Session 7D: Well-Being Over Time I
Time: Friday, August 10, 2012 AM

Paper Prepared for the 32nd General Conference of
The International Association for Research in Income and Wealth

Boston, USA, August 5-11, 2012

Intertemporal Poverty Comparisons

Florent Bresson and Jean-Yves Duclos

For additional information please contact:

Name: Florent Bresson
Affiliation: Université d'Orléans, France

Email Address: florent.bresson@unice.fr

Intertemporal poverty comparisons

Florent Bresson[*] and Jean-Yves Duclos[†]

July 19, 2012
version 0.16.2α[‡]

Abstract

The paper deals with poverty orderings when the value of multidimensional attributes can be compared on a same scale, such as with income of different types or from different members of the same household. The dominance criteria extend the power of earlier multidimensional dominance tests (see Duclos, Sahn and Younger 2006) by making (reasonable) assumptions on the relative marginal contributions of each dimensional attribute to poverty. The paper focuses on an important special case of this, that is comparisons of poverty over time. In contrast to earlier work on intertemporal poverty comparisons, this paper proposes procedures to check for whether poverty comparisons can be made robust to wide classes of aggregation procedures and to broad areas of intertemporal poverty frontiers.

Keywords: Poverty comparisons, intertemporal well-being, household inequalities, stochastic dominance, multidimensional poverty.
JEL Classification: D63, I3.

This research was partly funded by Canada's SSHRC and by Québec's FQRSC. It was also carried out with support from the Poverty and Economic Policy (PEP) Research Network, which is financed by the Government of Canada through the International Development Research Centre (IDRC) and the Canadian International Development Agency (CIDA), and by the Australian Agency for International Development (AusAID).

---

[*] Laboratoire d'Économie d'Orléans (LÉO), Université d'Orléans; email: florent.bresson@unice.fr.
[†] Département d'économique and CIRPÉE, Université Laval, Canada, G1V 0A6; email: jyves@ecn.ulaval.ca.
[‡] Please note that this version is an alpha release. It is incomplete and may contain notable mistakes and typos. As a consequence, it is likely to be slightly modified in the future.

1 Introduction

This paper deals with the problem of making general comparisons of well-being when well-being is measured in multiple dimensions. We note at the outset that much of the literature on the measurement of well-being incorporates multiple dimensional indicators by adding them up, such as when food and non-food expenditures are aggregated to compute total expenditures and assess monetary poverty — essentially returning to a univariate analysis. In some cases, these procedures may be perfectly appropriate. In other cases, however, it could be that the specific aggregation rules used to sum up the dimensions may be deemed somewhat arbitrary or objectionable, especially when the dimensions cannot be considered evidently comparable or perfectly substitutable in generating overall well-being. This then leaves open the possibility that two equally admissible rules for aggregating across several dimensions of well-being could lead to contradictory rankings of well-being and/or results for policy guidance.

One way to address this problem is through the use of multidimensional dominance procedures, as found in Atkinson and Bourguignon (1982), Bourguignon (1989), or Duclos, Sahn and Younger (2006). These are indeed useful procedures that make relatively few assumptions on the structure of the procedures used to measure and compare well-being. Their first-order multidimensional dominance comparisons suppose, for instance, that overall well-being should increase with dimensional well-being, but that the importance of these increases cannot be ranked across dimensions. Such comparisons do not impose any assumption of cardinality on the dimensional indicators of well-being. Because of this, they can generate very robust multidimensional comparisons of well-being from a normative point of view.

These weak assumptions come often, however, at the cost of a limited power to order distributions of multidimensional well-being. It would seem that they could be strengthened in several settings. One such setting is when the dimensional indicators have values that are ordinally comparable. Examples include the measurement of household poverty, using the incomes of the members of the same household as dimensions, but without assuming perfect income pooling; the measurement of child well-being, using the health or the nutritional status of children of the same household as dimensions, but without assuming that there is perfect substitutability of such status across the children; or the measurement of household education, using the education of members of the same household as dimensions, but again without assuming that for measurement purposes we can impose perfect substitutability of educational achievements across members of the same household. We build in this paper on the natural cardinality of multi-period incomes, which makes it possible to compare them in more specific ways than has been done until now.

Thus, although the methods we develop have broader applicability, the paper focuses on intertemporal poverty comparisons, that is comparisons of poverty over different time periods.[1]

In contrast to some of the earlier work,[2] the paper's objective is to develop procedures for

[1] Tough intertemporal poverty is the most widely used name for that concept, it is sometime also called "longitudinal poverty" (Busetta, Mendola and Milito, 2011, Busetta and Mendola, 2012) or "lifetime poverty" (Hoy, Thompson and Zheng, forthcoming) in the related literature.

[2] See for instance Foster (2007), Calvo and Dercon (2009), Hoy et al. (forthcoming), Duclos, Araar and Giles (2010), and Bossert, Chakravarty and d'Ambrosio (2011), but analogously to Hoy and Zheng (2008), though within

checking for whether intertemporal poverty comparisons are robust to aggregation procedures and to choices of multi-period poverty frontiers. Intertemporal poverty comparisons can then be made "poverty-measure robust," namely, valid for broad classes of aggregation rules across individuals and also for broad classes of aggregation rules across time. The comparisons can then also be made "poverty-line robust," in the sense of being valid for *any* temporal poverty frontier over broad areas. Given the difficulty involved in choosing poverty frontiers and poverty indices, and given the frequent sensitivity of poverty comparisons to these choices, this would appear to be a potentially useful contribution.

One of the first conceptual challenges of temporal poverty analysis is deciding who is "time poor." Measuring well-being across two time periods, say, a person can be considered intertemporally poor if her income falls below an income poverty line in *both* periods or in *either* period. This can be defined respectively as *intersection* and *union* definitions of temporal poverty. The procedures that we develop are valid for both definitions — and also valid for any choice of intermediate definitions for which the poverty line at one time period is a function of well-being at the other.

The paper also considers the role of mobility in the measurement of intertemporal poverty, both across time and across individuals. With the increased availability of longitudinal data sets, it is now well known that there are often significant movements in and out of poverty, as well as within poverty itself. Such income mobility has at least two welfare impacts.[3] The first is to make the distribution of "permanent" incomes across individuals more equal than the distribution of temporal incomes. Measures of poverty that are averse to inequality across individuals will therefore tend to be lower when based on permanent incomes. Mobility also introduces temporal variability. If individuals would prefer their incomes to be distributed as equally as possible across time (because they are risk averse or because they have limited access to credit and hence cannot smooth their consumption), then income mobility will also decrease well-being and thus increase poverty.

The rest of the paper is organized in the following manner. The next section elaborates on Duclos et al.'s (2006) multidimensional dominance criteria so as to extend the power of their procedures without going beyond first-order dominance checks. Increases in the power of dominance tests are traditionally obtained by emphasizing the importance of attribute-specific inequality across individuals. Section 2 uses instead across-attribute symmetry and asymmetry properties and introduces assumptions on how permutations of multi-period income profiles should affect poverty. Since it is often supposed that individuals prefer smoothed income patterns, Section 3 explicitly takes into account intertemporal inequalities. The links between the corresponding classes of poverty indices and the classes described in Section 2 are highlighted in Section 4. The results are illustrated using EU-SILC data in Section 5. Section 6 concludes.

a rather different — time-additive — framework.

[3] See, among some of the recent work, Atkinson, Cantillon, Marlier and Nolan (2002), Chaudhuri, Jalan and Suryahadi (2002), Ligon and Schechter (2003), Cruces and Wodon (2003), Bourguignon, Goh and Kim (2004), Christiaensen and Subbarao (2004), and Kamanou and Morduch (2004).

2 Intertemporal poverty

Let overall well-being be a function of two indicators, x_1 and x_2 , and be given by $\lambda(x_1, x_2)$.[4] This function is a member of Λ, defined as the set of continuous and non-decreasing functions of x_1 and x_2. For our purposes, we will typically think of x_t as income at time t; the vector (x_1, x_2) is called an income profile. For instance, x_1 may denote an individual's income during his working life, while x_2 could be his income when retired. Without loss of generality, we assume that incomes are defined on the set of positive real numbers, so that $\lambda : \Re^2_+ \to \Re$.

Similarly to Duclos et al. (2006), we assume that an unknown poverty frontier separates the poor from the rich. We can think of this frontier as a set of points at which the well-being of an individual is precisely equal to a "poverty level" of well-being, and below which individuals are in poverty. This frontier is assumed to be defined implicitly by a locus of the form $\lambda(x_1, x_2) = 0$, and is analogous to the usual downward-sloping indifference curves in the (x_1, x_2) space. Intertemporal poverty is then defined by states in which $\lambda(x_1, x_2) \leq 0$, and the poverty domain is consequently obtained as:

$$\Gamma(\lambda) := \left\{(x_1, x_2) \in \Re^2_+ | \lambda(x_1, x_2) \leq 0\right\}. \tag{1}$$

Let the joint cumulative distribution function of x_1 and x_2 be denoted by $F(x_1, x_2)$. For analytical simplicity, we focus on classes of additive bidimensional poverty indices, which are the kernels of broader classes of subgroup-consistent bidimensional poverty indices.[5] Such bidimensional indices can be defined generally as $P(\lambda)$:

$$P(\lambda) = \iint_{\Gamma(\lambda)} \pi(x_1, x_2; \lambda)\, dF(x_1, x_2), \tag{2}$$

where $\pi(x_1, x_2; \lambda)$ is the contribution to overall poverty of an individual whose income at period 1 and 2 is respectively x_1 and x_2. The well-known "focus axiom" entails that:

$$\pi(x_1, x_2; \lambda) \begin{cases} \geq 0 & \text{if } (x_1, x_2) \in \Gamma(\lambda), \\ = 0 & \text{otherwise.} \end{cases} \tag{3}$$

Our definitions of both the poverty domain and the poverty indices are consistent with different types of aggregation procedures. In a recent paper, Ravallion (2011) contrasted two different approaches to aggregation at the individual level, that is the "attainment aggregation" and the "deprivation aggregation." With the first approach, the values of the different attributes are blended together into a single well-being value, [6] the resulting value being directly compared to some poverty threshold. In the context of intertemporal poverty, that approach is used for instance by Rodgers and Rodgers (1993), and Jalan and Ravallion (1998) for the measurement of chronic poverty. With the second "deprivation aggregation" approach,

[4]For expositional simplicity, we focus on the case of two dimensions of individual well-being. Extensions to cases with more than two dimensions are discussed in footnotes.

[5]For the unidimensional case, see Foster and Shorrocks (1991).

[6] Ravallion (2011) only deals with the case of linear aggregation using a fixed set of prices, but the use of well-being functions like λ could also be considered.

deprivations in each dimension our first assessed and are then aggregated into a composite index. This is exemplified by Foster (2007), Hoy and Zheng (2008), Duclos et al. (2010) and Bossert et al. (2011). The first approach generally allows deprivations in some dimension to be compensated by "surpluses" in some other dimension; compensation effects are generally not admitted with the "deprivation aggregation" approach. The respective merits of each approach are discussed notably in Ravallion (2011) and Alkire and Foster (2011*b*). The tools proposed in this paper encompass both approaches.

For ease of exposition, let the derivatives of π be defined as:

- $\pi^{(i)}(a,b)$, $i=1,2$, for the first-order derivative of π with respect to its ith argument,
- and as $\pi^{(a)}(a,b)$, for the first-order derivative of π with respect to the variable a, so that $\pi^{(u)}\big(a(u),b(u)\big) = \pi^{(1)}\big(a(u),b(u)\big)\frac{\partial a}{\partial u} + \pi^{(2)}\big(a(u),b(u)\big)\frac{\partial b}{\partial u}$.

Then, define the class $\ddot{\Pi}(\lambda^+)$ of monotone poverty indices $P(\lambda)$ as:

$$\ddot{\Pi}(\lambda^+) = \left\{ P(\lambda) \;\middle|\; \begin{array}{l} \Gamma(\lambda) \subset \Gamma(\lambda^+), \\ \pi(x_1,x_2;\lambda) = 0, \text{ whenever } \lambda(x_1,x_2)=0, \\ \pi^{(1)}(x_1,x_2;\lambda) \le 0 \text{ and } \pi^{(2)}(x_1,x_2;\lambda) \le 0 \ \forall x_1,x_2, \\ \pi^{(1,2)}(x_1,x_2;\lambda) \ge 0, \forall x_1,x_2. \end{array} \right\} \tag{4}$$

The class $\ddot{\Pi}(\lambda^+)$ includes *inter alia* the families of bidimensional poverty indices proposed by Chakravarty, Mukherjee and Ranade (1998), Tsui (2002), and Chakravarty, Deutsch and Silber (2008), as well as some members of the family introduced by Bourguignon and Chakravarty (2003). The first condition in equation (4) indicates that the poverty domain for each member $P(\lambda)$ should lie within the poverty domain defined by the well-being function λ^+ (λ^+ then representing the maximum admissible poverty frontier). The second condition in equation (4) says that the poverty measures are continuous along the poverty frontier. Continuity is often assumed in order to prevent small measurement errors from resulting in non-marginal variations of the poverty index.[7] The third condition in (4) corresponds to the monotonicity axiom, *i.e.*, that an income increment in any period should never increase poverty.[8]

Finally, the last condition in (4) captures the axiom of non-decreasing poverty after a "correlation increasing switch", an axiom introduced by Atkinson and Bourguignon (1982). It is then supposed that the marginal benefit of an income increment at period 1 (2) decreases with the income level at period 2 (1). Intuitively, this property also says that a permutation of the incomes of two poor individuals during a given period should not decrease poverty if one of them becomes more deprived than the other in both periods. This can be seen on Figure 1, where it is supposed that profile I moves to profile I', and profile J moves to profile J'. This does not change the distribution of incomes at each time period, but it does increase the temporal correlation of incomes across individuals. The axiom of non-decreasing poverty after a correlation increasing switch says that poverty should not fall after these two movements.

[7] This continuity assumption therefore precludes most members of the Alkire and Foster (2011*a*) family of poverty indices from being part of $\ddot{\Pi}(\lambda^+)$.

[8] As noted in Duclos et al. (2006), we must also have that $\pi^{(1)} < 0$, $\pi^{(2)} < 0$, and $\pi^{(1,2)} > 0$ over some ranges of x_1 and x_2 for the indices to be non-degenerate.

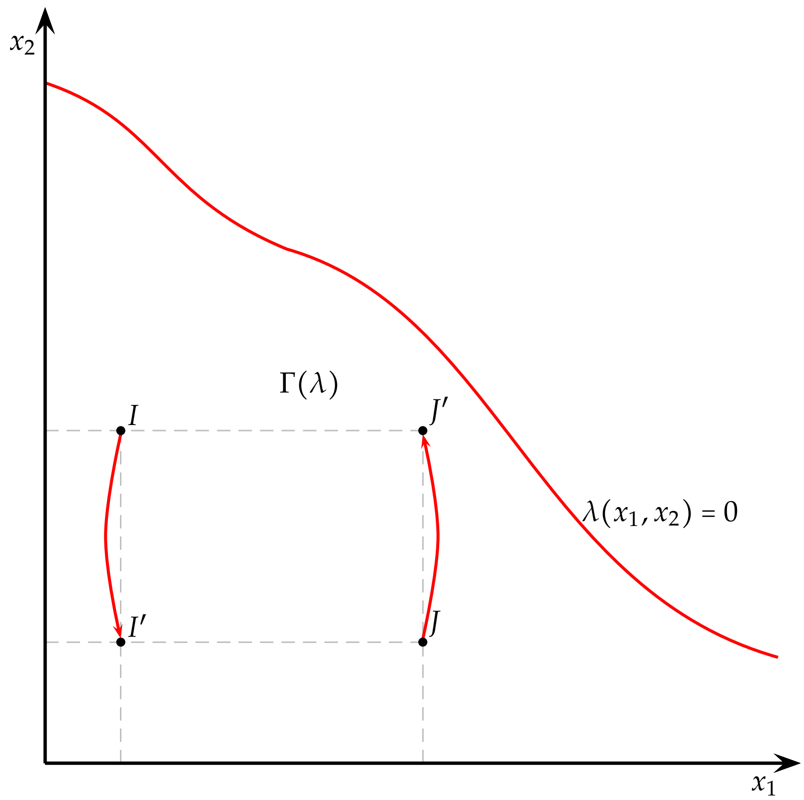


Figure 1: An increase in temporal correlation cannot decrease temporal poverty

Note that this axiom entails that incomes at time 1 and 2 are substitutes in producing overall well-being, which would seem to be a natural assumption.

A bidimensional stochastic dominance surface can now be defined using:

$$P^{\alpha,\beta}(z_u, z_v) := \int_0^{z_u} \int_0^{z_v} (z_u - u)^{\alpha-1} (z_v - v)^{\beta-1} \, dF(u, v). \tag{5}$$

where α and β refer to the dominance order in each dimension. The present paper focusses on first-order dominance, so that α and β are set equal to 1. The function $P^{1,1}(z_u, z_v)$ is the *intersection* bidimensional poverty headcount index.

Duclos et al. (2006) then show:

Proposition 1. (Duclos et al., 2006)

$$P_A(\lambda) \geq P_B(\lambda), \ \forall P(\lambda) \in \ddot{\Pi}(\lambda^+), \tag{6}$$

$$\textit{iff} \quad P_A^{1,1}(x_1, x_2) \geq P_B^{1,1}(x_1, x_2), \ \forall (x_1, x_2) \in \Gamma(\lambda^+). \tag{7}$$

Proposition 1 says that poverty is unambiguously larger for population A than for population B for all poverty sets within $\Gamma(\lambda^+)$ and for all members of the class of monotone bidimensional poverty measures $\ddot{\Pi}(\lambda^+)$ if and only if the bidimensional poverty headcount $P^{1,1}$ is never greater in B than in A for all intersection poverty frontiers in $\Gamma(\lambda^+)$. This is illustrated in Figure 2, which shows both the position of the upper poverty frontier λ^+ and some of the rectangular areas over which $P_A^{1,1}$ and $P_B^{1,1}$ must be computed. If $P_A^{1,1}(x_1, x_2)$ is larger than $P_B^{1,1}(x_1, x_2)$ for all of the rectangles that fit within $\Gamma(\lambda^+)$, then (6) is obtained.

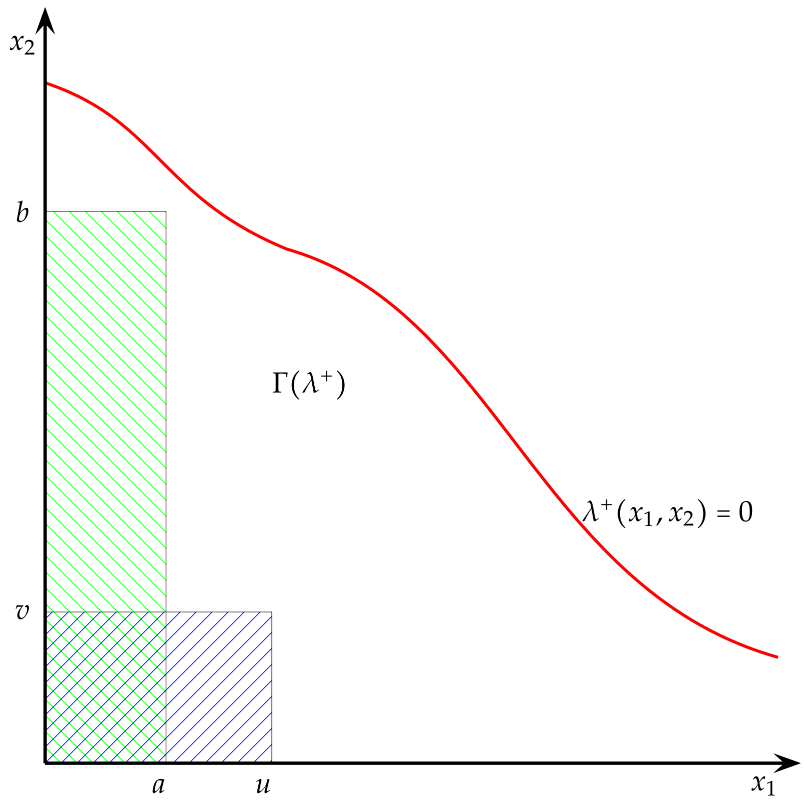


Figure 2: Bidimensional poverty dominance.

In the next pages, the power of the dominance criterion found in Proposition 1 is increased by adding assumptions on the poverty effects of income changes at each time period. For this, it is useful to distinguish between profiles with a lower income in the first period and profiles with a lower income in the second period. The poverty domain can be split into $\Gamma_1(\lambda) := \{(x_1, x_2) \in \Gamma(\lambda) | x_1 < x_2\}$, the set of poverty profiles whose minimal income is found in the first period, and $\Gamma_2(\lambda) := \{(x_1, x_2) \in \Gamma(\lambda) | x_1 \geq x_2\}$, the set of poverty profiles whose minimal income is found in the second period. Equation (2) can then be written as:

$$P(\lambda) = \iint_{\Gamma_1(\lambda)} \pi(x_1, x_2; \lambda)\, dF(x_1, x_2) + \iint_{\Gamma_2(\lambda)} \pi(x_1, x_2; \lambda)\, dF(x_1, x_2), \tag{8}$$

that is, the sum of relatively low-x_1 poverty and of relatively low-x_2 poverty. It is worth noting that the use of equation (8) makes sense only if incomes can be compared. Cost of living differences between the two periods and/or discounting preferences of the social evaluator may thus have to be taken into account before proceeding to (8) and to the symmetry and asymmetry properties that we are about to introduce. It is worth stressing that the need to compare ordinally the different values of x_1 and x_2 does not require that they be cardinal — such as, for instance, when x_1 and x_2 represent the health status of an individual at two points in time, given by categorical ordered data, where we would need to ensure that a value a at the first period is comparable to a value a at the second period.

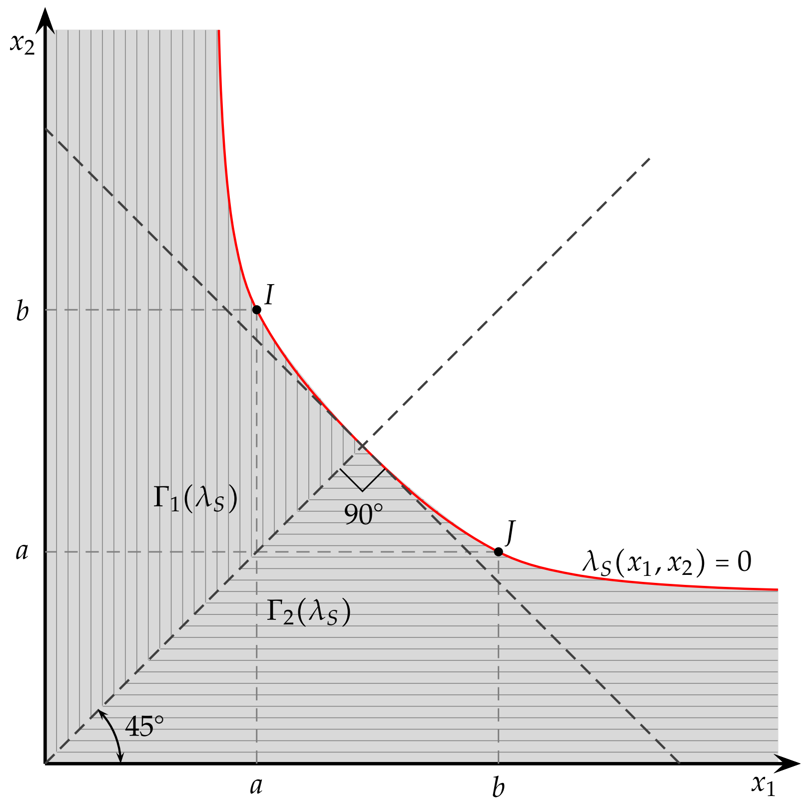


Figure 3: A poverty domain with symmetry

2.1 Symmetry

We now impose symmetry in the treatment of incomes, so that switching the values of the intertemporal income profile of any individual does not change poverty. This is a rather strong assumption since it means that the social evaluator is indifferent to the period at which incomes are enjoyed (again, after possibly adjusting for price differences and discounting preferences). Symmetry may, however, be regarded as reasonable for intertemporal poverty comparisons when the analysis focuses on a relatively short-time span and when we are yet interested in the variability of incomes. It may also be appropriate when one wishes to relax the assumption of perfect substitutability of temporal incomes (made in the univariate analysis of the sum of periodic incomes) without imposing asymmetry in the treatment of incomes.

The symmetry assumption implies that the poverty frontier is symmetric with respect to the line of perfect temporal income equality. As a consequence, the poverty domain is defined with respect to the functions λ_S that are symmetric at the poverty frontier: $\lambda_S(x_1, x_2) = \lambda_S(x_2, x_1)\ \forall(x_1, x_2)$, such that $\lambda_S(x_1, x_2) = 0$. Figure 3 illustrates with the case of two income profiles, $I := (a, b)$ and $J := (b, a)$, both on the poverty frontier. The poverty frontier that links I and J is symmetric along the 45-degree line, the line of temporal income equality. So is the straight line that is perpendicular to that same 45-degree line. That straight line is, however, a special case of all of the symmetric poverty frontiers; it is a poverty frontier that assumes perfect substitutability of temporal incomes. As we will discuss later, the use of those symmetric and straight poverty frontiers is equivalent to measuring temporal poverty using the sum of temporal incomes; associated with parallel iso-poverty lines, it thus effectively reduces

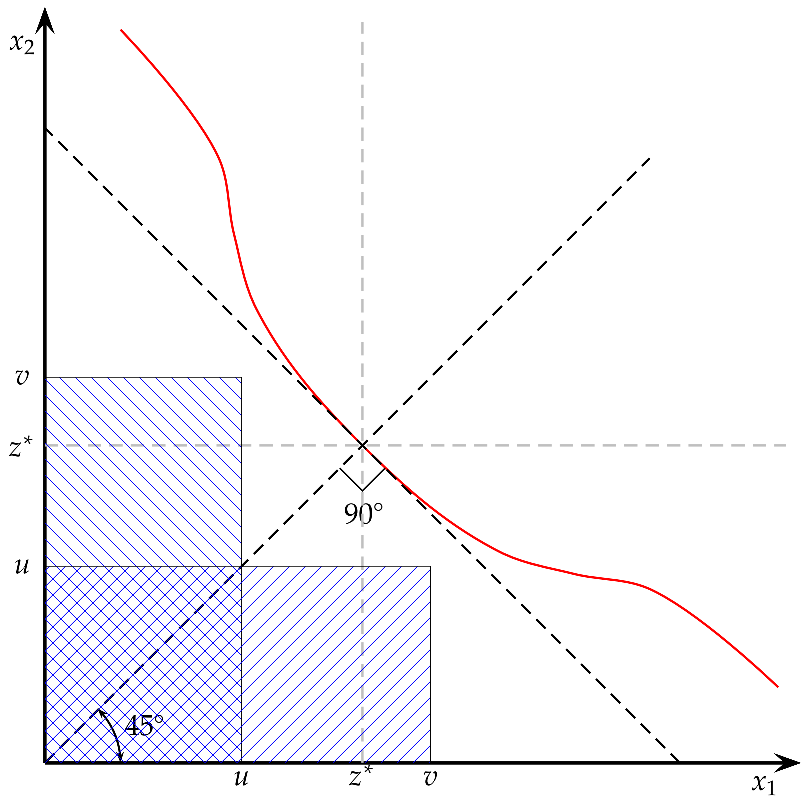


Figure 4: Symmetric property dominance

intertemporal income comparisons to univariate total income comparisons.

Let Λ_S be the subset of Λ whose members are symmetric, and consider the class $\ddot{\Pi}_S$ of bidimensional symmetric poverty measures defined as:

$$\ddot{\Pi}_S(\lambda_S^+) = \left\{P(\lambda_S) \in \ddot{\Pi}(\lambda_S^+) \,\middle|\, \pi(x_1,x_2;\lambda_S) = \pi(x_2,x_1;\lambda_S), \forall(x_1,x_2) \in \Gamma(\lambda_S)\right\}. \tag{9}$$

The additional restriction imposed to define $\ddot{\Pi}_S$ within the set $\ddot{\Pi}$ entails that the marginal effect of an income increment at the first period equals the marginal effect of the same increment at the second period, for two symmetric income profiles ($\pi^{(1)}(x_1,x_2;\lambda_S) = \pi^{(2)}(x_2,x_1;\lambda_S)$, $\forall(x_1,x_2) \in \Gamma(\lambda_S)$). In a similar manner, the variation of the marginal contribution of an income increment is symmetric for symmetric income profiles ($\pi^{(1,2)}(x_1,x_2;\lambda_S) = \pi^{(1,2)}(x_2,x_1;\lambda_S)$, $\forall(x_1,x_2) \in \Gamma(\lambda_S)$).

We now show how robust comparisons of bidimensional poverty can be made with symmetry.

Proposition 2.

$$P_A(\lambda_S) > P_B(\lambda_S),\ \forall P(\lambda_S) \in \ddot{\Pi}_S(\lambda_S^+), \tag{10}$$

$$\textit{iff}\quad P_A^{1,1}(x_1,x_2) + P_A^{1,1}(x_2,x_1) > P_B^{1,1}(x_1,x_2) + P_B^{1,1}(x_2,x_1),\ \forall(x_1,x_2) \in \Gamma(\lambda_S^+). \tag{11}$$

Proof. See appendix A. □

Proposition 2 says that poverty dominance can be checked by adding up two intersection headcounts, the first at a poverty line (x_1, x_2) and the second at (x_2, x_1).[9] With symmetric intertemporal poverty indices, we must therefore compare the sum of two intersection intertemporal headcounts that have symmetric poverty lines. Figure 4 shows graphically what this means: we must sum the proportions of income profiles found within two symmetric rectangular areas, each of them capturing the importance of those with low incomes in one time period. This effectively double counts the number of individuals that are highly deprived in both periods, as the double-slashed rectangle in Figure 4 shows. Define z^* as the minimal permanent income value an individual should enjoy at each period in order to escape poverty, that is $\lambda_S(z^*, z^*) = 0$. The double counting in Proposition 2 only covers those individuals whose income is below z^* in each period.[10] Though the paper does not touches upon the distinction between chronic and transient poverty, it is worth stressing that if chronic poverty occurs when an individual's income falls short of z^* at each period (the "always poor" in Hulme and Shepherd, 2003), this double counting can be seen to weight the chronic poor more than the transient ones, namely, the poor that are deprived during only one period.

The power of Proposition 2 is then larger than that of Proposition 1. This is because (11) makes it possible to compensate between symmetric income profiles and gives greater importance to "more severe" intertemporal poverty, namely, poverty in both periods. To illustrate this, consider two income distributions, A and B, made of profiles $\{(2,1),(2,1),(3,4)\}$ and $\{(1,2),(4,3),(4,3)\}$ respectively. Using Proposition 1, one would not be able to order these two distributions since equation (7) is larger for A when evaluated at $(2,1)$ and larger for B when evaluated at $(1,2)$. We would, however, observe dominance using Proposition 2 since equation (11) at $(1,2)$ would now be larger for A, the distribution with more severe poverty.

2.2 Asymmetry

Symmetry may not be appropriate, however, in those cases in which we may not be (individually or socially) indifferent to a permutation of periodic incomes. We may yet feel that poverty is higher with income profile (x_1, x_2) than with (x_2, x_1) whenever $x_1 < x_2$. For instance, it may be deemed that low income may be more detrimental to well-being during childhood than during adulthood, since low income as a child can lead to poorer health and lower educational outcomes over the entire lifetime.

Such asymmetry may also be reasonable when there is uncertainty regarding the way incomes in a given period should be scaled up or down before applying symmetry. This may be the case when intertemporal price adjustments need to be made but when but true inflation is

[9] Extending Proposition 2 to cases with more than two dimensions is relatively straightforward. For instance, if symmetry is assumed with three dimensions, one has to compare the sum of the joint distributions for the six permutations of each possible set of temporal poverty lines, that is $F(u,v,w) + F(u,w,v) + F(v,u,w) + F(v,w,u) + F(w,u,v) + F(w,v,u)$.

[10] In the tridimensional case mentioned in footnote 9, multiple counting also occurs but in a more complex manner. Those individuals whose incomes are less than z^* at each period are counted six times when checking dominance. Double counting occurs for those poor individuals whose incomes are below z^* during only two periods of time. The multidimensional dominance criterion thus introduces weights on poor households that depend on the number of periods of deprivations that they experience. Because of this, the social benefit of decreasing individual deprivation increases with the number of income shortfalls (with respect to z^*): a two-period-deprived person is twice as important as a single-period-deprived person, and a three-period-deprived person is thrice as important as a two-period-deprived person.

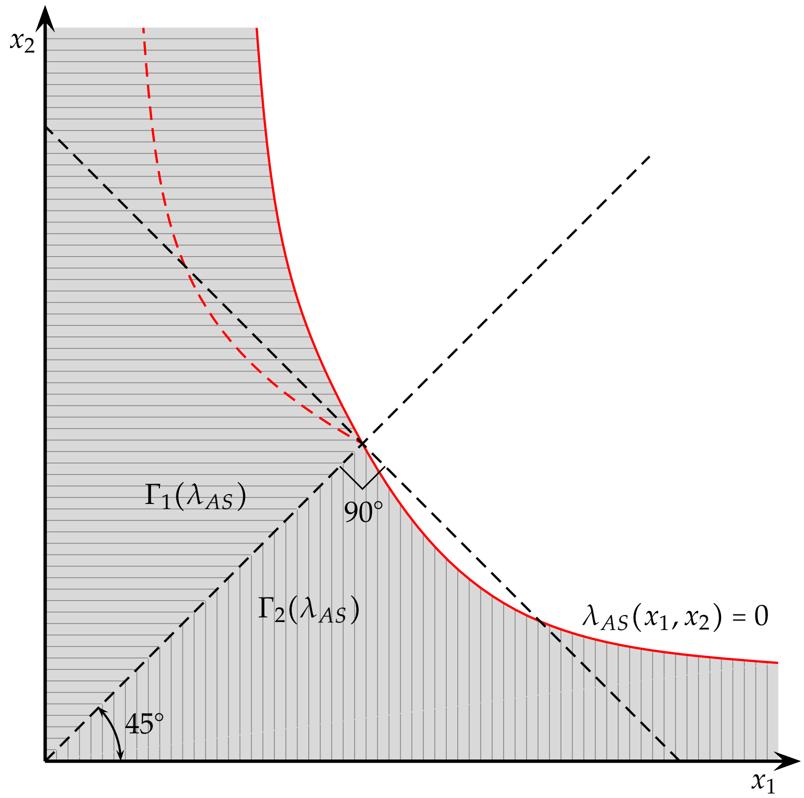


Figure 5: Asymmetric poverty measurement

unknown. If the purchasing power of money has decreased, but the extent of that fall is not known for sure, a prudent and yet substantive procedure may be to impose asymmetry on the treatment of the components of the income profiles. Asymmetry is also the general case in the class of intertemporal poverty indices proposed by Hoy and Zheng (2008) and Calvo and Dercon (2009), where weights at each period decrease as the final period is approached.

Without loss of generality, assume that income profiles within $\Gamma_1(\lambda)$ never yield less poverty than their symmetric image in $\Gamma_2(\lambda)$. The well-being functions λ_{AS} that are consistent with asymmetry are then members of the set Λ_{AS} of well-being functions defined by:

$$\Lambda_{AS} := \{\lambda \in \Lambda | \lambda(x_1, x_2) \leq \lambda(x_2, x_1) = 0, \quad \forall x_1 \leq x_2\}. \tag{12}$$

Figure 5 illustrates the possible shape of these functions. The asymmetry of $\lambda_{AS}(x_1, x_2)$ indicates that low x_1 is a source of greater poverty than low x_2. The poverty frontier ($\lambda_{AS}(x_1, x_2) = 0$, the continuous line) is chosen such that the poverty domain $\Gamma_1(\lambda_{AS})$ (the shaded area with horizontal lines) is larger than $\Gamma_2(\lambda_{AS})$ (the shaded area with vertical lines). In particular, the symmetric set of $\Gamma_2(\lambda_{AS})$ with respect to the line of perfect equality is a subset of $\Gamma_1(\lambda_{AS})$.

We can then consider the following class of asymmetric monotone poverty measures:

$$\ddot{\Pi}_{AS}(\lambda^+_{AS}) = \left\{ P(\lambda) \in \ddot{\Pi}(\lambda^+_{AS}) \;\middle|\; \begin{array}{ll} \pi^{(1)}(x_1, x_2; \lambda) \leq \pi^{(2)}(x_2, x_1; \lambda) & \text{if } x_1 \leq x_2 \\ \pi^{(1,2)}(x_1, x_2; \lambda) \geq \pi^{(1,2)}(x_2, x_1; \lambda) & \text{if } x_1 \leq x_2. \end{array} \right\} \tag{13}$$

The first line to the right of (13) implies that changes in the lowest income have a greater

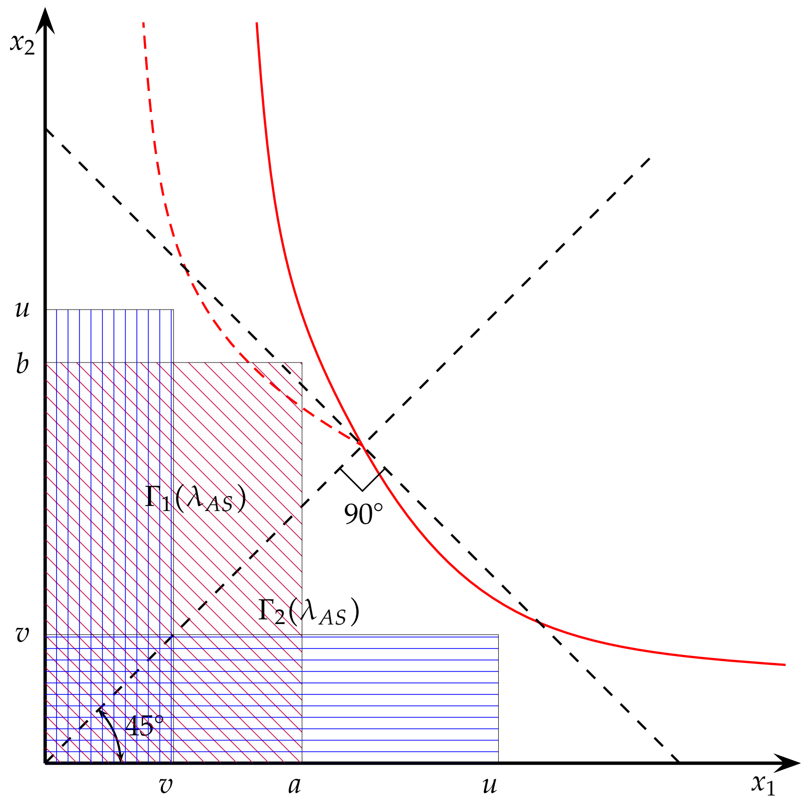


Figure 6: Asymmetric poverty dominance

impact on poverty when the lowest income is the first period income. Consequently, for equal values of x_1 and x_2, changes in x_1 have a greater impact on welfare than changes in x_2. The second line states that, considering two symmetric income profiles, the marginal poverty benefit of an increase in either x_1 or x_2 decreases the most with the value of the other variable when the income profile is the one with the lowest first period income. It also says that a correlation decreasing switch decreases poverty more when x_1 is lower, for the same total income. Both lines emphasize the greater normative importance of those with lower first-period incomes.

The necessary and sufficient conditions for robustly ordering asymmetric poverty measures are presented in Proposition 3:

Proposition 3.

$$P_A(\lambda_{AS}) > P_B(\lambda_{AS}),\ \forall P(\lambda_{AS}) \in \ddot{\Pi}_{AS}(\lambda^+_{AS}), \quad (14)$$

$$\textit{iff} \quad P^{1,1}_A(x_1,x_2) > P^{1,1}_B(x_1,x_2),\ \forall (x_1,x_2) \in \Gamma_1(\lambda^+_{AS}) \quad (15)$$

$$\textit{and} \quad P^{1,1}_A(x_1,x_2) + P^{1,1}_A(x_2,x_1) > P^{1,1}_B(x_1,x_2) + P^{1,1}_B(x_2,x_1),\ \forall (x_1,x_2) \in \Gamma_2(\lambda^+_{AS}). \quad (16)$$

Proof. See appendix A. □

The first condition in Proposition (3) indicates that dominance should first hold for each point in $\Gamma_1(\lambda_{AS})$. That condition is illustrated in Figure 6. For any (a,b) with $a < b$, poverty dominance with asymmetry implies that the share of the population whose incomes are simultaneously less than a and b respectively at period 1 and 2 (those in the rectangle with slanting lines on Figure 6) should be lower in B than in A. Thus, contrary to the symmetric case, poverty cannot be considered to be lower in B if the intersection headcount with a relatively low threshold for the first period is higher in B. Condition (16) is the same as condition (11) in Proposition 2, but for income profiles within $\Gamma_2(\lambda_{AS})$. Since symmetric poverty indices can be regarded as limiting cases of asymmetric ones, dominance with asymmetry logically implies dominance with symmetry, so long as the symmetric poverty frontiers lie within the asymmetric ones.

The power of asymmetric dominance tests is larger than with bidimensional poverty measures ÏI. To illustrate this difference in ranking power, consider two income distributions, A and B, with distribution A made of profiles $\{(1,2),(1,2)\}$ and distribution B made of profiles $\{(2,1),(6,6)\}$, and with z^*=5. Using Proposition 1, one would not be able to order these two distributions since equation (7) is larger for A when evaluated at $(1,2)$ and larger for B when evaluated at $(2,1)$; although A may look poorer than B at first glance, one of the profiles in B has the lowest income at time 2. We would, however, observe asymmetric dominance since equation (15) at $(2,1)$ would now be larger for A.

Note, however, that with the example (used on page 10) of distributions A set to $\{(2,1),(2,1),(3,4)\}$ and B set to $\{(1,2),(4,3),(4,3)\}$ no ranking can be obtained using asymmetric poverty dominance. The stronger symmetry assumption of Proposition 2 are then needed to rank these two distributions.

The conditions in Proposition 3 may thus hold even if B has a larger proportion of poor with low x_2, so long as this is compensated by a lower proportion with low x_1. This is reminiscent of the sequential stochastic dominance conditions found in Atkinson and Bourguignon (1987) and Atkinson (1992) and in subsequent work. Although similar, the two frameworks and their respective orderings conditions are different, however. The literature on sequential dominance makes assumptions only on the *signs* of different orders of derivatives; the conditions in (13) *compare* the value of these derivatives across dimensions, a procedure that is possible only when the dimensions are comparable. Such comparability assumptions have typically not been possible in the sequential dominance literature since the dimensions involved (income and family size, for instance) typically did not have comparable measurement units.

3 Aversion to intertemporal variability

Up to now, the classes of intertemporal poverty measures that we have considered have been defined with respect to the values of income at the first and second periods. The monotonicity assumption then yields an unambiguous poverty ranking of two income profiles, if one of them exhibits a lower income at both periods. But the monotonicity assumption cannot otherwise rank two income profiles. To see this, consider for instance the income profiles $I := (a,b)$ and $J := (u,v)$ drawn in Figure 7. By projecting these two profiles on the diagonal of perfect temporal equality, it can be seen that both profiles are characterized by the same total temporal income, so that the only difference between them is the way total income is allocated across the two periods. We may feel that individuals are better off when the distribution of a given total amount is smoothed across periods; we should then infer that poverty is unambiguously

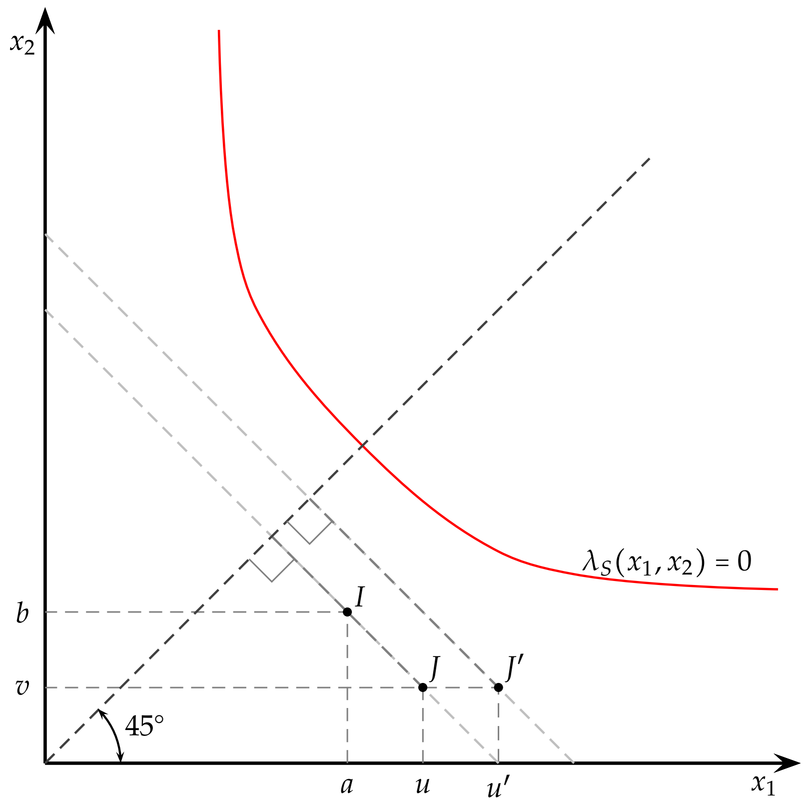


Figure 7: Ranking income profiles with aversion to intertemporal inequalities

lower with income profile I than with J (since $|a - b| < |u - v|$). This, however, cannot be inferred with any of the previous propositions.

Comparing two income profiles that differ in their total (or mean) income is more complicated. For instance, let us assume that an income profile J sees an increase in its first-period income. Let the new income profile be $J' := (u', v)$, as in Figure 7. Both intertemporal variability and average income have increased. To compare I and J', we could think of a lexicographic assumption that either mean income or distance from the mean prevails on the other. We can also use results from the social welfare literature when both inequality and mean income differ.

As noted by Kolm (1976) in a unidimensional context, views differ as to how additional income should be shared among different people so as to leave inequality unchanged. One view is that sharing this additional income according to the initial income shares of individuals would preserve the initial level of inequality; another view is that inequality is maintained if the same absolute amount of income is distributed to everyone. These correspond to relative and absolute inequality aversion views.

With this in mind, let us define poverty with respect to average income and income deviations from that average. An income profile (x_1, x_2) is then described by the coordinates (μ, τ), with μ being mean income and τ some measure of the distance of the lowest income to the mean. One reasonable property to impose on τ is unit-consistency; this states that changing the income measurement scale (using euros instead of cents, for instance) should not change the ranking properties of the measure (Zheng, 2007). Within our setting, unit consistency demands that multiplying each element of income profiles by the same scalar should not change the intertemporal inequality ranking of the income profiles.

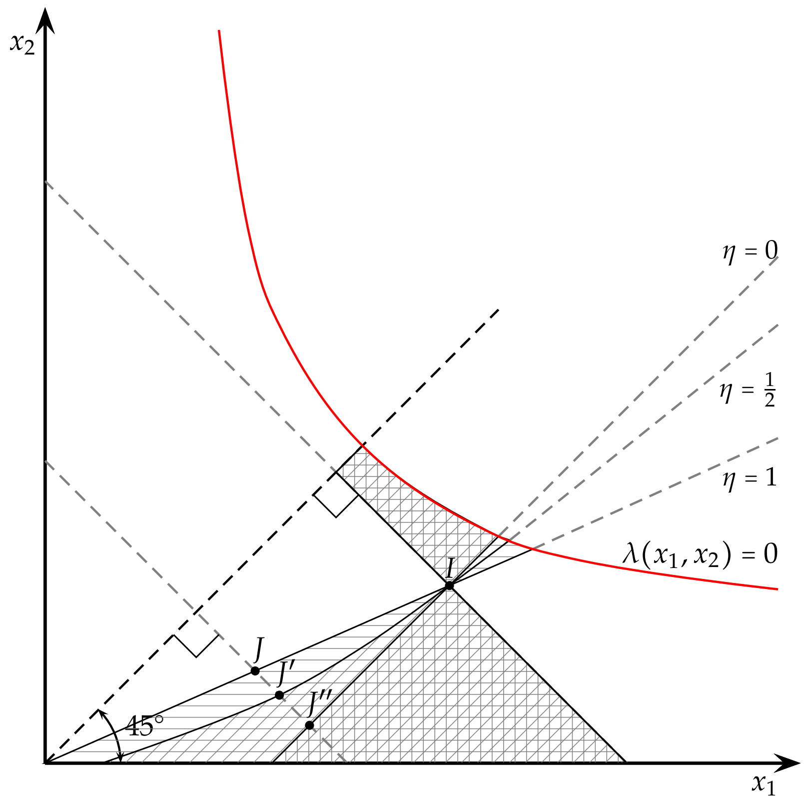


Figure 8: Bidimensional poverty with relative, intermediate and absolute variability aversion views.

With this in mind, we make use of a particular definition of τ, that is $\tau_\eta = \frac{\min\{x_1,x_2\}-\mu}{\mu^\eta}$, $\eta \in [0,1]$, (Krtscha, 1994, Zoli, 2003, Yoshida, 2005), so that $\tau_1 = \frac{\min\{x_1,x_2\}}{\mu} - 1$ for a relative inequality aversion view and $\tau_0 = \min\{x_1, x_2\} - \mu$ for an absolute inequality aversion view.[11] For a given μ, τ ranges from $-\mu^{1-\eta} \leq 0$ (extreme inequality) to 0 (perfect equality). Poverty is reasonably assumed to decrease with both μ and τ_η (which we term "variability", as a shorthand for temporal inequality).

Figure 8 illustrates the influence of η on the orderings of an income profile I with profiles with a lower mean income and located on the same side of the diagonal of equality. The areas below I with horizontal, slanting and vertical hatches correspond to the set of income profiles with unambiguously higher poverty than I when η is respectively set to 1, 0.5, and 0.[12] The areas above I but inside $\Gamma_2(\lambda)$ are those income profiles that are better than I for all values of η. Whatever the location of I, the relative view ranks more income profiles as worse than the intermediate and absolute views. For instance, income profiles J, J', and J'' exhibit the same distance τ_η as I with respect to the first diagonal when η is respectively set equal to 1, 0.5, and 0, but average income is lower. I is preferred to J, J', and J'' for $\eta = 1$, but cannot be ranked with J and J' when $\eta < 0.5$. Relative views also increase the set of income profiles that are preferred to I. In that sense, absolute views rely on the weakest measurement assumptions

[11] See Zheng (2007) for a detailed review of this.

[12] While the cases of η equal to 1 and 0 can easily be understood, the intermediate cases deserve some explanation. For instance, with $\eta = 0.5$, inequality will be preserved when moving from μ_1 to μ_2 if each additional dollar is distributed in the following manner: fifty cents are equally shared between the two period and the remaining fifty cents are distributed according to the last pre-increment income shares of each period.

and also induce the weakest power for ranking income profiles.

We can also express the poverty frontier as a function of both μ and τ_η. Let $\tilde{\lambda}$ be defined as:

$$\tilde{\lambda}(\mu, \tau_\eta, j) = \begin{cases} \tilde{\lambda}(\mu, \tau_\eta, 1) & \text{if } x_1 < x_2, \\ \tilde{\lambda}(\mu, \tau_\eta, 2) & \text{otherwise.} \end{cases} \tag{17}$$

Recall that both μ and τ_η are functions of x_1 and x_2. We can also assume that $\tilde{\lambda}(\mu, \tau_\eta, j) = \lambda(x_1, x_2)$, namely, that each function $\tilde{\lambda}$ has a unique representation λ in the space (x_1, x_2), and that $\frac{\partial \tilde{\lambda}}{\partial \mu} > 0$ and $\frac{\partial \tilde{\lambda}}{\partial \tau_\eta} > 0$. Let $\tilde{\Lambda}$ be the set of mean-income increasing and variability-decreasing well-being functions. It is worth indicating that non-increasingness with respect to variability not only entails that the poverty frontier is convex; it also has to be never below the straight line through (z^*, z^*) that is orthogonal to the line of perfect equality.[13] For convenience, we can express the poverty domain in the space (x_1, x_2) as:

$$\Gamma(\tilde{\lambda}) := \left\{ (x_1, x_2) \in \Re_+^2 \,\middle|\, \tilde{\lambda}(\mu, \tau_\eta, j) \leq 0 \right\}, \tag{18}$$

where, as previously, $\Gamma(\tilde{\lambda})$ can be divided into $\Gamma_1(\tilde{\lambda})$ and $\Gamma_2(\tilde{\lambda})$ to distinguish relatively low-x_1 income profiles from relatively low-x_2 income profiles.

3.1 The general case

To use the above setting for poverty measurement, let $q := \text{prob}(x_1 < x_2)$ be the share of the population whose first-period income is lower than second-period income. Let ρ_1 (ρ_2) be the individual poverty measure when $x_1 < x_2$ ($x_1 \geq x_2$), and let F_1 (F_2) denote the joint cumulative distribution function of μ and τ_η conditional on $x_1 < x_2$ ($x_1 \geq x_2$). A variability-averse poverty measure is given by

$$\begin{aligned} \tilde{P}(\tilde{\lambda}) = q & \iint_{\Gamma_1(\tilde{\lambda})} \rho_1(\mu, \tau_\eta)\, dF_1(\mu, \tau_\eta), \\ & + (1-q) \iint_{\Gamma_2(\tilde{\lambda})} \rho_2(\mu, \tau_\eta)\, dF_2(\mu, \tau_\eta). \end{aligned} \tag{19}$$

As in Section 2, equation (19) corresponds to a general definition of an additive intertemporal poverty measure, *i.e.*, overall poverty is simply the average individual poverty level.[14]

[13] Would these conditions not be met, it would then be possible for some income profiles to leave the poverty domain by increasing intertemporal variability.

[14] Although not as straightforward as with the poverty indices of Section 2, extending this mean-variability framework to $T > 2$ periods can be done. Let μ_k be the average value of the $k = 1, \ldots T$ lowest values of an income profile. μ_1 is thus the minimal value of the income profile and $\mu_T = \mu$ is average income. Then, define $\tau_{k,\eta} := \frac{\mu_k - \mu}{\mu^\eta}$, with $\tau_{k,\eta} \in [-\mu^{1-\eta}, 0]$. It can be seen that for each income profile of size T, only $T-1$ observations of inequality are needed to describe all relevant intertemporal inequalities. So an income profile $(x_1, x_2 \ldots, x_T)$ can be fully described in terms of intertemporal inequalities and average income by the T-vector $(\tau_{1,\eta}, \tau_{2,\eta} \ldots, \tau_{T-1,\eta}, \mu)$.

If income timing matters for poverty assessment (as for asymmetric poverty indices), this vector will not be sufficient. For instance, in the three-period case, it would be necessary to make use of $3! = 6$ possibly different individual poverty indices $\rho_{s,t}$, where s indicates the period of the lowest income and t is the period for the second-lowest income. Once this is done, generalizing Propositions 4 to 7 is relatively straightforward.

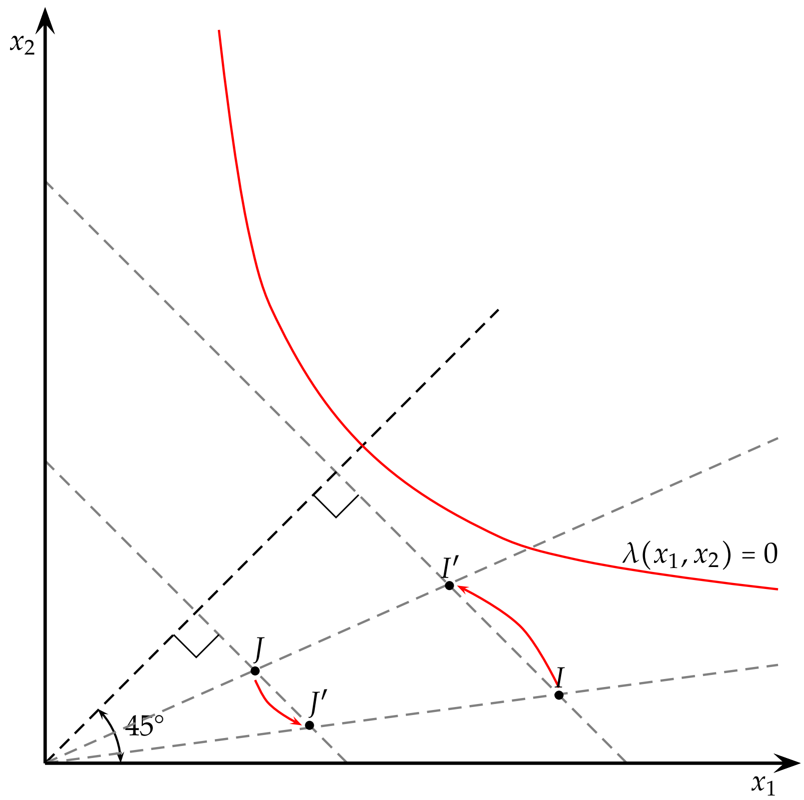


Figure 9: A correlation-increasing switch in the space (μ, τ_1) (relative variability aversion).

Let the class $\tilde{\Pi}_\eta(\tilde{\lambda}^+)$ of mean-variability poverty indices be defined as:

$$\tilde{\Pi}_\eta(\tilde{\lambda}^+) = \left\{ P(\tilde{\lambda}) \left| \begin{array}{l} \Gamma(\tilde{\lambda}) \subset \Gamma(\tilde{\lambda}^+) \\ \rho_t(\mu, \tau_\eta; \tilde{\lambda}) = 0, \text{ whenever } \tilde{\lambda}(\mu, \tau_\eta) = 0 \ \forall t \\ \rho_1(\mu, 0, \tilde{\lambda}) = \rho_2(\mu, 0, \tilde{\lambda}) \forall \mu \\ \rho_t^{(1)}(\mu, \tau_\eta; \tilde{\lambda}) \leq 0 \text{ and } \rho_t^{(2)}(\mu, \tau_\eta; \tilde{\lambda}) \leq 0 \ \forall \mu, \tau_\eta, t \\ \rho_t^{(1,2)}(\mu, \tau_\eta; \tilde{\lambda}) \geq 0, \forall \mu, \tau_\eta, \forall t. \end{array} \right. \right\} \tag{20}$$

As in the case of the class $\ddot{\Pi}(\lambda^+)$ defined in equation (4), the first two conditions say that the chosen poverty frontier should be nowhere above the maximum admissible poverty frontier $\tilde{\lambda}^+$, and that ρ_t is continuous at the poverty frontier. The third condition says that poverty measurement is continuous at the diagonal of perfect temporal equality. The fourth condition states that intertemporal variability-preserving income increments and mean-preserving variability-increasing transfers should not increase poverty.

The last condition in (20) says that the more variable are income profiles, the more effective are variability-preserving income increments in reducing poverty. Similarly, the benefits of a mean-preserving variability-decreasing income change falls with mean income. This condition can also be interpreted in terms of correlation-increasing switches in the (μ, τ_η) space. Indeed, the last condition in (20) implies that permuting the values of either μ or τ_η of two poor individuals, so that one of them becomes unambiguously poorer that the other, cannot reduce poverty. Figure 9 illustrates this in the case of relative variability aversion. The permutation of τ_1 that moves I and J to I' and J' respectively necessarily improves the situation of individual

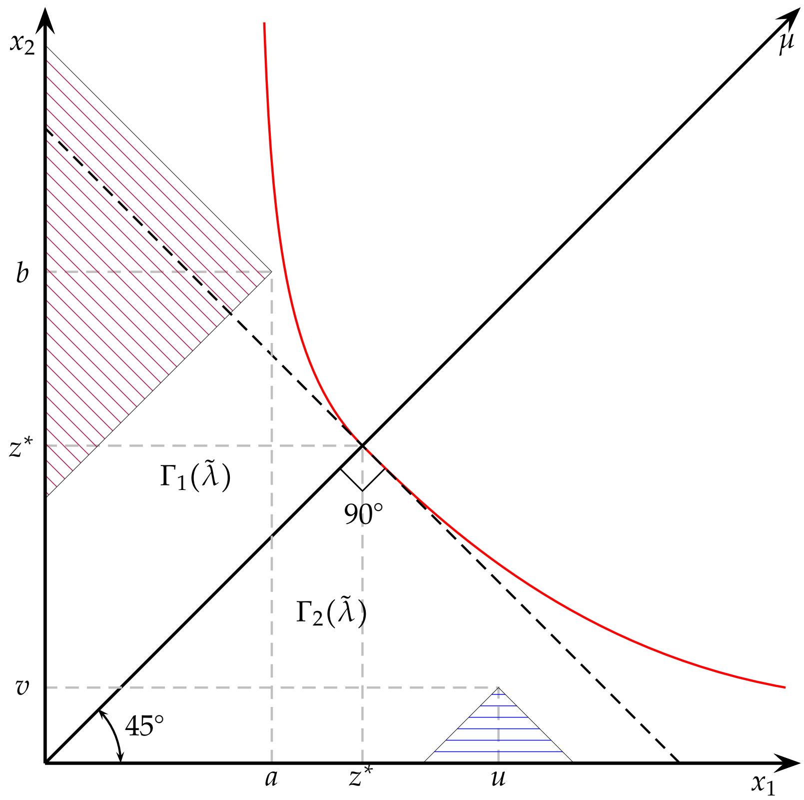


Figure 10: Poverty dominance criteria with absolute variability aversion.

I what worsens that of J. The permutation does not affect the marginal distributions of μ and τ_1, but nevertheless results in an increased correlation between them. The two different forms of deprivation then cumulating over the same person, it seems natural to regard such a change as worsening poverty.

This leads to the following general result.

Proposition 4.

$$P_A(\tilde{\lambda}) > P_B(\tilde{\lambda}),\ \forall P(\tilde{\lambda}) \in \tilde{\Pi}_\eta(\tilde{\lambda}^+), \tag{21}$$

$$\textit{iff} \quad q_A P_A^{1,1}(\mu, \tau_\eta | x_1 < x_2) > q_B P_B^{1,1}(\mu, \tau_\eta | x_1 < x_2), \quad \forall(\mu, \tau_\eta) \in \Gamma_1(\tilde{\lambda}^+), \tag{22}$$

$$\textit{and} \quad (1 - q_A) P_A^{1,1}(\mu, \tau_\eta | x_1 \geq x_2) > (1 - q_B) P_B^{1,1}(\mu, \tau_\eta | x_1 \geq x_2), \quad \forall(\mu, \tau_\eta) \in \Gamma_2(\tilde{\lambda}^+). \tag{23}$$

Proof. See appendix B. □

Proposition 4 says that a distribution A exhibits more poverty than a distribution B over the class of mean-variability poverty indices if and only if the share of the population with low mean income and high variability is greater in A than in B, whatever (μ, τ_η) within the poverty domain is used and considering separately each low x_1 and low x_2 region. Figure 10 illustrates the dominance criteria in the case of absolute variability aversion. Each income profile in $\Gamma(\tilde{\lambda}^+)$ defines a rectangular triangle whose hypotenuse is either the x_1 or the x_2-axis, for $x_1 > x_2$ and $x_1 < x_2$ respectively. Poverty is larger for the distribution that shows a larger population share within each one of those triangular areas that fit within $\Gamma(\tilde{\lambda}^+)$. It can be seen by inspection

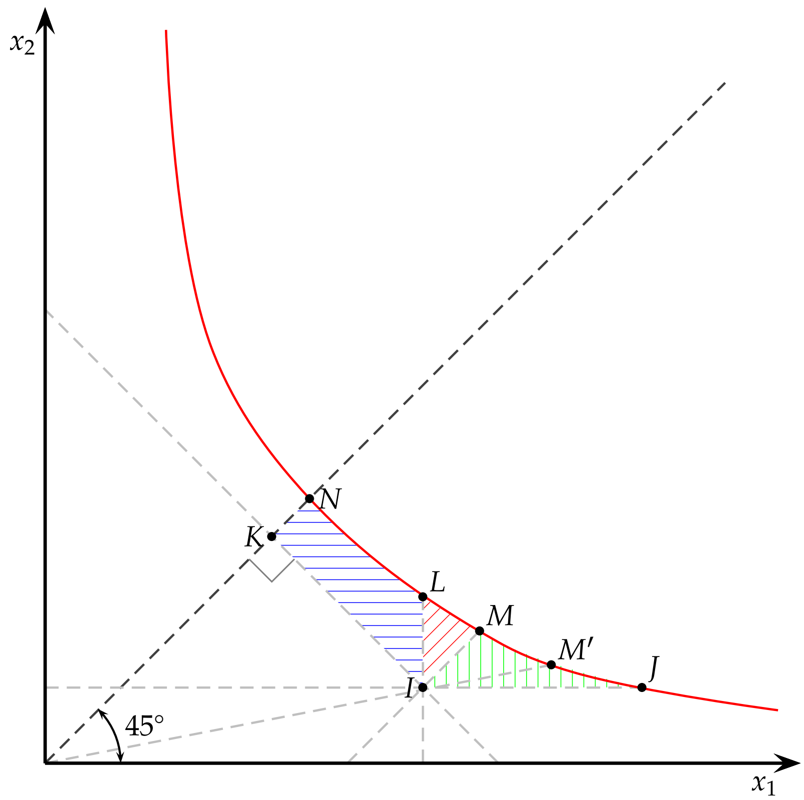


Figure 11: Effect of changes in temporal incomes

that a necessary (but not sufficient) condition for dominance of B over A is that the marginal distribution of μ for A is nowhere below that for B at each value of μ below z^*.

It is useful to compare the ability of propositions 1 and 4 to rank distributions. Suppose that distributions A and B are respectively defined by the income profiles $\{(3,1),(1,5)\}$ and $\{(3,1),(3,4)\}$. These two distributions cannot be ordered if all profiles lie within the poverty domain $\Gamma(\lambda^+)$ and if poverty is measured over $\ddot{\Pi}(\lambda^+)$: the intersection headcount is larger for B when evaluated at $(3,4)$, but lower when evaluated at $(1,5)$. In the space (μ, τ_0), the ordinates of the two distributions become $\{(2,-1),(3,-2)\}$ and $\{(2,-1),(3.5,-0.5)\}$. It can then be seen that the joint distribution function of (μ, τ_0) is larger for A when evaluated at $(3,-2)$ and nowhere lower when evaluated at any other point of the poverty domain. Consequently, A exhibits more poverty than B whatever poverty index is chosen within $\tilde{\Pi}(\tilde{\lambda}^+)$.

This does not mean that the ordering power of Proposition 4 is larger than that of Proposition 1. Proposition 1 orders $\{(3,1),(3,5)\}$ and $\{(3,1),(3,4)\}$), but Proposition 4 does not. This is also visible from Figure 7. Profile I is judged better than J by Proposition 4 but not by Proposition 1; Profile J' is judged better than J by Proposition 1 but not by Proposition 4.

Figure 11 provides an alternative illustration of the differences in the measurement assumptions behind each of Proposition 1 and Proposition 4 in the case of the absolute approach to income variability. A movement from point I to point J (or to any other point in the triangular area IJM) is deemed to decrease poverty according to the usual multidimensional poverty indices covered by Proposition 1, but not with respect those of Proposition 4. A movement from point I to point K is deemed to decrease poverty according to Proposition 4, but not with respect to Proposition 1. This is also true of a movement from point I to any of the points in the area $IKNL$ with horizontal stripes. It is only to the points in the triangular area IKM that a movement from point I will be judged to decrease poverty according to both Proposition 1 and Proposition 4. It is worth noting that, as η increases, that surfaces increases and so the probability to get the same conclusion from both propositions for any given increment in total income. In the limiting case of the relative view of variability, this area extends in the present case to IKM'.

Let $\tilde{\succcurlyeq}_{\eta,\lambda}$ denote dominance over the class $\tilde{\Pi}_\eta(\lambda)$, so that $A \tilde{\succcurlyeq}_{\eta,\tilde{\lambda}^+} B$ means that distribution A is preferred to distribution B according to Proposition 4. The next proposition considers how the dominance relationships $\succcurlyeq_{\eta,\tilde{\lambda}^+}$ are nested.

Proposition 5.

$$\textit{If } A \tilde{\succcurlyeq}_{\eta,\tilde{\lambda}^+} B, \quad \textit{then} \quad A \tilde{\succcurlyeq}_{\eta',\tilde{\lambda}^+} B \quad \forall \eta' \in [\eta, 1]. \tag{24}$$

$$\textit{If } A \tilde{\not\succcurlyeq}_{\eta,\tilde{\lambda}^+} B, \quad \textit{then} \quad A \tilde{\not\succcurlyeq}_{\eta',\tilde{\lambda}^+} B \quad \forall \eta' \in [0, \eta]. \tag{25}$$

Proof. The proof is straightforward since, for any couple of profiles (μ, τ_η) and (μ', τ_η') from $\Gamma_i(\tilde{\lambda})$, $i = 1$ or 2, the first one is preferred iff $\mu' \leq \mu$ and $\tau_\eta' \leq \tau_\eta$. This implies that $\frac{x_i'-\mu'}{\mu'^\eta} \leq \frac{x_i-\mu}{\mu^\eta} \leq 0$; it can then be seen that $\frac{x_i'-\mu'}{\mu'^\eta \mu'^\varepsilon} \leq \frac{x_i-\mu}{\mu^\eta \mu'^\varepsilon} \leq \frac{x_i-\mu}{\mu^\eta \mu^\varepsilon}$ $\forall \varepsilon > 0$, since $\mu' \leq \mu$ and the variability measure is negative. Consequently $(\mu, \tau_{\eta+\varepsilon})$ is preferred to $(\mu', \tau_{\eta+\varepsilon}')$. □

The first part of Proposition 5 states that, using our mean-variability framework, a sufficient condition for A to dominate B for some η is to observe such a dominance relationship for a lower value of η. An immediate consequence is that dominance holds for all values of η when dominance is observed for $\eta = 0$. This property is useful since it makes it possible to obtain poverty comparisons that are robust with respect to views of variability aversion without having to perform dominance tests for all such views.

The second part of Proposition 5 is a corollary of the first part: it is useless to check for whether A dominates B for some η if dominance does not hold for a larger η. This is again practically useful: the inability to order two distributions with a relative variability aversion view means that there is no hope of obtaining dominance with intermediate or absolute views.

3.2 Symmetry

As in Section 2.1, symmetry can be assumed, so that poverty depends only on the absolute gaps between incomes as well as on mean income. As a consequence, an income profile (x_1, x_2) is strictly equivalent to an income profile (x_2, x_1); both can be described by the same coordinates (μ, τ_η). We then have:

$$\tilde{\Pi}_{\eta S}(\tilde{\lambda}_S^+) = \left\{ P(\tilde{\lambda}_S) \in \tilde{\Pi}_\eta(\tilde{\lambda}_S^+) \middle| \rho_1(\mu, \tau_\eta; \tilde{\lambda}_S) = \rho_2(\mu, \tau_\eta; \tilde{\lambda}_S), \forall (\mu, \tau_\eta) \in \Gamma(\tilde{\lambda}_S) \right\}. \tag{26}$$

Proposition 6.

$$P_A(\tilde{\lambda}_S) > P_B(\tilde{\lambda}_S), \ \forall P(\tilde{\lambda}_S) \in \tilde{\Pi}_{\eta S}(\tilde{\lambda}_S^+), \tag{27}$$

$$\textit{iff} \quad P_A^{1,1}(\mu, \tau_\eta) > P_B^{1,1}(\mu, \tau_\eta), \quad \forall (\mu, \tau_\eta) \in \Gamma(\tilde{\lambda}_S^+). \tag{28}$$

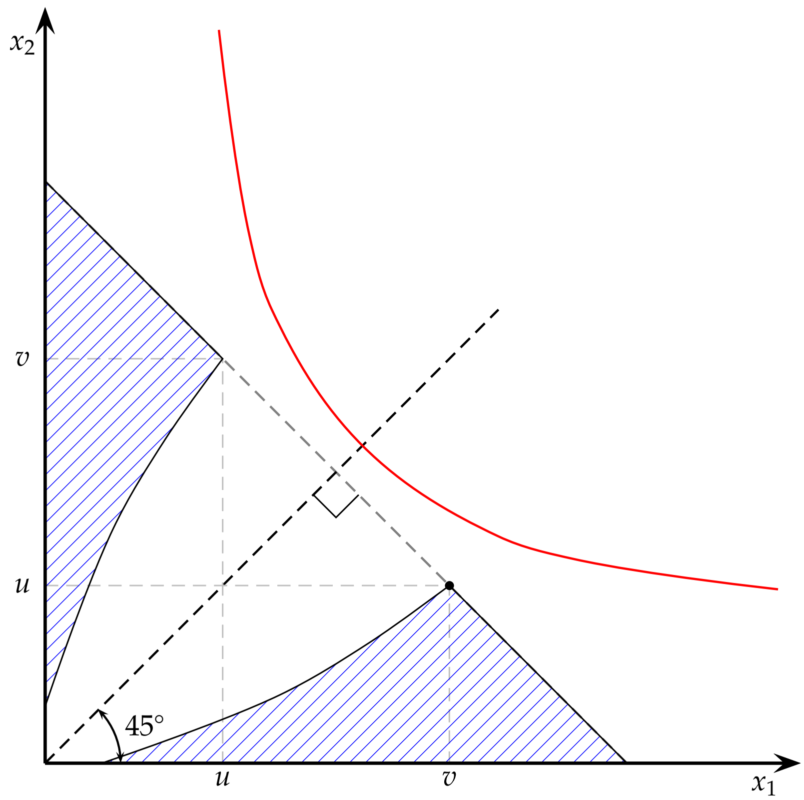


Figure 12: Poverty dominance criteria with intermediate aversion to availability ($\eta = 0.5$) and symmetry

Proof. See appendix B. □

Dominance of A over B for all measures within $\tilde{\Pi}$ requires that the joint distribution of mean income and (the negative of) the distance of income to the mean for distribution A first-order dominates that for B, $\forall(\mu, \tau_\eta) \in \tilde{\Gamma}(\tilde{\lambda}^+)$. Figure 12 shows the two areas over which the joint distributions are assessed for $\mu = (u+v)/2$ and $\tau_{0.5} = (u-\mu)/\mu^{0.5}$. As in the case of the class of poverty indices studied in Section 2.1, symmetry implies that a larger share of the population in one area can be compensated by a lower share in the other.

3.3 Asymmetry

As in Section 2.2, we can relax the symmetry assumption and suppose that income profile (x_1, x_2), with $x_1 < x_2$, leads to greater poverty than (x_2, x_1). With our mean-variability framework, this means that the cost of variability depends on the timing of deprivations. Since profiles within $\Gamma_1(\tilde{\lambda})$ are then worse than their symmetric image within $\Gamma_2(\tilde{\lambda})$, we can consider the following class of intertemporal poverty measures:

$$\tilde{\Pi}_{\eta AS}(\tilde{\lambda}^+_{AS}) = \left\{ P(\lambda) \in \tilde{\Pi}_\eta(\tilde{\lambda}^+_{AS}) \;\middle|\; \begin{array}{l} \rho_1^{(2)}(\mu, \tau_\eta; \tilde{\lambda}) \le \rho_2^{(2)}(\mu, \tau_\eta; \tilde{\lambda}) \\ \rho_1^{(1,2)}(\mu, \tau_\eta; \tilde{\lambda}) \ge \rho_2^{(1,2)}(\mu, \tau_\eta; \tilde{\lambda}). \end{array} \right\} \tag{29}$$

The first condition says that, for given μ, shrinking risk reduces poverty most when income is lowest in the first period. The second condition says that the shrinking effect decreases more

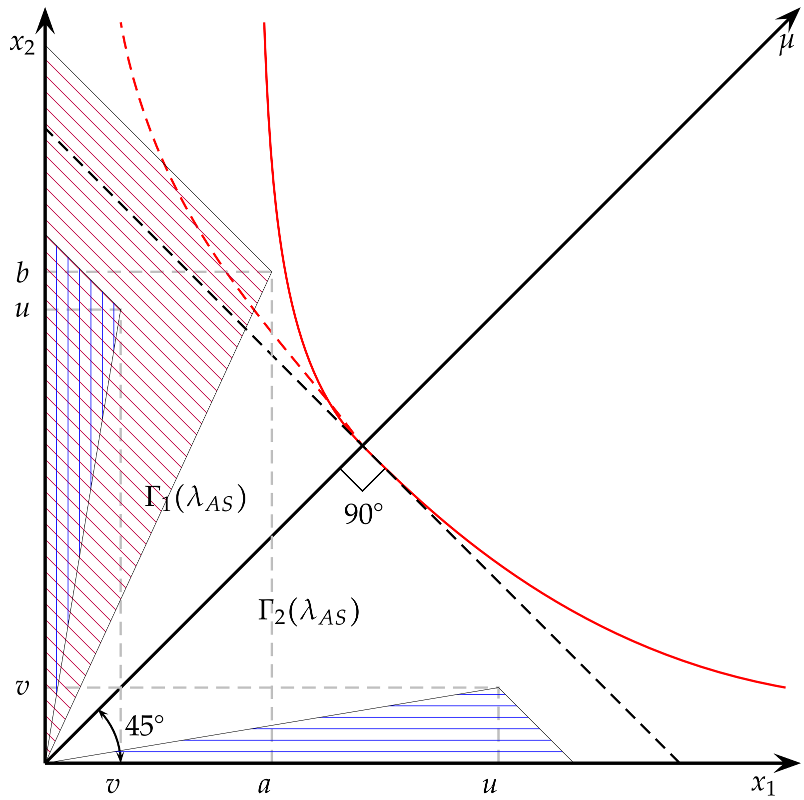


Figure 13: Poverty dominance criteria with relative variability aversion and asymmetry

rapidly with μ when incomes are lower in the first period.

Proposition 7.

$$P_A(\tilde{\lambda}_{AS}) > P_B(\tilde{\lambda}_{AS}),\ \forall P(\tilde{\lambda}_{AS}) \in \tilde{\Pi}_{\eta AS}(\tilde{\lambda}^+_{AS}), \tag{30}$$

$$\textit{iff} \quad q_A P^{1,1}_A(\mu, \tau_\eta | x_1 < x_2) > q_B P^{1,1}_B(\mu, \tau_\eta | x_1 < x_2), \quad \forall(\mu, \tau_\eta) \in \Gamma_1(\tilde{\lambda}^+_{AS}) \tag{31}$$

$$\textit{and} \quad P^{1,1}_A(\mu, \tau_\eta) > P^{1,1}_B(\mu, \tau_\eta), \quad \forall(\mu, \tau_\eta) \in \Gamma_2(\tilde{\lambda}^+_{AS}). \tag{32}$$

Proof. See appendix B. □

Figure 13 illustrates the areas over which dominance tests are performed for asymmetric mean-variability poverty measures and relative variability aversion. Such tests first entail comparing the share of the population that belongs to each triangular area with a side along the x_2 axis and that fits within $\Gamma_1(\tilde{\lambda}_{AS})$. If that share is nowhere lower for each $(a, b) \in \Gamma_1(\tilde{\lambda}_{AS})$, then one turns to the second condition in Proposition 7 and compares the share of the population within the union of two triangular areas, such as those defined by (u, v) and (v, u), for each $(u, v) \in \Gamma_2(\tilde{\lambda}_{AS})$. If this never results in a lower share for A than for B, then it can be concluded that A shows more poverty than B over the class of asymmetric mean-variability poverty indices and over the set of poverty frontiers lying within the maximum poverty domain $\Gamma\left(\tilde{\lambda}^+_{AS}\right)$. As in the case of the asymmetric poverty indices of Proposition 3, the dominance criteria of Proposition 7 have a greater ranking power than those for the general class of mean-variability poverty indices (Proposition 4). This power is weaker, however, than for the subclass of symmetric mean-variability indices considered in Proposition 6.

4 On the relationships between the dominance criteria

Each of the classes $\tilde{\Pi}(\tilde{\lambda}^+)$ and $\ddot{\Pi}(\lambda^+)$ (and they are their symmetric and asymmetric subclasses) of poverty measures presents appealing properties, but may not be individually regarded as fully satisfying in an intertemporal context. Take for instance an income profile (a, b), with $b > a$. If b increases, average income also increases but variability τ_η rises for all η, so that the net poverty effect is ambiguous over the class $\tilde{\Pi}(\tilde{\lambda}^+)$. Conversely, a transfer $\iota > 0$ that leads to $(a + \iota, b - \iota)$, with $2\iota < b - a$, reduces variability without affecting mean income, but cannot be regarded as favorable over the class $\ddot{\Pi}(\lambda^+)$ since it leads to a fall in one of the two income values.

We may seek to address this difficulty by considering poverty indices $P(\lambda)$ that simultaneously belong to the above two classes. Define $\breve{\Pi}_\eta(\tilde{\lambda}^+)$ as their intersection, that is:

$$\breve{\Pi}_\eta(\tilde{\lambda}^+) = \left\{P(\tilde{\lambda}) \in \ddot{\Pi}(\tilde{\lambda}^+) \cap \tilde{\Pi}_\eta(\tilde{\lambda}^+)\right\}. \tag{33}$$

As an illustration of membership into the class $\breve{\Pi}_\eta(\tilde{\lambda}^+)$, we can consider some members of the family of union bidimensional poverty indices P_{BC} suggested by Bourguignon and Chakravarty (2003). For a population of size n, P_{BC} is defined as:

$$P_{BC} = \frac{1}{n}\sum_{i=1}^{n}\left(a\,(1 - x_{1i})_+^{\beta} + (1 - a)\,(1 - x_{2i})_+^{\beta}\right)^{\frac{\alpha}{\beta}}, \tag{34}$$

where x_{ji} denotes the income of the ith poor person at time j, $(y)_+ = \max(0, y)$, and where poverty lines have been normalized to 1 at each period. For P_{BC} to be a member of $\ddot{\Pi}(\lambda^+)$, it is necessary that $\beta \geq 1$ and $\alpha \geq \beta$. It can be shown that for $a = 0.5$,[15] one then obtains a family of measures $\breve{P}_{BC}$ that is included in $\breve{\Pi}_\eta(\tilde{\lambda}^+)$ since the measure can alternatively be expressed as:

$$\breve{P}_{BC} = \frac{1}{n}\sum_{i=1}^{n}\left(0.5\,(1 - \mu_i - \tau_{0i})_+^{\beta} + 0.5\,(1 - \mu_i + \tau_{0i})_+^{\beta}\right)^{\frac{\alpha}{\beta}}. \tag{35}$$

Now consider the additional restrictions that need to be imposed on members of $\ddot{\Pi}(\tilde{\lambda}^+)$ to be also members of the subclass $\breve{\Pi}_\eta(\tilde{\lambda}^+)$. Since the elements of $\breve{\Pi}_\eta(\tilde{\lambda}^+)$ also belong to $\tilde{\Pi}_\eta(\tilde{\lambda}^+)$, the derivatives of π with respect to μ and τ_η have to comply with the restrictions imposed on ρ. While the condition $\pi^{(\mu)}(x_1, x_2) \leq 0$ is systematically obeyed with the restrictions imposed on $\pi^{(1)}$ and $\pi^{(2)}$ (see appendix C), the conditions $\pi^{(\tau_\eta)}(x_1, x_2) \leq 0$ and $\pi^{(\mu,\tau_\eta)}(x_1, x_2) \leq 0$ respectively require (whenever $x_i \leq x_j$) that:

$$\pi^{(i)}(x_1, x_2) - \pi^{(j)}(x_1, x_2) \leq 0, \tag{36}$$

$$\begin{aligned}\eta\left(\pi^{(i)}(x_1, x_2) - \pi^{(j)}(x_1, x_2)\right) + \left(\mu + \eta\tau_0\right)\left(\pi^{(i,i)}(x_1, x_2) - \pi^{(i,j)}(x_1, x_2)\right)& \\ +\left(\mu - \eta\tau_0\right)\left(\pi^{(i,j)}(x_1, x_2) - \pi^{(j,j)}(x_1, x_2)\right) \geq 0,&\end{aligned} \tag{37}$$

Condition (37) shows no direct interpretation but condition (36) says that the effect on the

[15] Equal weights for each deprivation are necessary in order to obtain individual poverty indices that are decreasing with respect to τ_0.

lower income of decreasing variability dominates the effect on the larger one. In the case of symmetric poverty measures, condition (36) can also be stated as $\pi^{(1,1)}(x_1, x_2) = \pi^{(2,2)}(x_2, x_1) \geq 0$, which is a well-known convexity property for poverty functions.[16] Since all second-order derivatives are then positive, it can be shown that members from $\breve{\Pi}_{\eta S}(\tilde{\lambda}^+)$ comply with a multidimensional extension of the Pigou-Dalton transfer, *i.e.* a progressive transfer at any period between to individuals that can unambiguously be ranked in terms of poverty do not increase poverty.

Consider now the members of $\tilde{\Pi}_\eta(\tilde{\lambda}^+)$ that also belong to the subclass $\breve{\Pi}_\eta(\tilde{\lambda}^+)$. For these indices, ρ must be such that $\rho_t^{(x_1)}(\mu, \tau_\eta) \leq 0$, $\rho_t^{(x_2)}(\mu, \tau_\eta) \leq 0$, and $\rho_t^{(x_1,x_2)}(\mu, \tau_\eta) \geq 0 \; \forall t = 1, 2$. The first two conditions are automatically respected when ρ_t is derived with respect to the lower value of the income profile; increasing that income simultaneously raises average income and reduces variability, so that such an income increment would undoubtedly decrease poverty. With an increase of the larger income, the conditions on the first-order derivatives of ρ_t are satisfied if and only if $\forall t$:

$$\rho_t^{(1)}(\mu, \tau_\eta) \leq \left(\mu^{-\eta} + \frac{\eta \tau_\eta}{\mu}\right) \rho_t^{(2)}(\mu, \tau_\eta), \tag{40}$$

which says that the mean effect dominates the risk effect, as would be the case for all members of $\ddot{\Pi}(\lambda)$. Regarding the cross-derivative condition, its sign is positive if and only if:

$$\rho_t^{(1,1)}(\mu, \tau_\eta) - 2\frac{\eta \tau_\eta}{\mu} \rho_t^{(1,2)}(\mu, \tau_\eta) - \frac{\eta(\eta+1)\tau_\eta}{2\mu^2} \rho_t^{(2)}(\mu, \tau_\eta) + \left(\left(\frac{\eta \tau_\eta}{\mu}\right)^2 - \mu^{-2\eta}\right) \rho_t^{(2,2)}(\mu, \tau_\eta) \geq 0. \tag{41}$$

which, in the case of absolute-variability aversion, states that the poverty-reducing effect of mean increases should decrease more rapidly with mean income than does the poverty reducing effect of lowering variability with respect to variability.

Let $A \ddot{\succcurlyeq}_\lambda B$ indicate dominance of A with respect to B over $\ddot{\Pi}(\lambda)$ (*cf.* Proposition 1).

Proposition 8.

$$P_A(\tilde{\lambda}) > P_B(\tilde{\lambda}), \; \forall P(\tilde{\lambda}) \in \breve{\Pi}_\eta(\tilde{\lambda}^+), \tag{42}$$

$$\textit{if} \quad A \ddot{\succcurlyeq}_{\tilde{\lambda}^+} B \tag{43}$$

$$\textit{or} \quad A \tilde{\succcurlyeq}_{\eta, \tilde{\lambda}^+} B. \tag{44}$$

Proof. See Appendix C. ☐

Proposition 8 highlights the complementary nature of the dominance relationships $\ddot{\succcurlyeq}_\lambda$ and $\tilde{\succcurlyeq}_{\eta, \tilde{\lambda}}$, shown through the "hybrid" class of intertemporal poverty indices $\breve{\Pi}_\eta(\tilde{\lambda}^+)$. If one fails to

[16] Assuming $x_1 < x_2$, symmetry means that condition (36) can be expressed as:

$$\pi^{(1)}(x_1, x_2) \leq \pi^{(1)}(x_2, x_1). \tag{38}$$

At the same time, we know that $\pi^{(1,2)}(x_1, x_2) \geq 0$, *i.e.* $\pi^{(1)}(x_1, x_1) - \pi^{(1)}(x_1, x_2) \leq 0$. Combining this expression with (38) yields:

$$\pi^{(1)}(x_1, x_1) \leq \pi^{(1)}(x_2, x_1). \tag{39}$$

which entails that second-order derivatives of π are non-negative $\forall (x_1, x_2)$.

observe a dominance relationship using Proposition 1, dominance my still be obtained using Proposition 4, and vice-versa. Consider for instance a distribution A made of two poor income profiles $(3,4)$ and $(7,1)$. Suppose that a distribution B is obtained by changing the second income profile into $(6,2)$ using some variability reducing transfer. It can be checked that the two distributions A and B cannot be compared using the cumulative distribution function of (x_1, x_2) as in Proposition 1. However, whatever the value of η, the cumulative distribution functions at (μ, τ_η) are never larger for B than for A, so that it can be concluded that B has less poverty than A for all poverty indices in $\breve{\Pi}_\eta(\tilde{\lambda}^+)$, some of them members of $\ddot{\Pi}(\lambda^+)$.

Corollary 1. *Assuming* $\exists (x_1, x_2) \in \Gamma(\tilde{\lambda}^+)$ *such that* $P_A^{1,1}(x_1, x_2) \neq P_B^{1,1}(x_1, x_2)$, *the following result cannot be obtained:*

$$A \ddot{\succcurlyeq}_{\tilde{\lambda}^+} B \text{ and } B \tilde{\succcurlyeq}_{\eta,\tilde{\lambda}^+} A. \tag{45}$$

Proof. See appendix C. □

Corollary 1 says that if one observes that A is dominated by B over $\ddot{\Pi}(\tilde{\lambda}^+)$ ($\tilde{\Pi}_\eta(\tilde{\lambda}^+)$), one would try in vain to infer that B is dominated by A over $\tilde{\Pi}_\eta(\tilde{\lambda}^+)$ ($\ddot{\Pi}(\tilde{\lambda}^+)$). Checking dominance of the type $\ddot{\succcurlyeq}_{\tilde{\lambda}^+}$ ($\tilde{\succcurlyeq}_{\eta,\tilde{\lambda}^+}$) can thus provide information on dominance of type $\tilde{\succcurlyeq}_{\eta,\tilde{\lambda}^+}$ ($\ddot{\succcurlyeq}_{\tilde{\lambda}^+}$) since both dominance criteria apply to classes of poverty measures that include the set of "hybrid" indices $\breve{\Pi}_\eta(\tilde{\lambda}^+)$.

5 An illustration with European data

To illustrate the issues that arise when performing intertemporal poverty comparisons and the usefulness of the tools proposed in section 2 to 4, we apply those tools to income data from 23 European countries.[17] These data come from the 2009 version of the EU-SILC (European Union Survey on Income and Living Conditions) database. For each country, we select individuals that were surveyed both in 2006 and 2009. The 2006-09 period is interesting since the European crisis may have resulted in a greater variability of income, with an intensity that may, however, have been different across countries due to differences in social safety net systems.

The dominance checks are performed using adult-equivalent disposable income obtained with the OECD equivalence scale. Purchasing power differences were taken into account using Eurostat PPP indices, and CPI indices were also used to compare income across both periods.[18] The maximum poverty domain is defined using a "union" approach; individuals are thus regarded as poor if they were suffering from monetary deprivation either in 2006 or 2009. The

[17] Data for the following countries are used in the present paper: Austria (AT), Belgium (BE), Bulgaria (BG), Cyprus(CY), Czech Republic (CZ), Denmark (DK), Estonia (EE), Spain (ES), Finland (FI), France (FR), Hungary (HU), Iceland (IS), Italy (IT), Latvia (LV), Lithuania (LT), Malta (MT), Netherlands (NL), Norway (NO), Poland (PL), Portugal (PT), Slovenia (SI), Sweden (SE), and United Kingdom (UK).

[18] Incomes were censored at the bottom, so that our results should be regarded as restricted dominance tests (see Davidson and Duclos, 2006, for the theoretical and practical arguments). Censoring was applied at the second centile and below of the pooled distribution of incomes in 2006 and 2009, that is, at around 2,100€ per person and per year.

maximal deprivation line was set to 120% of the overall median income, that is about 15,350€ per person and per year.[19]

5.1 Symmetric and asymmetric dominance within the Duclos et al.'s (2006) framework

The results of the 253 pairwise comparisons performed using these samples and the dominance criteria proposed in Section 2 are presented in Table 1. It is worth remembering that symmetry is a limiting case of asymmetry, so that observing a dominance relationship with asymmetry entails that dominance necessarily also holds with symmetry. As asymmetry is a special case of the general case described by Proposition 1, we also know that dominance with asymmetry can be observed when the usual first-order general dominance is observed. For these reasons, Table 1 only reports the dominance results that correspond to the broadest classes of intertemporal indices. With our setting, we note that about 46% of the comparisons result in a robust result using the Duclos et al.'s (2006) first-order dominance procedure (*cf.* Proposition 1).

Since asymmetry can be understood in different manners, the dominance checks are performed using two rival versions, reflecting different tastes with respect to the patterns of income profiles. The first version of the asymmetry assumption considers that an income profile (a, b) with $a > b$ is never poverty-preferred to a symmetric profile (b, a). Such a view is consistent with the concept of loss aversion. Loss aversion is an important assumption in prospect theory and suggests that losses (of a given magnitude) outweigh gains (of the same magnitude) in terms of well-being. In other words, it is supposed that individuals prefer income profiles with upward patterns. The second version of the asymmetry assumption supports the opposite view, that is, that an income profile (a, b) with $a < b$ is never poverty-preferred to a symmetric profile (b, a). This view is consistent with aversion to early poverty. Income deprivations may have long-lasting effects on people's abilities to enjoy a valuable life, particularly if experienced during childhood. Consequently, the earlier a deprivation occurs, the longer its effects may last. It can then be put forward that intertemporal poverty is harsher when the more severe deprivations occur during the first period(s). Both versions of asymmetry rely on reasonable and documented arguments, so that it cannot easily be said which one is most appropriate in the present context.

Asymmetry increases our ability to order our set of European countries in a significant manner in comparison with the general case; the ordering power increases from 46% to 52% with loss aversion and to 49% with aversion to early poverty. The increase in the ordering power is higher with loss aversion, hence indicating that it is more difficult to compare the countries represented in our sample over income profiles in the subset $\Gamma_1(\lambda_S^+)$, that is, over relatively low first-period income profiles. With symmetry, the ordering power increases up to 55%. In most cases, the dominance relationships with symmetry corresponds to comparisons that are also robust either with loss aversion or with aversion to early poverty. Indeed, symme-

[19] This figure is almost equal to the value of the median income for Italy over the period. This choice is quite conservative but would undoubtedly meet unanimous agreement as a value above which an individual cannot be considered as deprived in the European context. Moreover, it is worth emphasizing that the exercise is merely illustrative.

Table 1: First-order dominance tests within the (x_1, x_2)-space.

Country	BE	BG	CY	CZ	DK	EE	ES	FI	FR	HU	IS	IT	LT	LV	MT	NL	NO	PL	PT	SE	SI	UK
AT	$\varnothing$	$\ddot{\succcurlyeq}$	$\ddot{\preccurlyeq}^S$	$\varnothing$	$\varnothing$	$\ddot{\succcurlyeq}$	$\ddot{\succcurlyeq}$	$\varnothing$	$\ddot{\succcurlyeq}^S$	$\ddot{\succcurlyeq}^+$	$\varnothing$	$\ddot{\succcurlyeq}$	$\ddot{\succcurlyeq}$	$\ddot{\succcurlyeq}$	$\ddot{\succcurlyeq}^+$	$\varnothing$	$\varnothing$	$\ddot{\succcurlyeq}$	$\ddot{\succcurlyeq}$	$\varnothing$	$\varnothing$	$\ddot{\succcurlyeq}^+$
BE	$\dots$	$\ddot{\succcurlyeq}$	$\ddot{\preccurlyeq}^+$	$\varnothing$	$\varnothing$	$\ddot{\succcurlyeq}$	$\ddot{\succcurlyeq}$	$\varnothing$	$\varnothing$	$\ddot{\succcurlyeq}$	$\varnothing$	$\ddot{\succcurlyeq}$	$\ddot{\succcurlyeq}$	$\ddot{\succcurlyeq}$	$\varnothing$	$\varnothing$	$\varnothing$	$\ddot{\succcurlyeq}$	$\ddot{\succcurlyeq}$	$\varnothing$	$\varnothing$	$\varnothing$
BG	$\dots$	$\dots$	$\ddot{\preccurlyeq}$	$\ddot{\preccurlyeq}$	$\ddot{\preccurlyeq}$	$\ddot{\preccurlyeq}$	$\ddot{\preccurlyeq}$	$\ddot{\preccurlyeq}$	$\ddot{\preccurlyeq}$	$\ddot{\preccurlyeq}^+$	$\ddot{\preccurlyeq}$	$\ddot{\preccurlyeq}$	$\ddot{\preccurlyeq}$	$\ddot{\preccurlyeq}^-$	$\ddot{\preccurlyeq}$	$\ddot{\preccurlyeq}$	$\ddot{\preccurlyeq}$	$\ddot{\preccurlyeq}$	$\ddot{\preccurlyeq}$	$\ddot{\preccurlyeq}$	$\ddot{\preccurlyeq}$	$\ddot{\preccurlyeq}$
CY	$\dots$	$\dots$	$\dots$	$\ddot{\succcurlyeq}$	$\varnothing$	$\ddot{\succcurlyeq}$	$\ddot{\succcurlyeq}$	$\ddot{\succcurlyeq}^+$	$\ddot{\succcurlyeq}^+$	$\ddot{\succcurlyeq}$	$\varnothing$	$\ddot{\succcurlyeq}$	$\ddot{\succcurlyeq}$	$\ddot{\succcurlyeq}$	$\ddot{\succcurlyeq}$	$\varnothing$	$\varnothing$	$\ddot{\succcurlyeq}$	$\ddot{\succcurlyeq}$	$\ddot{\succcurlyeq}^+$	$\ddot{\succcurlyeq}$	$\ddot{\succcurlyeq}$
CZ	$\dots$	$\dots$	$\dots$	$\dots$	$\varnothing$	$\ddot{\succcurlyeq}^-$	$\varnothing$	$\varnothing$	$\varnothing$	$\varnothing$	$\varnothing$	$\varnothing$	$\ddot{\succcurlyeq}$	$\varnothing$	$\varnothing$	$\varnothing$	$\varnothing$	$\varnothing$	$\varnothing$	$\varnothing$	$\varnothing$	$\varnothing$
DK	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\varnothing$	$\varnothing$	$\varnothing$	$\varnothing$	$\varnothing$	$\varnothing$	$\varnothing$	$\ddot{\succcurlyeq}$	$\ddot{\succcurlyeq}$	$\varnothing$	$\varnothing$	$\varnothing$	$\varnothing$	$\ddot{\succcurlyeq}$	$\varnothing$	$\varnothing$	$\varnothing$
EE	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\varnothing$	$\ddot{\preccurlyeq}$	$\ddot{\preccurlyeq}$	$\varnothing$	$\ddot{\preccurlyeq}$	$\ddot{\preccurlyeq}$	$\ddot{\succcurlyeq}$	$\varnothing$	$\ddot{\preccurlyeq}$	$\ddot{\preccurlyeq}$	$\ddot{\preccurlyeq}$	$\varnothing$	$\varnothing$	$\ddot{\preccurlyeq}$	$\ddot{\preccurlyeq}$	$\ddot{\preccurlyeq}$
ES	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\ddot{\preccurlyeq}$	$\ddot{\preccurlyeq}$	$\varnothing$	$\ddot{\preccurlyeq}$	$\ddot{\preccurlyeq}$	$\ddot{\succcurlyeq}^+$	$\ddot{\succcurlyeq}^+$	$\varnothing$	$\ddot{\preccurlyeq}$	$\ddot{\preccurlyeq}$	$\varnothing$	$\varnothing$	$\ddot{\preccurlyeq}$	$\varnothing$	$\ddot{\preccurlyeq}$
FI	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\varnothing$	$\ddot{\succcurlyeq}$	$\varnothing$	$\ddot{\succcurlyeq}$	$\ddot{\succcurlyeq}$	$\ddot{\succcurlyeq}$	$\varnothing$	$\varnothing$	$\varnothing$	$\ddot{\succcurlyeq}$	$\ddot{\succcurlyeq}$	$\varnothing$	$\varnothing$	$\varnothing$
FR	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\varnothing$	$\varnothing$	$\ddot{\succcurlyeq}$	$\ddot{\succcurlyeq}$	$\ddot{\succcurlyeq}$	$\varnothing$	$\varnothing$	$\varnothing$	$\ddot{\succcurlyeq}$	$\ddot{\succcurlyeq}$	$\varnothing$	$\varnothing$	$\varnothing$
HU	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\ddot{\preccurlyeq}$	$\varnothing$	$\varnothing$	$\varnothing$	$\varnothing$	$\ddot{\preccurlyeq}^+$	$\ddot{\preccurlyeq}$	$\varnothing$	$\varnothing$	$\varnothing$	$\ddot{\preccurlyeq}^-$	$\varnothing$
IS	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\ddot{\succcurlyeq}$	$\ddot{\succcurlyeq}$	$\ddot{\succcurlyeq}$	$\varnothing$	$\varnothing$	$\varnothing$	$\ddot{\succcurlyeq}$	$\ddot{\succcurlyeq}$	$\varnothing$	$\varnothing$	$\ddot{\succcurlyeq}^-$
IT	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\ddot{\succcurlyeq}$	$\ddot{\succcurlyeq}$	$\varnothing$	$\ddot{\preccurlyeq}$	$\ddot{\preccurlyeq}^-$	$\varnothing$	$\ddot{\succcurlyeq}$	$\ddot{\preccurlyeq}$	$\varnothing$	$\ddot{\preccurlyeq}$
LT	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\varnothing$	$\ddot{\preccurlyeq}$	$\ddot{\preccurlyeq}$	$\ddot{\preccurlyeq}$	$\ddot{\preccurlyeq}$	$\ddot{\preccurlyeq}$	$\ddot{\preccurlyeq}$	$\ddot{\preccurlyeq}$	$\ddot{\preccurlyeq}$
LV	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\ddot{\preccurlyeq}$	$\ddot{\preccurlyeq}$	$\ddot{\preccurlyeq}$	$\varnothing$	$\ddot{\preccurlyeq}$	$\ddot{\preccurlyeq}$	$\ddot{\preccurlyeq}$	$\ddot{\preccurlyeq}$
MT	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\ddot{\preccurlyeq}^-$	$\varnothing$	$\ddot{\succcurlyeq}^+$	$\ddot{\succcurlyeq}$	$\varnothing$	$\varnothing$	$\varnothing$
NL	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\varnothing$	$\ddot{\succcurlyeq}$	$\ddot{\succcurlyeq}$	$\varnothing$	$\varnothing$	$\varnothing$
NO	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\ddot{\succcurlyeq}$	$\ddot{\succcurlyeq}$	$\varnothing$	$\varnothing$	$\varnothing$
PL	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\varnothing$	$\ddot{\preccurlyeq}^+$	$\ddot{\preccurlyeq}$	$\ddot{\preccurlyeq}^+$
PT	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\ddot{\preccurlyeq}$	$\ddot{\preccurlyeq}$	$\ddot{\preccurlyeq}$
SE	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\varnothing$	$\varnothing$
SI	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\varnothing$

Note: $\ddot{\succcurlyeq}$ ($\ddot{\preccurlyeq}$) indicates that the first distribution dominates (is dominated by) the second distribution at the correspond order of dominance. Ø denotes a non-conclusive test. The "+", "-", and "S" subscripts indicate that dominance holds only when the asymmetry (loss aversion), asymmetry (early poverty), and symmetry assumptions are used.

try is necessary to obtain a dominance relationship only in two cases, that is, when comparing Austria with Cyprus and France.

Since symmetry and asymmetry are ways of extending the ordering power for intertemporal poverty comparisons, we also contrast the results presented in Table 1 with those obtained with second order dominance. In a multidimensional framework, second-order dominance means that poverty comparisons are made with respect to members from a subclass of $\ddot{\Pi}(\lambda^+)$ that are sensitive to inequalities between the poor (more details in Duclos et al., 2006). It is then supposed that a progressive transfer at a given period between two individuals that can unambiguously be ranked in terms of well-being reduces poverty. Moreover, the second-order derivatives of the individual poverty index π are decreasing and convex with respect to the level of other periods income, *i.e.*, the poverty reducing effect of a progressive transfer at a given period is decreasing (at a decreasing rate) with the level of income at the other period. Results of second-order dominance tests are provided by Table 2 and show that 62% of the comparisons are now conclusive.

A few remarks are useful in this regard. First, second-order dominance requires cardinality of the different attributes used to assess poverty, whereas ordinality is sufficient for first-order dominance checks. So, if intertemporal poverty comparisons were performed for instance on health statuses at different periods, then second-order dominance checks could not plausibly be used, and symmetry and asymmetry assumptions would be more natural avenues to extend the ordering power of first-order dominance tests. Secondly, for many pairwise comparisons, first-order dominance tests with asymmetry or symmetry yield robust comparisons that cannot be observed with second-order dominance, and *vice versa*. Such situations are observed for 9% of the comparisons from our samples, 22 out of the 28 additional orderings being observed with the help of second-order dominance procedures.[20] Finally, even tough the results of Table 1 look like those of Table 2, the two approaches should be considered as complements, not as substitutes, since observing for instance both first-order dominance with loss aversion and second-order dominance means two populations can be ranked for a class of poverty indices that is larger than the class of distribution-sensitive poverty indices, thus widening the scope of agreement on how intertemporal poverty be ranked across two populations.

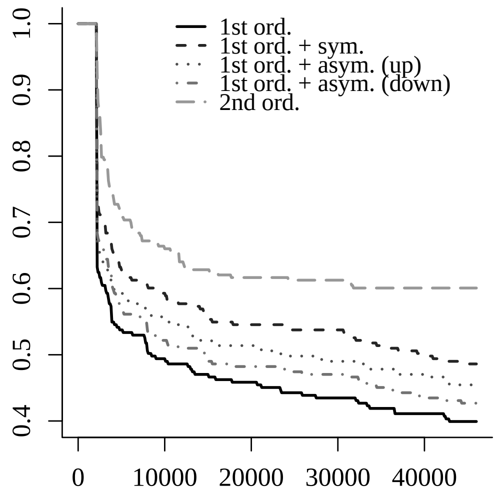
The ordering power is likely to be contingent on the definition of the maximum poverty domain λ_S^+. To look into this, we estimate the share of pairwise comparisons yielding dominance relationships for different maximum values of the deprivation lines, up to 45,000€. The results are reported on Figure 14. Notice first that the absolute difference in ordering power between the different first-order dominance procedures does not significantly change with the value of the maximum deprivation line. In particular, asymmetry with loss aversion performs systematically better than asymmetry with aversion to early poverty. This result is likely to be due to bad economic performances in many countries from our sample during the period 2006-09; increasing unemployment and lower incomes yield joint distributions with a larger

[20] Although the propositions from Section 2 only deal with first-order dominance, they could be extended to second-order dominance procedures, yielding greater ranking power. Since second-order dominance with symmetry should be observed for those pairwise comparisons that shows first-order dominance with symmetry or second-order dominance, we could expect the corresponding ordering power to be at least equal to 64%. The analysis of the related class of poverty indices and its properties would of course deserve more scrutiny.

Table 2: Second-order dominance tests within the (x_1, x_2)-space.

Country	BE	BG	CY	CZ	DK	EE	ES	FI	FR	HU	IS	IT	LT	LV	MT	NL	NO	PL	PT	SE	SI	UK
AT	∅	≽	≼	∅	∅	≽	≽	∅	∅	≽	∅	≽	≽	≽	≽	∅	∅	≽	≽	∅	∅	≽
BE	…	≽	∅	∅	∅	≽	≽	∅	∅	≽	≼	≽	≽	≽	≽	∅	∅	≽	≽	∅	∅	∅
BG	…	…	≼	≼	≼	≼	≼	≼	≼	≼	≼	≼	≼	≼	≼	≼	≼	≼	≼	≼	≼	≼
CY	…	…	…	≽	∅	≽	≽	∅	∅	≽	∅	≽	≽	≽	≽	∅	∅	≽	≽	∅	≽	≽
CZ	…	…	…	…	∅	≽	∅	∅	∅	≽	≼	∅	≽	≽	∅	≼	∅	≽	∅	∅	∅	∅
DK	…	…	…	…	…	∅	≽	∅	∅	∅	≼	∅	≽	≽	∅	∅	∅	∅	≽	≼	∅	∅
EE	…	…	…	…	…	…	∅	≼	≼	∅	≼	≼	≽	≽	≼	≼	≼	∅	∅	≼	≼	≼
ES	…	…	…	…	…	…	…	≼	≼	∅	≼	≼	≽	≽	≼	≼	≼	∅	∅	≼	≼	≼
FI	…	…	…	…	…	…	…	…	∅	≽	∅	≽	≽	≽	≽	∅	∅	≽	≽	∅	∅	≽
FR	…	…	…	…	…	…	…	…	…	∅	≼	≽	≽	≽	∅	∅	∅	≽	≽	∅	∅	∅
HU	…	…	…	…	…	…	…	…	…	…	≼	∅	≽	≽	∅	≼	≼	∅	∅	∅	≼	∅
IS	…	…	…	…	…	…	…	…	…	…	…	≽	≽	≽	≽	∅	∅	≽	≽	∅	≽	≽
IT	…	…	…	…	…	…	…	…	…	…	…	…	≽	≽	∅	≼	≼	≽	≽	≼	∅	≼
LT	…	…	…	…	…	…	…	…	…	…	…	…	…	∅	≼	≼	≼	≼	≼	≼	≼	≼
LV	…	…	…	…	…	…	…	…	…	…	…	…	…	…	≼	≼	≼	≼	≼	≼	≼	≼
MT	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	≼	∅	≽	≽	∅	∅	∅
NL	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	∅	≽	≽	∅	∅	≽
NO	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	≽	≽	∅	∅	∅
PL	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	∅	≼	≼	≼
PT	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	≼	≼	≼
SE	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	∅	∅
SI	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	∅

Note: ≽ (≼) indicates that the first distribution dominates (is dominated by) the second distribution at the correspond order of dominance. Ø denotes a non-conclusive test.



Note: Each point gives the proportion of cross-country dominance relationships observed with the EU-SILC data for a maximal union poverty domain whose bounds are given by the x-axis.

Figure 14: Ordering power with different maximal deprivation lines.

population within the relatively low second-period income domain $\Gamma_2(\lambda_S^+)$, that is the set of poverty profiles on which emphasis is given with loss aversion. An other interesting result is that the gap between the second-order dominance procedure and the different first-order dominance procedures is relatively constant, and widens significantly when the deprivation line increases above 30,000€.

5.2 Mean-variability dominance

Our second set of dominance tests are made on the classes of mean-variability intertemporal poverty indices presented in Section 3. A potential problem with these classes deals with the choice of a value for the parameter η. Proposition 5 shows, however, that much can be said by focussing on the absolute ($\eta = 0$) and relative ($\eta = 1$) bounds. The results (not shown here) are somewhat surprising: we are unable to obtain any robust comparison using absolute risk aversion, even when imposing symmetry. The relative rankings should in principle be stronger, since the ordering power of relative risk aversion is theoretically greater (as shown by Proposition 5). Our results shows, however, that the ordering power increases little with relative risk aversion since only one dominance relationship is obtained, between Cyprus and Spain. This limited ranking power is in large part due to the presence in each distribution of highly volatile income profiles, which make it difficult to establish dominance over large areas of mean-variability thresholds.

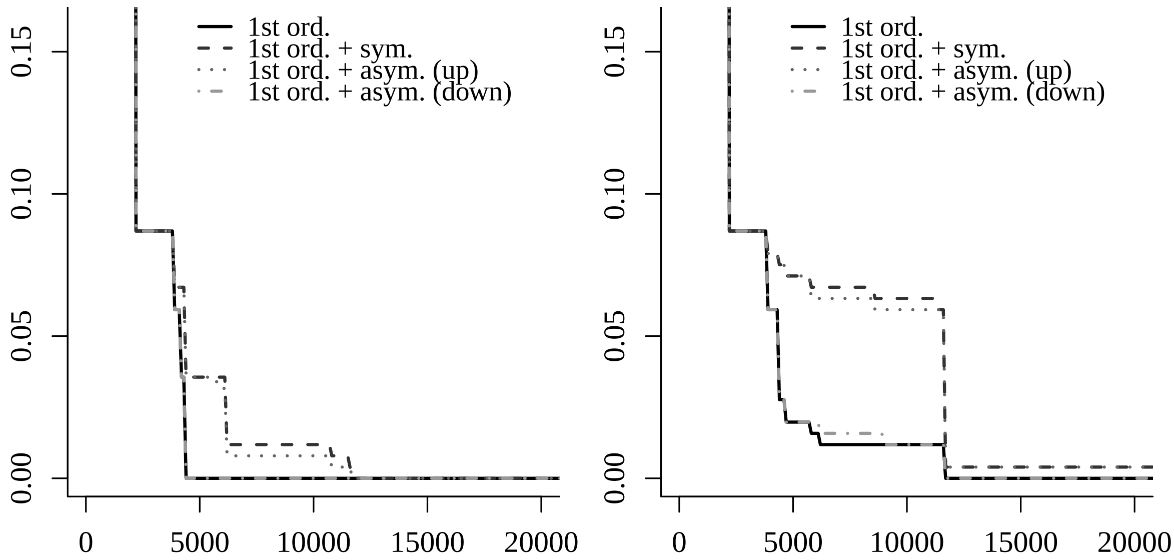
The role of income variability in explaining these results can be seen by comparing these results with the ones presented in Table 3. Table 3 keeps the same value for the deprivation line but consider a subset of the class of intertemporal poverty indices $\tilde{\Pi}(\tilde{\lambda}_S)$ for which individual poverty is not affected by income variability; said differently, perfect individual-level pooling of incomes is assumed, so that mean temporal income is all that matters for assessing the well-being of the poor. The dominance criterion compares the distribution functions of

Table 3: First-order dominance tests with mean-variability poverty indices: perfect pooling.

Country	BE	BG	CY	CZ	DK	EE	ES	FI	FR	HU	IS	IT	LT	LV	MT	NL	NO	PL	PT	SE	SI	UK
AT	∅	$\succcurlyeq_\mu$	$\preccurlyeq_\mu$	∅	∅	$\succcurlyeq_\mu$	$\succcurlyeq_\mu$	∅	$\succcurlyeq_\mu$	$\succcurlyeq_\mu$	∅	$\succcurlyeq_\mu$	$\succcurlyeq_\mu$	$\succcurlyeq_\mu$	$\succcurlyeq_\mu$	∅	∅	$\succcurlyeq_\mu$	$\succcurlyeq_\mu$	∅	∅	$\succcurlyeq_\mu$
BE	…	$\succcurlyeq_\mu$	$\preccurlyeq_\mu$	$\succcurlyeq_\mu$	∅	$\succcurlyeq_\mu$	$\succcurlyeq_\mu$	∅	∅	$\succcurlyeq_\mu$	$\preccurlyeq_\mu$	$\succcurlyeq_\mu$	$\succcurlyeq_\mu$	$\succcurlyeq_\mu$	$\succcurlyeq_\mu$	$\preccurlyeq_\mu$	∅	$\succcurlyeq_\mu$	$\succcurlyeq_\mu$	∅	$\succcurlyeq_\mu$	∅
BG	…	…	$\preccurlyeq_\mu$	$\preccurlyeq_\mu$	$\preccurlyeq_\mu$	$\preccurlyeq_\mu$	$\preccurlyeq_\mu$	$\preccurlyeq_\mu$	$\preccurlyeq_\mu$	$\preccurlyeq_\mu$	$\preccurlyeq_\mu$	$\preccurlyeq_\mu$	$\preccurlyeq_\mu$	$\preccurlyeq_\mu$	$\preccurlyeq_\mu$	$\preccurlyeq_\mu$	$\preccurlyeq_\mu$	$\preccurlyeq_\mu$	$\preccurlyeq_\mu$	$\preccurlyeq_\mu$	$\preccurlyeq_\mu$	$\preccurlyeq_\mu$
CY	…	…	…	$\succcurlyeq_\mu$	∅	$\succcurlyeq_\mu$	$\succcurlyeq_\mu$	$\succcurlyeq_\mu$	$\succcurlyeq_\mu$	$\succcurlyeq_\mu$	∅	$\succcurlyeq_\mu$	$\succcurlyeq_\mu$	$\succcurlyeq_\mu$	$\succcurlyeq_\mu$	∅	∅	$\succcurlyeq_\mu$	$\succcurlyeq_\mu$	$\succcurlyeq_\mu$	$\succcurlyeq_\mu$	$\succcurlyeq_\mu$
CZ	…	…	…	…	∅	∅	∅	$\preccurlyeq_\mu$	∅	$\succcurlyeq_\mu$	$\preccurlyeq_\mu$	∅	$\succcurlyeq_\mu$	$\succcurlyeq_\mu$	∅	$\preccurlyeq_\mu$	∅	$\succcurlyeq_\mu$	∅	∅	∅	∅
DK	…	…	…	…	…	∅	∅	∅	∅	∅	$\preccurlyeq_\mu$	∅	$\succcurlyeq_\mu$	$\succcurlyeq_\mu$	∅	∅	∅	∅	$\succcurlyeq_\mu$	∅	∅	∅
EE	…	…	…	…	…	…	∅	$\preccurlyeq_\mu$	$\preccurlyeq_\mu$	∅	$\preccurlyeq_\mu$	$\preccurlyeq_\mu$	$\succcurlyeq_\mu$	$\succcurlyeq_\mu$	$\preccurlyeq_\mu$	$\preccurlyeq_\mu$	$\preccurlyeq_\mu$	∅	∅	$\preccurlyeq_\mu$	$\preccurlyeq_\mu$	$\preccurlyeq_\mu$
ES	…	…	…	…	…	…	…	$\preccurlyeq_\mu$	$\preccurlyeq_\mu$	∅	$\preccurlyeq_\mu$	$\preccurlyeq_\mu$	$\succcurlyeq_\mu$	$\succcurlyeq_\mu$	∅	$\preccurlyeq_\mu$	$\preccurlyeq_\mu$	∅	$\succcurlyeq_\mu$	$\preccurlyeq_\mu$	∅	$\preccurlyeq_\mu$
FI	…	…	…	…	…	…	…	…	∅	$\succcurlyeq_\mu$	$\preccurlyeq_\mu$	$\succcurlyeq_\mu$	$\succcurlyeq_\mu$	$\succcurlyeq_\mu$	∅	∅	∅	$\succcurlyeq_\mu$	$\succcurlyeq_\mu$	∅	∅	∅
FR	…	…	…	…	…	…	…	…	…	∅	$\preccurlyeq_\mu$	$\succcurlyeq_\mu$	$\succcurlyeq_\mu$	$\succcurlyeq_\mu$	∅	$\preccurlyeq_\mu$	∅	$\succcurlyeq_\mu$	$\succcurlyeq_\mu$	∅	∅	∅
HU	…	…	…	…	…	…	…	…	…	…	$\preccurlyeq_\mu$	∅	∅	∅	∅	$\preccurlyeq_\mu$	$\preccurlyeq_\mu$	∅	∅	∅	$\preccurlyeq_\mu$	∅
IS	…	…	…	…	…	…	…	…	…	…	…	$\succcurlyeq_\mu$	$\succcurlyeq_\mu$	$\succcurlyeq_\mu$	$\succcurlyeq_\mu$	∅	∅	$\succcurlyeq_\mu$	$\succcurlyeq_\mu$	$\succcurlyeq_\mu$	$\succcurlyeq_\mu$	$\succcurlyeq_\mu$
IT	…	…	…	…	…	…	…	…	…	…	…	…	$\succcurlyeq_\mu$	$\succcurlyeq_\mu$	∅	$\preccurlyeq_\mu$	$\preccurlyeq_\mu$	∅	$\succcurlyeq_\mu$	$\preccurlyeq_\mu$	∅	$\preccurlyeq_\mu$
LT	…	…	…	…	…	…	…	…	…	…	…	…	…	∅	$\preccurlyeq_\mu$	$\preccurlyeq_\mu$	$\preccurlyeq_\mu$	$\preccurlyeq_\mu$	$\preccurlyeq_\mu$	$\preccurlyeq_\mu$	$\preccurlyeq_\mu$	$\preccurlyeq_\mu$
LV	…	…	…	…	…	…	…	…	…	…	…	…	…	…	$\preccurlyeq_\mu$	$\preccurlyeq_\mu$	$\preccurlyeq_\mu$	∅	$\preccurlyeq_\mu$	$\preccurlyeq_\mu$	$\preccurlyeq_\mu$	$\preccurlyeq_\mu$
MT	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	$\preccurlyeq_\mu$	∅	$\succcurlyeq_\mu$	$\succcurlyeq_\mu$	∅	∅	∅
NL	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	∅	$\succcurlyeq_\mu$	$\succcurlyeq_\mu$	∅	$\succcurlyeq_\mu$	$\succcurlyeq_\mu$
NO	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	$\succcurlyeq_\mu$	$\succcurlyeq_\mu$	∅	∅	∅
PL	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	∅	$\preccurlyeq_\mu$	$\preccurlyeq_\mu$	$\preccurlyeq_\mu$
PT	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	$\preccurlyeq_\mu$	$\preccurlyeq_\mu$	$\preccurlyeq_\mu$
SE	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	∅	∅
SI	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	∅

Note: $\succcurlyeq_\mu$ ($\preccurlyeq_\mu$) indicates that the first distribution dominates (is dominated by) the second distribution at the correspond order of dominance. ∅ denotes a non-conclusive test.

(a) Absolute view ($\eta = 0$). (b) Relative view ($\eta = 1$).



Note: Each point gives the proportion of cross-country dominance relationships observed with the EU-SILC data for a maximal union poverty domain with bounds given by the x-axis.

Figure 15: Ordering power with different maximal deprivation lines for mean-variability indices.

mean temporal income.[21] Table 3 confirms that income variability accounts for the low ranking power of the mean-variability dominance tests: the ordering power increases to 63% when no risk aversion is assumed. We note by the way that the ordering power is larger than the one observed with symmetry using the joint distribution of income (55%), but the gain can be regarded as low considering the large losses in ethical robustness due to the assumption of perfect individual-level pooling.

The effect of income volatility can also be inferred from Figures 15a and 15b. As for Figure 14, the curves show the proportions of dominance relationships observed for different maximum deprivation thresholds (up to 20,000€. The rapid collapse of the ordering power below 5,000€, in particular when income variability is assessed in absolute terms, confirms that income volatility limits the power of mean-variability dominance relationships.

6 Conclusion

This paper assesses the intertemporal poverty ranking of populations. More generally, it considers the welfare comparisons of populations when multidimensional attributes of interest can be measured along comparable scales. The orderings are obtained with assumptions that do not require cardinality of the variables, as is required for instance with Pigou-Dalton-like transfer axioms. The role of symmetric and asymmetric assumptions is more particularly in-

[21] More specifically, we consider poverty indices from $\tilde{\Pi}_S(\tilde{\lambda}_S)$ such that $\rho_t^{(2)}(\mu,\tau_\eta) = \rho_t^{(1,2)}(\mu,\tau_\eta) = 0\ \forall t = 1,2$. It can then be easily be seen from equation (81) in appendix B that the poverty domain is necessarily defined as the set of income profiles such that $\mu < z^*$. Moreover, the corresponding dominance relationship $\succcurlyeq_\mu$ entails the sole comparison of the cumulative distribution of mean income up to z^+ for the two populations to be compared.

vestigated. Symmetry supposes that the social evaluator is sensitive to the overall distribution of temporal deprivations, but not about the pattern of income profiles, so that switching two incomes within an individual income profile is supposed not to affect overall well-being. The less demanding asymmetry assumption requires intertemporal patterns to be ordered in a systematic manner. An empirical illustration on a large set of European countries for the period 2006-2009 shows that such assumptions can significantly improve the outcome of first-order dominance tests.

The paper also introduces classes of poverty indices that depend on mean temporal income, income variability and income "patterns." This framework is more demanding in terms of indicator comparability than the previous one as it requires full cardinality of the variables used to measure poverty, but it makes it possible to incorporate a natural intertemporal progressive transfer (without having to deal with interpersonal transfers). Moreover, it mirrors economists' usual practices as mean-and-variance frameworks are often used to describe the distributions of economic phenomenons. This framework can also incorporate symmetry and asymmetry axioms and can be applied to a continuum of different views of risk aversion, with the usual relative and absolute views as limiting cases. This common mean-variability framework for thinking about intertemporal welfare does not, however, have empirical strength when applied to our data. Dominance tests with a subset of indices that do no not display variability-sensitivity show that this low ordering power is mostly due to income variability within observed distribution. On the one hand, this result suggest that the popular mean-variance approach may not be as relevant for intertemporal poverty comparisons as the Duclos et al.'s (2006) approach which is more consistent with the frameworks developed within the field of welfare analyses. On the other hand the results without variability-sensitivity suggests that considerable gains may potentially be obtained by adding some more restrictions to the corresponding classes of intertemporal poverty indices without having to suppose perfect pooling at the individual level.

Finally, the framework developed in the paper lends itself nicely to making a distinction between "chronic" and "transient" poverty. Distinguishing between these two types of poverty can be important for both descriptive and policy purposes. Chronic poverty is usually deemed (normatively speaking) worse than transient poverty. Furthermore, not only is chronic poverty different from a measurement perspective from transient poverty, but the difference between the two is also likely to call for distinct policy responses, as argued for instance in Jalan and Ravallion (1998) — see also Duclos et al. 2010. From a theoretical point of view, two approaches of the chronic/transient distinction have been proposed in the literature, namely the "spell" and "component" approaches. The "spell" approach considers that the chronic or transient poor ought to be identified on the basis of the number or the extent of deprivations they endure. The "component" approach says that the poor are likely to experience simultaneously chronic and transient poverty, chronic poverty being determined by average income and transient poverty by deviations from that chronic poverty. Chronic and transient poverty dominance relationships do not seem to have been considered in the literature, either for the spell or for the component approach; the tools developed in this paper would appear to be a promising start for that.

A Proof of propositions from Section 2

Let $z_1(x_2)$ and $z_2(x_1)$ be respectively the value of the first and second income such that $\lambda\big(x_1, z_2(x_1)\big) = 0$ and $\lambda\big(z_1(x_2), x_2\big) = 0$. Thus, $z_1\big(z_2(x_1)\big) = x_1$, and z_1 is then the inverse of z_2. Let z^* be the value of income such that $\lambda(z^*, z^*) = 0$. We than can define a two-period poverty index as a sum of low x_1 (with respect to x_2) and of low x_2 (with respect to x_1) time poverty:

$$P(\lambda) = \int_0^{z^*} \int_{x_2}^{z_1(x_2)} \pi(x_1, x_2, \lambda) f(x_1, x_2) dx_1 dx_2 + \int_0^{z^*} \int_{x_1}^{z_2(x_1)} \pi(x_1, x_2; \lambda) f(x_1, x_2) dx_2 dx_1. \tag{46}$$

We first proceed with the first part of the right-hand term of (46). Integrating that expression by parts with respect to x_1, we find:

$$\begin{aligned}\int_0^{z^*} \int_{x_2}^{z_1(x_2)} \pi(x_1, x_2, \lambda) f(x_1, x_2) dx_1 dx_2 = &\int_0^{z^*} \big[\pi(x_1, x_2) F(x_1|x_2)\big]_{x_1=x_2}^{x_1=z_1(x_2)} f(x_2)\, dx_2 \\ &- \int_0^{z^*} \int_{x_2}^{z_1(x_2)} \pi^{(1)}(x_1, x_2) F(x_1|x_2) f(x_2)\, dx_1 dx_2. \end{aligned} \tag{47}$$

Rearranging the first element of (47), we find

$$\int_0^{z^*} [\pi(x_1, x_2) F(x_1|x_2)]_{x_1=x_2}^{x_1=z_1(x_2)} f(x_2)\, dx_2$$

$$= \int_0^{z^*} \Big(\pi\big(z_1(x_2), x_2\big) F\big(z_1(x_2)|x_2\big) - \pi(x_2, x_2) F(x_1 = x_2|x_2)\Big) f(x_2)\, dx_2 \tag{48}$$

$$= -\int_0^{z^*} \pi(x_2, x_2) F(x_1 = x_2|x_2) f(x_2)\, dx_2, \tag{49}$$

since $\pi\big(z_1(x_2), x_2\big) = 0$.

To integrate the second part of the right-hand term of (47) by parts with respect to x_2, let $K(x_2) = \int_{x_2}^{z_1(x_2)} \pi^{(1)}(x_1, x_2)\, F(x_1, x_2)\, dx_1$. We then get:

$$\begin{aligned}\frac{\partial K(x_2)}{\partial x_2} = &\, z_1'(x_2) \pi^{(1)}\big(z_1(x_2), x_2\big) F\big(z_1(x_2), x_2\big) \\ &- \pi^{(1)}(x_2, x_2) F(x_2, x_2) \\ &+ \int_{x_2}^{z_1(x_2)} \pi^{(1,2)}(x_1, x_2) F(x_1, x_2)\, dx_1 \\ &+ \int_{x_2}^{z_1(x_2)} \pi^{(1)}(x_1, x_2) F(x_1|x_2) f(x_2)\, dx_1. \end{aligned} \tag{50}$$

Integrating that expression along x_2 and over $[0, z^*]$ and rearranging, we have:

$$\int_0^{z^*} \int_{x_2}^{z_1(x_2)} \pi^{(1)}(x_1, x_2) F(x_1|x_2) f(x_2)\, dx_1 dx_2 \tag{51}$$

$$\begin{aligned}= &\big[K(x_2)\big]_0^{z^*} - \int_0^{z^*} z_1'(x_2) \pi^{(1)}\big(z_1(x_2), x_2\big) F\big(z_1(x_2), x_2\big)\, dx_2 \\ &+ \int_0^{z^*} \pi^{(1)}(x_2, x_2) F(x_2, x_2)\, dx_2 - \int_0^{z^*} \int_{x_2}^{z_1(x_2)} \pi^{(1,2)}(x_1, x_2) F(x_1, x_2)\, dx_1 dx_2, \end{aligned} \tag{52}$$

$$\begin{aligned}= &-\int_0^{z^*} z_1'(x_2) \pi^{(1)}\big(z_1(x_2), x_2\big) F\big(z_1(x_2), x_2\big)\, dx_2 \\ &+ \int_0^{z^*} \pi^{(1)}(x_2, x_2) F(x_2, x_2)\, dx_2 - \int_0^{z^*} \int_{x_2}^{z_1(x_2)} \pi^{(1,2)}(x_1, x_2) F(x_1, x_2)\, dx_1 dx_2, \end{aligned} \tag{53}$$

since $z_1(z^*) = z^*$ (hence $K(z^*) = 0$) and $F(x_1, 0) = 0\ \forall x_1$ (hence $K(0) = 0$). Using (49) and (53), we obtain:

$$\begin{aligned}
&\int_0^{z^*}\int_{x_2}^{z_1(x_2)} \pi(x_1, x_2, \lambda) f(x_1, x_2) dx_1 dx_2 \\
&= -\int_0^{z^*} \pi(x_2, x_2) F(x_1 = x_2 | x_2) f(x_2)\, dx_2 + \int_0^{z^*} z_1'(x_2)\pi^{(1)}\big(z_1(x_2), x_2\big) F\big(z_1(x_2), x_2\big)\, dx_2 \\
&- \int_0^{z^*} \pi^{(1)}(x_2, x_2) F(x_2, x_2)\, dx_2 + \int_0^{z^*}\int_{x_2}^{z_1(x_2)} \pi^{(1,2)}(x_1, x_2) F(x_1, x_2)\, dx_1 dx_2. \qquad (54)
\end{aligned}$$

Proceeding similarly with the second part of the right-hand term of (46) and adding the above, we obtain:

$$\begin{aligned}
P(\lambda) = &-\int_0^{z^*} \pi(x_2, x_2) F(x_1 = x_2 | x_2) f(x_2)\, dx_2 + \int_0^{z^*} z_1'(x_2)\pi^{(1)}\big(z_1(x_2), x_2\big) F\big(z_1(x_2), x_2\big)\, dx_2 \\
&- \int_0^{z^*} \pi^{(1)}(x_2, x_2) F(x_2, x_2)\, dx_2 + \int_0^{z^*}\int_{x_2}^{z_1(x_2)} \pi^{(1,2)}(x_1, x_2) F(x_1, x_2)\, dx_1 dx_2 \\
&- \int_0^{z^*} \pi(x_1, x_1) F(x_2 = x_1 | x_1) f(x_1)\, dx_1 + \int_0^{z^*} z_2'(x_1)\pi^{(2)}\big(x_1, z_2(x_1)\big) F\big(x_1, z_2(x_1)\big)\, dx_1 \\
&- \int_0^{z^*} \pi^{(2)}(x_1, x_1) F(x_1, x_1)\, dx_1 + \int_0^{z^*}\int_{x_1}^{z_2(x_1)} \pi^{(1,2)}(x_1, x_2) F(x_1, x_2)\, dx_2 dx_1. \qquad (55)
\end{aligned}$$

It can be observed that $F(x_2 = x_1 | x_1) f(x_1) = \frac{\partial F(x_1, x_1)}{\partial x_1} - F(x_1 | x_2 = x_1) f(x_2 = x_1)$, so that:

$$\begin{aligned}
&\int_0^{z^*} \pi(x_1, x_1) F(x_2 = x_1 | x_1) f(x_1)\, dx_1 \\
&= \int_0^{z^*} \pi(x_1, x_1) \frac{\partial F(x_1, x_1)}{\partial x_1}\, dx_1 - \int_0^{z^*} \pi(x_1, x_1) F(x_1 | x_2 = x_1) f(x_2 = x_1)\, dx_1 \qquad (56) \\
&= \left[\pi(x_1, x_1) F(x_1, x_1)\right]_0^{z^*} - \int_0^{z^*} \left(\pi^{(1)}(x_1, x_1) + \pi^{(2)}(x_1, x_1)\right) F(x_1, x_1)\, dx_1 \\
&- \int_0^{z^*} \pi(x_1, x_1) F(x_1 | x_2 = x_1) f(x_2 = x_1)\, dx_1 \qquad (57) \\
&= -\int_0^{z^*} \left(\pi^{(1)}(x_1, x_1) + \pi^{(2)}(x_1, x_1)\right) F(x_1, x_1)\, dx_1 \\
&- \int_0^{z^*} \pi(x_2, x_2) F(x_1 = x_2 | x_2) f(x_2)\, dx_2. \qquad (58)
\end{aligned}$$

Using that result and changing the integration variable in $\int_0^{z^*} \pi^{(2)}(x_2, x_2) F(x_2, x_2)\ dx_2$, we then have:

$$\begin{aligned}
P(\lambda) = &\int_0^{z^*} z_2'(x_1)\pi^{(2)}\big(x_1, z_2(x_1)\big) F\big(x_1, z_2(x_1)\big)\, dx_1 \\
&+ \int_0^{z^*} z_1'(x_2)\pi^{(1)}\big(z_1(x_2), x_2\big) F\big(z_1(x_2), x_2\big)\, dx_2 \\
&+ \int_0^{z^*}\int_{x_1}^{z_2(x_1)} \pi^{(1,2)}(x_1, x_2) F(x_1, x_2)\, dx_2 dx_1 \\
&+ \int_0^{z^*}\int_{x_2}^{z_1(x_2)} \pi^{(1,2)}(x_1, x_2) F(x_1, x_2)\, dx_1 dx_2. \qquad (59)
\end{aligned}$$

A.1 Proof of Proposition 2

Symmetry implies the following properties:

$$\pi^{(1)}(x_1, x_2) = \pi^{(2)}(x_2, x_1) \quad \forall x_1, x_2, \tag{60}$$

$$\pi^{(1,2)}(x_1, x_2) = \pi^{(1,2)}(x_2, x_1) \quad \forall x_1, x_2. \tag{61}$$

At the poverty frontier, we also have $\lambda(x_1, x_2) = 0$ and $\pi^{(x_2)}\big(z_1(x_2), x_2\big) = 0$. Since:

$$\pi^{(x_2)}\big(z_1(x_2), x_2\big) = z_1'(x_2)\pi^{(1)}\big(z_1(x_2), x_2\big) + \pi^{(2)}\big(z_1(x_2), x_2\big), \tag{62}$$

we obtain:

$$z_1'(x_2)\pi^{(1)}\big(z_1(x_2), x_2\big) = -\pi^{(2)}\big(z_1(x_2), x_2\big). \tag{63}$$

Symmetry also leads to $z_1(x_2) = z_2(x_2)$ and $z_1'(x_2) = z_2'(x_2)$. Using (60), we find that:

$$z_1'(x_2)\pi^{(1)}\big(z_1(x_2), x_2\big) = z_2'(x_2)\pi^{(2)}\big(x_2, z_2(x_2)\big). \tag{64}$$

From the expression of $P(\lambda)$ in (59), the symmetry assumptions therefore lead to:

$$P(\lambda) = \int_0^{z^*} z_1^{(1)}(x_2)\pi^{(1)}\big(z_1(x_2), x_2\big)\Big(F\big(z_1(x_2), x_2\big) + F\big(x_2, z_2(x_2)\big)\Big)\, dx_2 \tag{65}$$

$$+ \int_0^{z^*} \int_{x_2}^{z_1(x_2)} \pi^{x_1,x_2}(x_1, x_2)\big(F(x_1, x_2) + F(x_2, x_1)\big)\, dx_1 dx_2 \tag{66}$$

The necessary and sufficient conditions for Proposition 2 follow upon inspection.

A.2 Proof of Proposition 3

With asymmetry, we assume that $z_2(x_1) \geq z_1(x_1)$ for all $x_1 \in [0, z^*]$. Equation (46) can then be rewritten as:

$$\begin{aligned} P(\lambda) = &\int_0^{z^*} \int_{x_1}^{z_2(x_1)} \pi(x_1, x_2; \lambda) f(x_1, x_2)\, dx_2 dx_1 \\ &+ \int_0^{z^*} \int_{x_2}^{z_2(x_2)} \pi(x_1, x_2, \lambda) f(x_1, x_2)\, dx_1 dx_2. \end{aligned} \tag{67}$$

Equation (59) then becomes:

$$\begin{aligned} P(\lambda) = &\int_0^{z^*} z_2'(x_1)\pi^{(2)}\big(x_1, z_2(x_1)\big)F\big(x_1, z_2(x_1)\big)\, dx_1 \\ &+ \int_0^{z^*} z_2'(x_2)\pi^{(1)}\big(z_2(x_2), x_2\big)F\big(z_2(x_2), x_2\big)\, dx_2 \\ &+ \int_0^{z^*} \int_{x_1}^{z_2(x_1)} \pi^{(1,2)}(x_1, x_2)F(x_1, x_2)\, dx_2 dx_1 \\ &+ \int_0^{z^*} \int_{x_2}^{z_2(x_2)} \pi^{(1,2)}(x_1, x_2)F(x_1, x_2)\, dx_1 dx_2. \end{aligned} \tag{68}$$

We obtain:

$$
\begin{aligned}
P(\lambda) = & \int_0^{z^*} z_2'(x_1)\pi^{(2)}\big(z_2(x_1),x_1\big)\left[F\big(z_2(x_1),x_1\big)+F\big(x_1,z_2(x_1)\big)\right]\,dx_1 \\
& + \int_0^{z^*} z_2'(x_1)\left[\pi^{(1)}\big(x_1,z_2(x_1)\big)-\pi^{(2)}\big(z_2(x_1),x_1\big)\right]F\big(x_1,z_2(x_1)\big)\,dx_1 \\
& + \int_0^{z^*}\int_{x_2}^{z_2(x_2)} \pi^{(1,2)}(x_1,x_2)\left[F(x_1,x_2)+F(x_2,x_1)\right]dx_1dx_2 \\
& + \int_0^{z^*}\int_{x_1}^{z_2(x_1)} \left[\pi^{(1,2)}(x_1,x_2)-\pi^{(1,2)}(x_2,x_1)\right]F(x_1,x_2)dx_2dx_1. \qquad (69)
\end{aligned}
$$

with, by assumption, $\pi^{(1)}\big(x_1,z_2(x_1)\big)-\pi^{(2)}\big(z_2(x_1),x_1\big)\leq 0$ and $\pi^{(1,2)}(x_1,x_2)-\pi^{(1,2)}(x_2,x_1)\geq 0$. The second and fourth terms of the right-hand side of (69) account for the first condition of Proposition 3, while the first and third terms account for its second condition.

B Proof of propositions from Section 3

Let the lowest value of mean income on the mean-variability poverty frontier be obtained for $\tau_\eta = 0$ at $\mu = z^*$, so that $\tilde{\lambda}(z^*,0) = \lambda(z^*,z^*) = 0$. At this point, it is also necessary to differentiate between the cases of $x_1 < x_2$ and $x_1 > x_2$. Let $\tau_\eta^{z1}(\mu)$ ($\tau_\eta^{z2}(\mu)$) be the value of τ_η such that $\tilde{\lambda}\big(\mu,\tau_\eta^{z}(\mu)\big) = 0$ when $x_1 < x_2$ ($x_1 > x_2$). Since individuals are supposed to be poor $\forall\tau^\eta$ if $\mu \leq z^*$, $\tau_\eta^{z1}(\mu)$ and $\tau_\eta^{z2}(\mu)$ are defined on the intervals $[z^*,+\infty)$. Due to the monotonicity assumptions, $\frac{\partial\tau_\eta^{z1}}{\partial\mu}\leq 0$ and $\frac{\partial\tau_\eta^{z2}}{\partial\mu}\leq 0$.

Let $q := \text{prob}(x_1 < x_2)$ and ρ_1 (ρ_2) be the individual poverty measure to be applied when $x_1 < x_2$ ($x_1 > x_2$). Let f_1 (f_2) denote the joint density function of μ and τ_η, conditional on $x_1 < x_2$ ($x_1 > x_2$). The same notation applies for the cdf, conditional cdf and marginal cdf and marginal density functions. With the above, lifetime poverty defined in (19) can alternatively be defined as:

$$
\begin{aligned}
\tilde{P}(\tilde{\lambda}) = & q\int_0^{z^*}\int_{-\mu^{1-\eta}}^{0}\rho_1(\mu,\tau_\eta,\tilde{\lambda})f_1(\mu,\tau_\eta)\,d\tau_\eta d\mu + q\int_{z^*}^{+\infty}\int_{-\mu^{1-\eta}}^{\tau_\eta^{z1}(\mu)}\rho_1(\mu,\tau_\eta,\tilde{\lambda})f_1(\mu,\tau_\eta)\,d\tau_\eta d\mu \qquad (70) \\
& + (1-q)\int_0^{z^*}\int_{-\mu^{1-\eta}}^{0}\rho_2(\mu,\tau_\eta,\tilde{\lambda})f_2(\mu,\tau_\eta)\,d\tau_\eta d\mu + (1-q)\int_{z^*}^{+\infty}\int_{-\mu^{1-\eta}}^{\tau_\eta^{z2}(\mu)}\rho_2(\mu,\tau_\eta,\tilde{\lambda})f_2(\mu,\tau_\eta)\,d\tau_\eta d\mu.
\end{aligned}
$$

For convenience, $\tilde{\lambda}$ is dropped from the expression of ρ. We first consider the first and third elements of the right-hand term of (70) and, integrating by parts, find $\forall j = 1,2$:

$$
\begin{aligned}
\int_0^{z^*}\int_{-\mu^{1-\eta}}^{0}\rho_j(\mu,\tau_\eta)f_j(\mu,\tau_\eta)d\tau_\eta d\mu = & \int_0^{z^*}\left[\rho_j(\mu,\tau_\eta)F_j(\tau_\eta|\mu)\right]_{\tau_\eta=-\mu^{1-\eta}}^{\tau_\eta=0} f_j(\mu)\,d\mu \\
& - \int_0^{z^*}\int_{-\mu^{1-\eta}}^{0}\rho_j^{(2)}(\mu,\tau_\eta)F_j(\tau_\eta|\mu)f_j(\mu)\,d\tau_\eta d\mu. \qquad (71)
\end{aligned}
$$

As $F_j(\tau_\eta = -\mu^{1-\eta}|\mu) = 0$ and $F_j(\tau_\eta = 0|\mu) = 1$, the first element on the right-hand side of (71) can be expressed as:

$$\begin{aligned}
&\int_0^{z^*} \left[\rho_j(\mu,\tau_\eta)F_j(\tau_\eta|\mu)\right]_{\tau_\eta=-\mu^{1-\eta}}^{\tau_\eta=0} f_j(\mu)\, d\mu \\
&= \int_0^{z^*} \rho_j(\mu,0)f_j(\mu)\, d\mu, \\
&= \left[\rho_j(\mu,0)F_j(\mu)\right]_{\mu=0}^{\mu=z^*} - \int_0^{z^*} \rho_j^{(1)}(\mu,0)F_j(\mu)\, d\mu \\
&= -\int_0^{z^*} \rho_j^{(1)}(\mu,0)F_j(\mu)\, d\mu, \qquad (72)
\end{aligned}$$

since $F_j(\mu = 0) = 0$ and the function ρ_j is zero at the poverty frontier ($\rho_j(z^*, 0) = 0$).

We now can turn to the second element of the right-hand term of (71). Define $Q_j(\mu) = \int_{-\mu^{1-\eta}}^{0} \rho_j^{(2)}(\mu,\tau_\eta)F_j(\mu,\tau_\eta)\, d\tau_\eta$. We have:

$$\begin{aligned}
\frac{\partial Q_j}{\partial \mu} &= (1-\eta)\mu^{-\eta}\rho_j^{(2)}\left(\mu,-\mu^{1-\eta}\right)F_j\left(\mu,-\mu^{1-\eta}\right) + \int_{-\mu^{1-\eta}}^{0} \rho_j^{(1,2)}(\mu,\tau_\eta)F_j(\mu,\tau_\eta)\, d\tau_\eta \\
&+ \int_{-\mu^{1-\eta}}^{0} \rho_j^{(2)}(\mu,\tau_\eta)F_j(\tau_\eta|\mu)f_j(\mu)\, d\tau_\eta \\
&= \int_{-\mu^{1-\eta}}^{0} \rho_j^{(1,2)}(\mu,\tau_\eta)F_j(\mu,\tau_\eta)\, d\tau_\eta + \int_{-\mu^{1-\eta}}^{0} \rho_j^{(2)}(\mu,\tau_\eta)F_j(\tau_\eta|\mu)f_j(\mu)\, d\tau_\eta, \qquad (73)
\end{aligned}$$

since $F_j(\mu, -\mu^{1-\eta}) = 0$. Integrating that expression along μ and over $[0, z^*]$ and rearranging, we have:

$$\begin{aligned}
\int_0^{z^*}\int_{-\mu^{1-\eta}}^{0} \rho_j^{(2)}(\mu,\tau_\eta)F_j(\tau_\eta|\mu)f_j(\mu)\, d\tau_\eta d\mu &= \left[Q_j(\mu)\right]_0^{z^*} - \int_0^{z^*}\int_{-\mu^{1-\eta}}^{0} \rho_j^{(1,2)}(\mu,\tau_\eta)F_j(\mu,\tau_\eta)\, d\tau_\eta d\mu \\
&= \int_{-z^{*1-\eta}}^{0} \rho_j^{(2)}(z^*,\tau_\eta)F_j(z^*,\tau_\eta)\, d\tau_\eta \\
&- \int_0^{z^*}\int_{-\mu^{1-\eta}}^{0} \rho_j^{(1,2)}(\mu,\tau_\eta)F_j(\mu,\tau_\eta)\, d\tau_\eta d\mu. \qquad (74)
\end{aligned}$$

We then consider the second and fourth elements on the right-hand side of (70) and, using once again integration by parts, find:

$$\begin{aligned}
\int_{z^*}^{+\infty}\int_{-\mu^{1-\eta}}^{\tau_\eta^{zj}(\mu)} \rho_j(\mu,\tau_\eta)f_j(\mu,\tau_\eta)\, d\tau_\eta d\mu &= \int_{z^*}^{+\infty} \left[\rho_j(\mu,\tau_\eta)F_j(\tau_\eta|\mu)\right]_{\tau_\eta=-\mu^{1-\eta}}^{\tau_\eta=\tau_\eta^{zj}(\mu)} f_j(\mu)\, d\mu \\
&- \int_{z^*}^{+\infty}\int_{-\mu^{1-\eta}}^{\tau_\eta^{zj}(\mu)} \rho_j^{(2)}(\mu,\tau_\eta)F_j(\tau_\eta|\mu)f_j(\mu)\, d\tau_\eta d\mu \\
&= -\int_{z^*}^{+\infty}\int_{-\mu^{1-\eta}}^{\tau_\eta^{zj}(\mu)} \rho_j^{(2)}(\mu,\tau_\eta)F_j(\tau_\eta|\mu)f_j(\mu)\, d\tau_\eta d\mu, \qquad (75)
\end{aligned}$$

as $\rho_j\left(\mu, \tau_\eta^{zj}(\mu)\right) = 0$ and $F_j(\tau_\eta = -\mu^{1-\eta}|\mu) = 0$.

Let $R_j(\mu) = \int_{-\mu^{1-\eta}}^{\tau_\eta^{zj}(\mu)} \rho_j^{(2)}(\mu,\tau_\eta)F_j(\mu,\tau_\eta)\, d\tau_\eta$. We have:

$$\begin{aligned}
\frac{\partial R_j}{\partial \mu} &= \tau_\eta^{zj\prime}(\mu)\rho_j^{(2)}\big(\mu,\tau_\eta^{zj}(\mu)\big)F_j(\mu,\tau_\eta^{zj}(\mu)\big) + (1-\eta)\mu^{-\eta}\rho_j^{(2)}(\mu,-\mu^{1-\eta})F_j(\mu,-\mu^{1-\eta}) \\
&+ \int_{-\mu^{1-\eta}}^{\tau_\eta^{zj}(\mu)} \rho_j^{(1,2)}(\mu,\tau_\eta)F_j(\mu,\tau_\eta)\, d\tau_\eta + \int_{-\mu^{1-\eta}}^{\tau_\eta^{zj}(\mu)} \rho_j^{(2)}(\mu,\tau_\eta)F_j(\tau_\eta|\mu)f_j(\mu)\, d\tau_\eta, \\
&= \tau_\eta^{zj\prime}(\mu)\rho_j^{(2)}\big(\mu,\tau_\eta^{zj}(\mu)\big)F_j(\mu,\tau_\eta^{zj}(\mu)\big) \\
&+ \int_{-\mu^{1-\eta}}^{\tau_\eta^{zj}(\mu)} \rho_j^{(1,2)}(\mu,\tau_\eta)F_j(\mu,\tau_\eta)\, d\tau_\eta + \int_{-\mu^{1-\eta}}^{\tau_\eta^{zj}(\mu)} \rho_j^{(2)}(\mu,\tau_\eta)F_j(\tau_\eta|\mu)f_j(\mu)\, d\tau_\eta. \qquad (76)
\end{aligned}$$

Integrating that expression along μ and over $[z^*,+\infty]$ and rearranging, we have:

$$\begin{aligned}
&\int_{z^*}^{+\infty} \int_{-\mu^{1-\eta}}^{\tau_\eta^{zj}(\mu)} \rho_j^{(2)}(\mu,\tau_\eta)F_j(\tau_\eta|\mu)f_j(\mu)\, d\tau_\eta d\mu \\
&= \big[R_j(\mu)\big]_{z^*}^{+\infty} - \int_{z^*}^{+\infty} \tau_\eta^{zj\prime}(\mu)\rho_j^{(2)}\big(\mu,\tau_\eta^{zj}(\mu)\big)F_j(\mu,\tau_\eta^{zj}(\mu)\big)\, d\mu \\
&- \int_{z^*}^{+\infty} \int_{-\mu^{1-\eta}}^{\tau_\eta^{zj}(\mu)} \rho_j^{(1,2)}(\mu,\tau_\eta)F_j(\mu,\tau_\eta)\, d\tau_\eta d\mu \\
&= \int_{-\infty^{1-\eta}}^{\tau_\eta^{zj}(+\infty)} \rho_j^{(2)}(+\infty,\tau_\eta)F_j(+\infty,\tau_\eta)\, d\tau_\eta - \int_{-z^{*1-\eta}}^{0} \rho_j^{(2)}(z^*,\tau_\eta)F_j(z^*,\tau_\eta)\, d\tau_\eta \\
&- \int_{z^*}^{+\infty} \tau_\eta^{zj\prime}(\mu)\rho_j^{(2)}\big(\mu,\tau_\eta^{zj}(\mu)\big)F_j(\mu,\tau_\eta^{zj}(\mu)\big)\, d\mu - \int_{z^*}^{+\infty} \int_{-\mu^{1-\eta}}^{\tau_\eta^{zj}(\mu)} \rho_j^{(1,2)}(\mu,\tau_\eta)F_j(\mu,\tau_\eta)\, d\tau_\eta d\mu. \qquad (77)
\end{aligned}$$

Using (72), (74), (77), and $\rho_1(\mu,0) = \rho_2(\mu,0)\ \forall \mu$, we finally obtain the following expression for $P(\lambda)$:

$$\begin{aligned}
P(\tilde{\lambda}) &= -\int_0^{z^*} \rho_2^{(1)}(\mu,0)F(\mu)\, d\mu - q\int_{-\infty^{1-\eta}}^{\tau_\eta^{z1}(+\infty)} \rho_1^{(2)}(+\infty,\tau_\eta)F_1(+\infty,\tau_\eta)\, d\tau_\eta \qquad (78) \\
&+ q\int_{z^*}^{+\infty} \tau_\eta^{z1\prime}(\mu)\rho_1^{(2)}\big(\mu,\tau_\eta^{z1}(\mu)\big)F_1(\mu,\tau_\eta^{z1}(\mu)\big)\, d\mu + q\int_0^{z^*}\int_{-\mu^{1-\eta}}^{0} \rho_1^{(1,2)}(\mu,\tau_\eta)F_1(\mu,\tau_\eta)\, d\tau_\eta d\mu \\
&+ q\int_{z^*}^{+\infty}\int_{-\mu^{1-\eta}}^{\tau_\eta^{z1}(\mu)} \rho_1^{(1,2)}(\mu,\tau_\eta)F_1(\mu,\tau_\eta)\, d\tau_\eta d\mu - (1-q)\int_{-\infty^{1-\eta}}^{\tau_\eta^{z2}(+\infty)} \rho_2^{(2)}(+\infty,\tau_\eta)F_2(+\infty,\tau_\eta)\, d\tau_\eta \\
&+ (1-q)\int_{z^*}^{+\infty} \tau_\eta^{z2\prime}(\mu)\rho_2^{(2)}\big(\mu,\tau_\eta^{z2}(\mu)\big)F_2(\mu,\tau_\eta^{z2}(\mu)\big)\, d\mu \\
&+ (1-q)\int_0^{z^*}\int_{-\mu^{1-\eta}}^{0} \rho_2^{(1,2)}(\mu,\tau_\eta)F_2(\mu,\tau_\eta)\, d\tau_\eta d\mu + (1-q)\int_{z^*}^{+\infty}\int_{-\mu^{1-\eta}}^{\tau_\eta^{z2}(\mu)} \rho_2^{(1,2)}(\mu,\tau_\eta)F_2(\mu,\tau_\eta)\, d\tau_\eta d\mu.
\end{aligned}$$

Proposition 4 then follows directly from (78) by inspection.

B.1 Proof of Proposition 6

Letting $\rho_1(\mu,\tau_\eta) = \rho_2(\mu,\tau_\eta)\ \forall(\mu,\tau_\eta) \in \Gamma_S(\tilde{\lambda})$, it follows that $\forall(\mu,\tau_\eta) \in \Gamma_S(\tilde{\lambda})$:

$$\rho_1^{(2)}(\mu,\tau_\eta) = \rho_2^{(2)}(\mu,\tau_\eta), \qquad (79)$$

$$\rho_1^{(1,2)}(\mu,\tau_\eta) = \rho_2^{(1,2)}(\mu,\tau_\eta). \qquad (80)$$

Moreover, $\tau_\eta^{z1}(\mu) = \tau_\eta^{z2}(\mu)$, so that $\tau_\eta^{z1\prime}(\mu) = \tau_\eta^{z2\prime}(\mu)$. Equation (78) can then be rewritten as:

$$\begin{aligned}P(\tilde{\lambda}) = &-\int_0^{z^*}\rho_1^{(1)}(\mu,0)F(\mu)\,d\mu - \int_{-\infty^{1-\eta}}^{\tau_\eta^{z1}(+\infty)}\rho_1^{(2)}(+\infty,\tau_\eta)F(+\infty,\tau_\eta)\,d\tau_\eta\\ &+\int_{z^*}^{+\infty}\tau_\eta^{z1\prime}(\mu)\rho_1^{(2)}\big(\mu,\tau_\eta^{z1}(\mu)\big)F\big(\mu,\tau_\eta^{z1}(\mu)\big)\,d\mu\\ &+\int_0^{z^*}\int_{-\mu^{1-\eta}}^{0}\rho_1^{(1,2)}(\mu,\tau_\eta)F(\mu,\tau_\eta)\,d\tau_\eta d\mu + \int_{z^*}^{+\infty}\int_{-\mu^{1-\eta}}^{\tau_\eta^{z}(\mu)}\rho_1^{(1,2)}(\mu,\tau_\eta)F(\mu,\tau_\eta)\,d\tau_\eta d\mu.\end{aligned} \tag{81}$$

The rest of the proof follows by inspection.

B.2 Proof of Proposition 7

With asymmetry, it is assumed that $\tau_\eta^{z1}(\mu) \geq \tau_\eta^{z2}(\mu)$ for all $\mu \in [z^*, +\infty)$. As a consequence, equation (46) can be rewritten as:

$$\begin{aligned}P(\tilde{\lambda}) = &\,q\int_0^{z^*}\int_{-\mu^{1-\eta}}^{0}\rho_1(\mu,\tau_\eta,\tilde{\lambda})f_1(\mu,\tau_\eta)\,d\tau_\eta d\mu + q\int_{z^*}^{+\infty}\int_{-\mu^{1-\eta}}^{\tau_\eta^{z1}(\mu)}\rho_1(\mu,\tau_\eta,\tilde{\lambda})f_1(\mu,\tau_\eta)\,d\tau_\eta d\mu \\ &+(1-q)\int_0^{z^*}\int_{-\mu^{1-\eta}}^{0}\rho_2(\mu,\tau_\eta,\tilde{\lambda})f_2(\mu,\tau_\eta)\,d\tau_\eta d\mu + (1-q)\int_{z^*}^{+\infty}\int_{-\mu^{1-\eta}}^{\tau_\eta^{z1}(\mu)}\rho_2(\mu,\tau_\eta,\tilde{\lambda})f_2(\mu,\tau_\eta)\,d\tau_\eta d\mu.\end{aligned} \tag{82}$$

Noting that $\rho_1^{(2)}(\mu,\tau_\eta) - \rho_2^{(2)}(\mu,\tau_\eta) \leq 0$ and $\rho_1^{(1,2)}(\mu,\tau_\eta) - \rho_2^{(1,2)}(\mu,\tau_\eta) \geq 0$, we obtain:

$$\begin{aligned}P(\tilde{\lambda}) = &-\int_0^{z^*}\rho_2^{(1)}(\mu,0)F(\mu)\,d\mu - \int_{-\infty^{1-\eta}}^{\tau_\eta^{z1}(+\infty)}\rho_2^{(2)}(+\infty,\tau_\eta)F(+\infty,\tau_\eta)\,d\tau_\eta\\ &-q\int_{-\infty^{1-\eta}}^{\tau_\eta^{z1}(+\infty)}\left(\rho_1^{(2)}(+\infty,\tau_\eta)-\rho_2^{(2)}(+\infty,\tau_\eta)\right)F_1(+\infty,\tau_\eta)\,d\tau_\eta\\ &+\int_{z^*}^{+\infty}\tau_\eta^{z1\prime}(\mu)\rho_2^{(2)}\big(\mu,\tau_\eta^{z1}(\mu)\big)F\big(\mu,\tau_\eta^{z1}(\mu)\big)\,d\mu\\ &+q\int_{z^*}^{+\infty}\tau_\eta^{z1\prime}(\mu)\left(\rho_1^{(2)}\big(\mu,\tau_\eta^{z1}(\mu)\big)-\rho_2^{(2)}\big(\mu,\tau_\eta^{z1}(\mu)\big)\right)F_1\big(\mu,\tau_\eta^{z1}(\mu)\big)\,d\mu\\ &+\int_0^{z^*}\int_{-\mu^{1-\eta}}^{0}\rho_2^{(1,2)}(\mu,\tau_\eta)F(\mu,\tau_\eta)\,d\tau_\eta d\mu + \int_0^{z^*}\int_{-\mu^{1-\eta}}^{0}\left(\rho_1^{(1,2)}(\mu,\tau_\eta)-\rho_2^{(1,2)}(\mu,\tau_\eta)\right)F_1(\mu,\tau_\eta)\,d\tau_\eta d\mu\\ &+\int_{z^*}^{+\infty}\int_{-\mu^{1-\eta}}^{\tau_\eta^{z1}(\mu)}\rho_2^{(1,2)}(\mu,\tau_\eta)F(\mu,\tau_\eta)\,d\tau_\eta d\mu\\ &+q\int_{z^*}^{+\infty}\int_{-\mu^{1-\eta}}^{\tau_\eta^{z1}(\mu)}\left(\rho_1^{(1,2)}(\mu,\tau_\eta)-\rho_2^{(1,2)}(\mu,\tau_\eta)\right)F_1(\mu,\tau_\eta)\,d\tau_\eta d\mu.\end{aligned} \tag{83}$$

The rest of the proof follows by inspection.

C Intersection of the different classes of poverty measures

C.1 The derivations of additional restrictions on the individual poverty measure π and ρ

We first consider the conditions that ρ must obey so that members of $\ddot{\Pi}(\lambda^+)$ are also members of $\tilde{\Pi}(\tilde{\lambda}^+)$. First, we have $\pi^{(x_i)}(x_1,x_2) \leq 0, \forall i = 1,2$, so that we should also observe $\rho_t^{(x_i)}(\mu,\tau_\eta) \leq 0\ \forall t$. When x_i is not the lowest income, an income increment increases variability ($|\tau_\eta|$ raises) while increasing mean income, so that the net effect of this is *a priori* not known. Assuming x_1 being the lowest income, if that net effect is supposed to correspond to a decrease in the level of poverty, we then should have:

$$
\begin{aligned}
& \rho_1^{(x_2)}(\mu,\tau_\eta) = \rho_1^{(1)}(\mu,\tau_\eta)\mu^{(x_2)} + \rho_1^{(2)}(\mu,\tau_\eta)\tau_\eta^{(x_2)} \leq 0 && (84)\\
\Rightarrow\quad & \frac{1}{2}\rho_1^{(1)}(\mu,\tau_\eta) + \frac{1}{2}\mu^{-\eta-1}\left(\frac{\eta}{2}(x_2-x_1)-\mu\right)\rho_1^{(2)}(\mu,\tau_\eta) \leq 0 && (85)\\
\Rightarrow\quad & \frac{1}{2}\rho_1^{(1)}(\mu,\tau_\eta) - \frac{1}{2}\left(\mu^{-\eta}+\frac{\eta\tau_\eta}{\mu}\right)\rho_1^{(2)}(\mu,\tau_\eta) \leq 0 && (86)\\
\Rightarrow\quad & \rho_1^{(1)}(\mu,\tau_\eta) \leq \left(\mu^{-\eta}+\frac{\eta\tau_\eta}{\mu}\right)\rho_1^{(2)}(\mu,\tau_\eta). && (87)
\end{aligned}
$$

In the same manner, it would also be necessary to observe $\rho^{(x_1,x_2)}(\mu,\tau_\eta) \geq 0$. Still supposing x_1 to be the lower income, we have:

$$
\begin{aligned}
\rho_1^{(x_1,x_2)}(\mu,\tau_\eta) &= \frac{1}{4}\rho_1^{(1,1)}(\mu,\tau_\eta) + \frac{1}{4}\mu^{-\eta-1}\left(\frac{\eta}{2}(x_2-x_1)+\mu\right)\rho_1^{(1,2)}(\mu,\tau_\eta)\\
&- \frac{\eta(\eta+1)}{2}2^{\eta-1}(x_1+x_2)^{-\eta-2}(x_2-x_1)\rho_1^{(2)}(\mu,\tau_\eta)\\
&+ \frac{1}{2}\mu^{-\eta-1}\left(\frac{\eta}{2}(x_2-x_1)-\mu\right)\left(\frac{1}{2}\rho_1^{(1,2)}(\mu,\tau_\eta) + \frac{1}{2}\mu^{-\eta-1}\left(\frac{\eta}{2}(x_2-x_1)+\mu\right)\rho_1^{(2,2)}(\mu,\tau_\eta)\right) && (88)\\
&= \frac{1}{4}\rho_1^{(1,1)}(\mu,\tau_\eta) + \frac{\eta}{4}\mu^{-\eta-1}(x_2-x_1)\rho_1^{(1,2)}(\mu,\tau_\eta)\\
&- \frac{\eta(\eta+1)}{2}2^{\eta-1}(x_1+x_2)^{-\eta-2}(x_2-x_1)\rho_1^{(2)}(\mu,\tau_\eta)\\
&+ \frac{1}{4}\mu^{-2(\eta+1)}\left(\frac{\eta^2}{4}(x_2-x_1)^2-\mu^2\right)\rho_1^{(2,2)}(\mu,\tau_\eta) && (89)\\
&= \frac{1}{4}\rho_1^{(1,1)}(\mu,\tau_\eta) - \frac{1}{2}\frac{\eta\tau_\eta}{\mu}\rho_1^{(1,2)}(\mu,\tau_\eta) - \frac{\eta(\eta+1)\tau_\eta}{8\mu^2}\rho_1^{(2)}(\mu,\tau_\eta)\\
&+ \frac{1}{4}\left(\left(\frac{\eta\tau_\eta}{\mu}\right)^2-\mu^{-2\eta}\right)\rho_1^{(2,2)}(\mu,\tau_\eta). && (90)
\end{aligned}
$$

Considering the case of absolute variability aversion ($\eta = 0$), ρ_t must exhibit the following two properties:

$$
\begin{cases}
\rho_t^{(1)}(\mu,\tau_0) \leq \rho_t^{(2)}(\mu,\tau_0),\\
\rho_t^{(1,1)}(\mu,\tau_0) \geq \rho_t^{(2,2)}(\mu,\tau_0).
\end{cases} \qquad (91)
$$

With relative the variability aversion ($\eta = 1$), the conditions become:

$$
\begin{cases}
\rho_t^{(1)}(\mu,\tau_1) \leq \frac{1+\tau_1}{\mu}\rho_t^{(2)}(\mu,\tau_1),\\
\rho_t^{(1,1)}(\mu,\tau_1) \geq \frac{2\tau_1}{\mu}\rho_t^{(1,2)}(\mu,\tau_1) + \frac{\tau_1}{\mu^2}\rho_t^{(2)}(\mu,\tau_1) + \frac{1-\tau_1^2}{\mu^2}\rho_t^{(2,2)}(\mu,\tau_1).
\end{cases} \qquad (92)
$$

It can also be shown that $x_1 = \mu + \tau_\eta\mu^\eta$ and $x_2 = \mu - \tau_\eta\mu^\eta$ if $x_1 < x_2$. It is then possible to compute $\pi^{(\mu)}$, $\pi^{(\tau_\eta)}$ and $\pi^{(\mu,\tau_\eta)}$ to see what conditions have to be met so that π respects the conditions imposed on ρ. First, considering the derivatives of π with respect to mean income, we should observe:

$$
\begin{aligned}
& \pi^{(\mu)}(x_1,x_2) = \pi^{(1)}(x_1,x_2)x_1^{(\mu)} + \pi^{(2)}(x_1,x_2)x_2^{(\mu)} \leq 0 && (93)\\
\Rightarrow\quad & (1+\eta\tau_\eta\mu^{\eta-1})\pi^{(1)}(x_1,x_2) + (1-\eta\tau_\eta\mu^{\eta-1})\pi^{(2)}(x_1,x_2) \leq 0. && (94)
\end{aligned}
$$

That condition is always fulfilled since $1+\eta\tau_\eta\mu^{\eta-1}$ and $1-\eta\tau_\eta\mu^{\eta-1}$ are positive for $\eta \in [0,1]$, and $\pi^{(1)}(x_1,x_2)$ and $\pi^{(2)}(x_1,x_2)$ are also non-negative. The result is intuitive. Increasing the mean without altering variability implies increasing income at both periods, so that poverty should logically fall.

Considering now a decrease in variability without a change in mean income, things are less clear since such a change raises the lower income but decreases the higher one. It is then necessary to consider the net sum of those opposite effects. Since $\rho_t^{(2)}(\mu,\tau_\eta) \leq 0$, we should obtain:

$$\pi^{(\tau_\eta)}(x_1,x_2) = \mu^\eta \pi^{(1)}(x_1,x_2) - \mu^\eta \pi^{(2)}(x_1,x_2) \leq 0 \tag{95}$$

$$\Rightarrow \qquad \pi^{(1)}(x_1,x_2) \leq \pi^{(2)}(x_1,x_2). \tag{96}$$

Finally, π has to be defined so as to respect $\pi^{(\mu,\tau_\eta)}(x_1,x_2) \geq 0$. We have:

$$\begin{aligned}
\pi^{(\mu,\tau_\eta)}(x_1,x_2) &= \eta\mu^{\eta-1}\pi^{(1)}(x_1,x_2) + \left(1+\eta\tau_\eta\mu^{\eta-1}\right)\left(\mu^\eta\pi^{(1,1)}(x_1,x_2) - \mu^\eta\pi^{(1,2)}(x_1,x_2)\right) \\
&\quad - \eta\mu^{\eta-1}\pi^{(2)}(x_1,x_2) + \left(1-\eta\tau_\eta\mu^{\eta-1}\right)\left(\mu^\eta\pi^{(1,2)}(x_1,x_2) - \mu^\eta\pi^{(2,2)}(x_1,x_2)\right) && (97) \\
&= \eta\mu^{\eta-1}\left(\pi^{(1)}(x_1,x_2) - \pi^{(2)}(x_1,x_2)\right) + \mu^\eta\left(1+\eta\tau_\eta\mu^{\eta-1}\right)\left(\pi^{(1,1)}(x_1,x_2) - \pi^{(1,2)}(x_1,x_2)\right) \\
&\quad + \mu^\eta\left(1-\eta\tau_\eta\mu^{\eta-1}\right)\left(\pi^{(1,2)}(x_1,x_2) - \pi^{(2,2)}(x_1,x_2)\right). && (98)
\end{aligned}$$

With absolute variability aversion ($\eta = 0$), π should be such that:

$$\begin{cases}
\pi^{(1)}(x_1,x_2) \leq \pi^{(2)}(x_1,x_2), \\
\pi^{(1,1)}(x_1,x_2) \geq \pi^{(2,2)}(x_1,x_2).
\end{cases} \tag{99}$$

With relative variability aversion ($\eta = 1$), we obtain, for $x_1 < x_2$:

$$\begin{cases}
\pi^{(1)}(x_1,x_2) \leq \pi^{(2)}(x_1,x_2), \\
\pi^{(1)}(x_1,x_2) - \pi^{(2)}(x_1,x_2) + \mu(1+\tau_1)\left(\pi^{(1,1)}(x_1,x_2) - \pi^{(1,2)}(x_1,x_2)\right) \\
+\mu(1-\tau_1)\left(\pi^{(1,2)}(x_1,x_2) - \pi^{(2,2)}(x_1,x_2)\right) \geq 0.
\end{cases} \tag{100}$$

C.2 Proof of Proposition 8 and Corollary 1

We have shown that it is possible to impose restrictions on the derivatives of both $\pi(x_1,x_2)$ and $\rho(\mu,\tau_\eta)$ to obtain measures that are included in both $\tilde{\Pi}(\tilde{\lambda}^+)$ and $\ddot{\Pi}(\lambda^+)$. Since the class of poverty measures $\breve{\Pi}_\eta(\tilde{\lambda}^+)$ is not empty, any measure $P(\tilde{\lambda}) \in \breve{\Pi}_\eta(\tilde{\lambda}^+)$ can equally be expressed using equation (8) or equation (19). Consequently, both (59) and (78) are valid expressions for $P(\tilde{\lambda})$. For Proposition 8 not to hold, it would be necessary to show that one can find two distributions A and B such that $A \ddot{\succcurlyeq}_{\tilde{\lambda}^+} B$ and $B \tilde{\succcurlyeq}_{\eta,\tilde{\lambda}^+} A$. However, with the restrictions imposed on the classes $\ddot{\Pi}(\lambda^+)$ and $\tilde{\Pi}(\tilde{\lambda}^+)$, such a situation would imply that for any poverty measure in $\breve{\Pi}_\eta(\tilde{\lambda}^+)$, the difference $P_A(\tilde{\lambda}) - P_B(\tilde{\lambda})$ should simultaneously be non-negative and non-positive. This will happen if and only if $P_A^{1,1}(x_1,x_2) = P_B^{1,1}(x_1,x_2)$, $\forall(x_1,x_2) \in \Gamma(\tilde{\lambda})$. This proves Proposition 8.

The demonstration of Corollary 1 is straightforward. As long as the class of poverty measures $\breve{\Pi}_\eta(\tilde{\lambda}^+)$ is not empty, observing dominance with respect to either $\ddot{\Pi}(\lambda^+)$ or $\tilde{\Pi}(\tilde{\lambda}^+)$ precludes observing an opposite strong dominance relationship with the other class of poverty measures, as both classes include $\breve{\Pi}_\eta(\tilde{\lambda}^+)$.

References

Alkire, S. and Foster, J. (2011*a*), 'Counting and multidimensional poverty measurement', *Journal of Public Economics* **95**(7-8), 476–487.

Alkire, S. and Foster, J. (2011*b*), 'Understandings and misunderstandings of multidimensional poverty measurement', *Journal of Economic Inequality* **9**(2), 289–314.

Atkinson, A. (1992), 'Measuring poverty and differences in family composition', *Economica* **59**(233), 1–16.

Atkinson, A. B. and Bourguignon, F. (1987), Income distribution and differences in needs, *in* G. R. Feiwel, ed., 'Arrow and the Foundations of the Theory of Economic Policy', Macmillan, New York, pp. 350–370.

Atkinson, A. and Bourguignon, F. (1982), 'The comparison of multi-dimensioned distributions of economic status', *Review of Economic Studies* **49**(2), 183–201.

Atkinson, A., Cantillon, B., Marlier, E. and Nolan, B. (2002), *Social Indicators – The EU and Social Inclusion*, Oxford University Press, Oxford.

Bossert, W., Chakravarty, S. and d'Ambrosio, C. (2011), 'Poverty and time', *Journal of Economic Inequality* **Online First**, 1–18.

Bourguignon, F. (1989), 'Family size and social utiliy: Income distribution dominance criteria', *Journal of Econometrics* **42**(1), 67–80.

Bourguignon, F., Goh, C.-C. and Kim, D. I. (2004), Estimating individual vulnerability to poverty with pseudo-panel data, Discussion Paper, World Bank, Washington DC.

Bourguignon, Fr. and Chakravarty, S. (2003), 'The measurement of multidimensional poverty', *Journal of Economic Inequality* **1**(1), 25–49.

Busetta, A. and Mendola, D. (2012), 'The importance of consecutive spells of poverty: A path-dependent index of longitudinal poverty', *Review of Income and Wealth* **58**(2), 355–374.

Busetta, A., Mendola, D. and Milito, A. M. (2011), 'Combining the intensity and sequencing of the poverty experience: A class of longitudinal poverty indices', **174**(4), 953–973.

Calvo, C. and Dercon, S. (2009), Chronic poverty and all that: The measurement of poverty over time, *in* T. Addison, D. Hulme and R. Kanbur, eds, 'Poverty Dynamics: Interdisciplinary Perspectives', Oxford University Press, Oxford, chapter 2, pp. 29–58.

Chakravarty, S., Deutsch, J. and Silber, J. (2008), 'On the watts multidimensional poverty index and its decomposition', *World Development* **36**(6), 1067–1077.

Chakravarty, S., Mukherjee, D. and Ranade, R. (1998), On the family of subgroup and factor decomposable measures of multidimensional poverty, *in* D. Slottje, ed., 'Research on Economic Inequality', Vol. 8, JAI Press, pp. 175–194.

Chaudhuri, S., Jalan, J. and Suryahadi, A. (2002), Assessing household vulnerability to poverty from cross-sectional data: A methodology and estimates from indonesia, Discussion Paper 0102-52, Columbia University.

Christiaensen, L. and Subbarao, K. (2004), Toward an understanding of household vulnerability in rural kenya, Policy Research Working Paper 3326, World Bank.

Cruces, G. and Wodon, Q. (2003), 'Transient and chronic poverty in turbulent times: Argentina 1995-2002', *Economics Bulletin* **9**(3), 1–12.

Davidson, R. and Duclos, J.-Y. (2006), Testing for restricted stochastic dominance, Discussion Paper 2047, IZA.

Duclos, J.-Y., Araar, A. and Giles, J. (2010), 'Chronic and transient poverty: Measurement and estimation, with evidence from china', *Journal of Development Economics* **91**(2), 266–277.

Duclos, J.-Y., Sahn, D. and Younger, S. (2006), 'Robust multidimensional poverty comparisons', *The Economic Journal* **116**, 943–968.

Foster, J. (2007), A class of chronic poverty measures, Working Paper 07-W01, Department of Economics, Vanderbilt University.

Foster, J. and Shorrocks, A. (1991), 'Subgroup consistent poverty indices', *Econometrica* **59**(3), 687–709.

Hoy, M., Thompson, B. and Zheng, B. (forthcoming), 'Empirical issues in lifetime poverty measurement', *Journal of Economic Inequality* pp. 1–27. 10.1007/s10888-011-9192-1.
URL: *http://dx.doi.org/10.1007/s10888-011-9192-1*

Hoy, M. and Zheng, B. (2008), Measuring lifetime poverty, Working Paper 0814, University of Guelph, Department of Economics.

Hulme, D. and Shepherd, A. (2003), 'Conceptualizing chronic poverty', *World Development* **31**(3), 403–423.

Jalan, J. and Ravallion, M. (1998), 'Transient poverty in postreform rural china', *Journal of Comparative Economics* **26**, 338–357.

Kamanou, G. and Morduch, J. (2004), *Measuring Vulnerability to Poverty*, Oxford University Press, Oxford.

Kolm, S.-C. (1976), 'Unequal inequalities I', *Journal of Economic Theory* **12**, 416–442.

Krtscha, M. (1994), A new compromise measure of inequality, *in* W. Eichhorn, ed., 'Models and Measurement of Welfare and Inequality', Springer-Verlag, pp. 111–119.

Ligon, E. and Schechter, L. (2003), 'Measuring vulnerability', *The Economic Journal* **113**, C95–C102.

Ravallion, M. (2011), 'On multidimensional indices of poverty', *Journal of Economic Inequality* **9**(2), 235–248.

Rodgers, J. and Rodgers, J. (1993), 'Chronic poverty in the United States', *Journal of Human Resources* **28**(1), 25–54.

Tsui, K.-Y. (2002), 'Multidimensional poverty indices', *Social Choice and Welfare* **19**(1), 69–93.

Yoshida, T. (2005), 'Social welfare rankings of income distributions. a new parametric concept of intermediate inequality', *Social Choice and Welfare* **24**(3), 555–574.

Zheng, B. (2007), 'Inequality orderings and unit-consistency', *Social Choice and Welfare* **29**(3), 515–538.

Zoli, C. (2003), Characterizing inequality equivalence criteria, Working Paper, University of Nottingham.