relationship between terms of trade and RGDP^o per capita (or with RGDP^e per capita). Figure 3 looks at the price levels for RGDP^e and RGDP^o, plotted against the log of RGDP^o per capita.[5] For individual countries, the differences are frequently sizeable, about 5 percent on average, but the overall cross-country pattern of price levels is comparable. This is reflected in the similarity of the relationship between price levels and RGDP^o per capita levels. Note that three countries (Bermuda, El Salvador and Zimbabwe) have been omitted from this figure because their price levels are so high that these would have distorted the overall pattern.

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[5] Both price levels are compared with RGDP^o per capita to facilitate the comparison of price levels for individual countries. The figure would have looked qualitatively similar if the price level of RGDP^e had been plotted against RGDP^e per capita.

Figure 2:
Terms of trade and GDP^{o} per capita in 2005

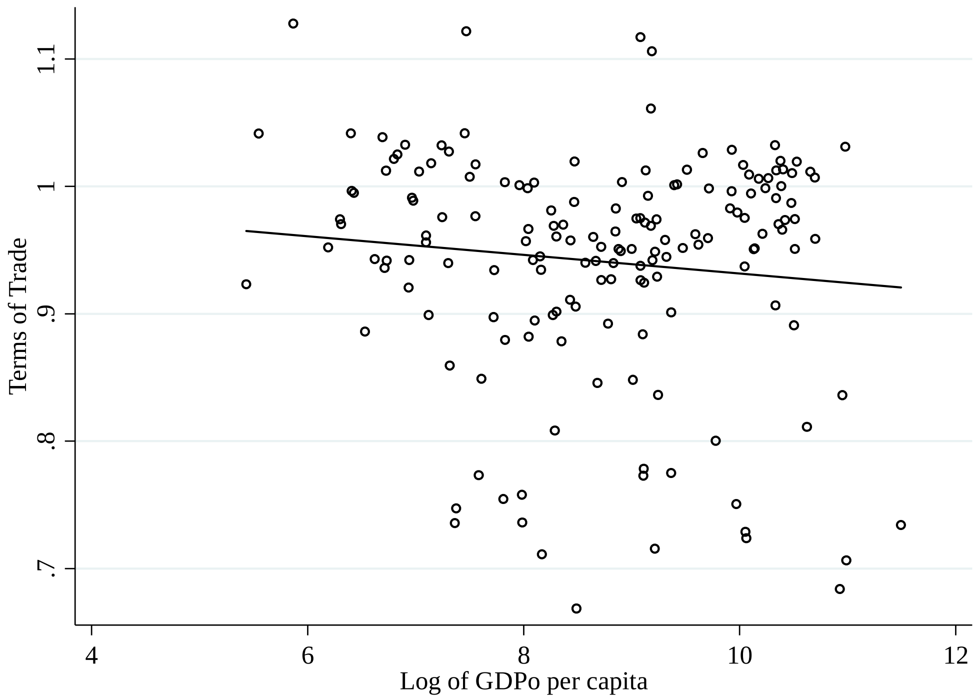

Figure 3:
RGDP^{e} and RGDP^{o} price levels and RGDP^{o} per capita in 2005

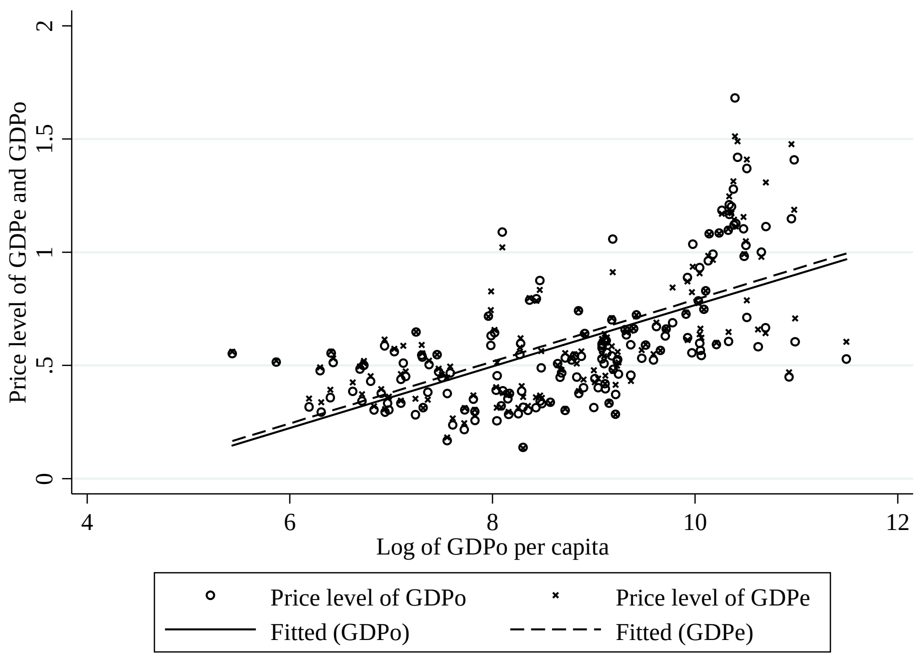

Figure 4:
PPP and National Accounts price for consumption in India, 1970-2010 (1970=1)

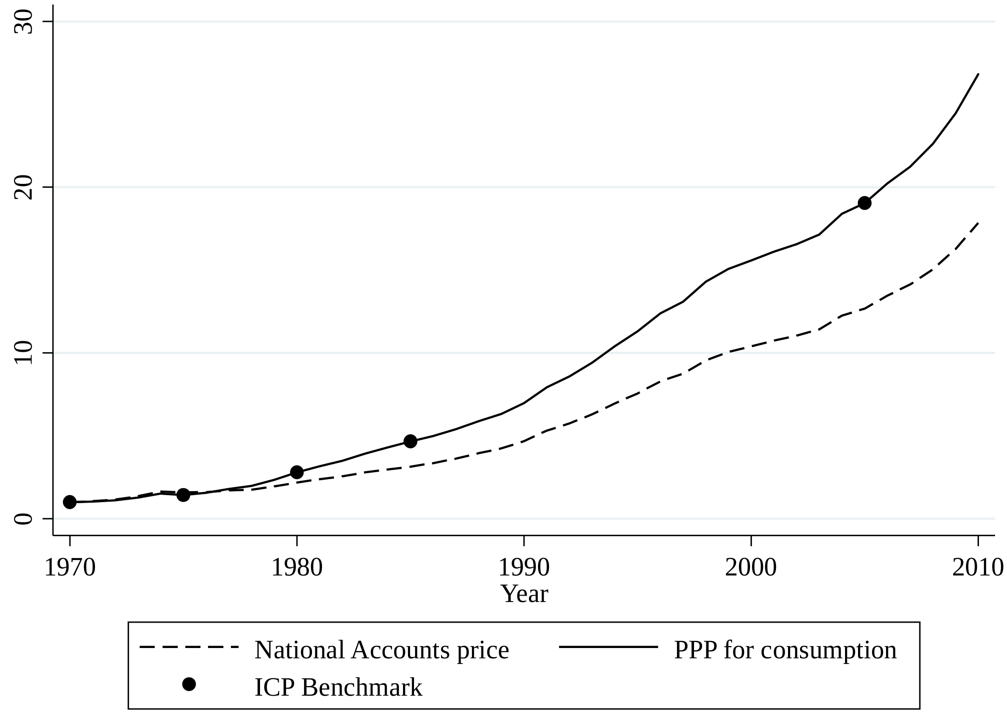

Figure 4 moves to a comparison of PPPs over time, focusing on the case of India where the difference between National Accounts prices and the PPP for household consumption is particularly marked. Between the 1970 and the 1975 benchmark, national consumption prices increased at approximately the same rate as the consumption PPP. A big gap opened up between 1975 and 1985: in 1985 the consumption PPP was more than three times higher than in 1975, while consumption prices from the national accounts were only twice as high. The implication is that ICP prices for consumption were *lower* in 1975 (and earlier) that indicated by starting in 1985 and back-casting from the national accounts prices. Having lower prices in 1975 means higher real income, and therefore lower future growth. The difference in trend between the two price series narrowed again in future years: between 1985 and 2005 the consumption PPP and consumption prices both increased by a factor of four. So the lower ICP-based growth rates for

India in Figure 4 is mainly due to the ICP benchmark of 1975, which showed relatively low prices.

As Deaton (2012) discusses in more detail, some of this pattern can be understood by considering that the overall consumption PPP is calculated using weights of both rich and poor countries. As a result, the change in PPP over time will reflect both price changes of goods and differences in budget shares. Poorer countries tend to consume relatively more food and other traded products, which tend to be relatively expensive, and fewer (non-traded) services, which tend to be less expensive. As countries grow richer, the price of services will tend to increase more rapidly so as a result, the prices of goods that are heavily consumed in poorer countries rise relatively less. This systematic difference in budget shares and price changes can explain at least some of the pattern shown in Figure 4. There will also be differences in reliability and representativeness of different PPP benchmarks. This also means that the 'right' real GDP series will depend on the question at hand. For a comparison of real GDP growth over time, National Accounts may well be most appropriate. Our GDP^e ICP series, in contrast, is best used for a comparison of the real GDP level in country A in year 1 to country B in year 2, for instance in convergence analysis. We hope that providing all four series in PWT 8.0 will stimulate additional research questions that have not yet been considered.

7. Conclusions

From its inception, the ICP only collected the prices of final products – for consumption, investment and the government – across countries. It was prohibitively expensive to further collect comparable prices for the whole range of industrial and intermediate inputs used in economies, many of which are also traded. This limitation means that the PWT calculations based on ICP prices are best thought of as representing the standard of living of countries rather than their production possibilities. Feenstra *et al* (2009) argued that a measure of the productive capacity of countries could be obtained by combining the ICP data with prices for exports and imports. These two approaches lead to real GDP on the expenditure-side and real GDP on the output-side, respectively, both of which will be included in PWT version 8.0.

The other contribution of PWT 8.0 will be to improve upon the growth of real GDP previously reported in PWT, which was based on National Accounts data. Johnson *et al* (2009) criticized that growth rate as being dependent on the benchmark year of ICP data, and thereby dependent on the version of PWT being used. That technical problem is resolved here in the national-accounts based growth rates. In addition, a second growth rate that is based on *multiple* ICP benchmarks will be reported in PWT 8.0. Under this second approach, countries can grow faster or slower than in their national accounts. India, for example, is found to have a higher standard of living in its 1975 ICP benchmark than predicted from the 1985 benchmark and back-casting using the growth of national accounts prices. It follows that its ICP-based growth from 1975 onwards is correspondingly reduced.

Our plans for revision to PWT beyond version 8.0 is to re-introduce measures of relative capital input as well as introduce measures of relative productivity. Many researchers have been faced with the absence of established and harmonized data on capital input and have had to estimate series based on the available total investment data (see e.g. Caselli, 2005). The capital data will distinguish between different assets to account for compositional changes over time and differences across countries. The capital data will be combined with labor input data; recent data on educational attainment (Barro and Lee, 2010); and new estimates of the labor share in GDP to come up with relative productivity measures. Especially in combination with the GDP^o series introduced in this paper, this will provide a new starting point for cross-country research.

Appendix A

Proof of Theorem 1:

Because the outputs y_k are feasible for country *k*, but not optimal at the prices P_j, it follows that $r_k(P_j, v_k) \geq P_j^{'} y_k$. This establishes the first inequality below and the second is established similarly:

$$\frac{r_j(P_j, v_j)}{r_k(P_j, v_k)} \leq \left(\frac{P_j^{'} y_j}{P_j^{'} y_k} \right) \quad \text{and} \quad \frac{r_j(P_k, v_j)}{r_k(P_k, v_k)} \geq \left(\frac{P_k^{'} y_j}{P_k^{'} y_k} \right),$$

Using the Gerschenkron effect it follows that:

$$\frac{r_j(P_j, v_j)}{r_k(P_j, v_k)} \leq \left(\frac{P_j^{'} y_j}{P_j^{'} y_k} \right) < \left[\left(\frac{P_j^{'} y_j}{P_j^{'} y_k} \right) \left(\frac{P_k^{'} y_j}{P_k^{'} y_k} \right) \right]^{0.5} < \left(\frac{P_k^{'} y_j}{P_k^{'} y_k} \right) \leq \frac{r_j(P_k, v_j)}{r_k(P_k, v_k)}. \qquad \text{(A1)}$$

Now consider the first condition in (5), which states that $y_k \delta_k$ is feasible using the technology of county *j*. Because these outputs are not optimally chosen for the prices P_j it is immediate that:

$$P_j^{'}(y_k \delta_k) \leq P_j^{'} y_j .$$

It follows that $\delta_k \leq P_j^{'} y_j / P_j^{'} y_k$. Then consider the second condition in (5), which states that (y_j / δ_j) is feasible using the technology of county *k*. Because these outputs are not optimally chosen for the prices P_k it is immediate that:

$$P_k^{'}(y_j / \delta_j) \leq P_k^{'} y_k .$$

It follows that $\delta_j \geq P_k^{'} y_j / P_k^{'} y_k$. Using the Gerschenkron effect again we have therefore shown:

$$\delta_k \leq \left(\frac{P_j^{'} y_j}{P_j^{'} y_k} \right) < \left[\left(\frac{P_j^{'} y_j}{P_j^{'} y_k} \right) \left(\frac{P_k^{'} y_j}{P_k^{'} y_k} \right) \right]^{0.5} < \left(\frac{P_k^{'} y_j}{P_k^{'} y_k} \right) \leq \delta_j . \qquad \text{(A2)}$$

Using (A1) and (A2), and by continuity of the function $r_j(\Pi, v_j) / r_k(\Pi, v_k)$, there exists a value for Π lying in-between P_j and P_k such that (6) holds. QED

Appendix B   EKS indexes for exports and imports

[to be written]

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