

## MEASURING THE SIZE OF THE WORLD ECONOMY

*A Framework, Methodology and Results from the  
International Comparison Program (ICP)*

Editors

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### **Chapter 8: Methods of Aggregation above the Basic Heading Level: Linking the Regions**

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#### **1. Introduction**

In this chapter, various possible methods for linking the five regions in the ICP in such a way that the relative country volumes within each region are preserved will be discussed.

Diewert (2004; 46-47) suggested a method for linking the regions in the 2005 ICP round that would preserve relative volumes for country aggregates within a region but at the same time would link the various regions so that volumes could be compared across all countries in the ICP comparison in a consistent manner. The basic idea behind the method is as follows: once information on country expenditures for each Basic Heading (BH) category in each country in a region is determined along with the corresponding Basic Heading Purchasing Power Parities (PPPs), then BH expenditures in the local currencies can be divided by the corresponding PPPs in order to obtain country volumes in consistent comparable units by Basic Heading category. These volumes (or loosely speaking, quantities) can then be added across countries within the region in order to obtain *total regional volumes* or quantities by Basic Heading category. Then with an appropriate choice for BH prices for each region, these BH regional prices and quantities can then be linked using any multilateral method such as the GEKS method<sup>1</sup> studied in chapter 7. Once regional real output shares have been determined by the chosen multilateral method, they can be combined with each region's country shares of regional real output to give each country's share of world real output. Note that this method will not affect the country shares of regional output that are determined independently by each region. Note also that this method is very similar to the methods used by national income accountants to determine the annual real output of a nation by aggregating over quarterly data.

The problem with the above class of methods for linking the regions is this: how exactly should the regional Basic Heading prices be chosen? These regional price or PPP vectors

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<sup>1</sup> This method is due to Gini (1924) (1931), Eltetö and Köves (1964) and Szulc (1964).

should be chosen so that the overall method leads to country (and regional) relative volumes that are independent of the choice of both the numeraire region and the numeraire countries within each region. In section 2 below, several alternative choices of the regional BH prices will be explored. Option 1 explained in section 2 does lead to relative volumes which are independent of the choice of numeraire regions and numeraire countries within each region but the resulting method involves the use of country exchange rates which is not a desirable feature. Option 2 explained in section 2 does not involve the use of exchange rates but the resulting method is dependent on the choice of the regional numeraire countries, which again, is not a desirable feature.<sup>2</sup> Finally, Option 3 explained in section 2, which was suggested by Sergeev (2009b), does lead to relative volumes which are independent of the choice of numeraire regions and numeraire countries within each region and this method for linking the regions does not involve the use of exchange rates.

It should be noted that all of the variants of the above method for linking the regions are based on two important properties within each region:

- Volumes or quantities are added up across countries in the region to obtain total regional volumes by BH category of expenditure and
- A single set of BH reference prices is applied to the above regional volumes for each region.

As mentioned above, the first property is analogous to practices in the System of National Accounts where annual quantities by expenditure category are obtained by adding up quarterly or monthly quantities and so this aspect of the linking method is not necessarily problematic. However, in the national accounting context, the structure of prices tends to be very similar going from quarter to quarter within a year. Thus in the national accounts, the annual price for a commodity is usually taken to be an annual unit value; i.e., total expenditure or revenue for the commodity is summed up over say quarters and divided by the total quantity used or produced during the year and this unit value is used as the annual price of the commodity.<sup>3</sup> Using an annual unit value price in the context of aggregating over time is generally not a problem, since the price of a commodity within a year generally does not vary too much. However, when one is aggregating quantities across countries within a region, variations in commodity prices can be very large and generally speaking, there will *not* be a single vector of commodity prices that are *representative* for the structure of commodity prices for *every* country in the region. Thus the property of these methods which calculates a regional quantity vector by addition over country quantity vectors means that all of these methods are essentially *additive methods* (within each region) and hence are subject to *substitution bias*.<sup>4</sup> Thus for ICP 2011, other methods for linking the regions, while respecting within region parities,

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<sup>2</sup> Sergey Sergeev (2009b) first noticed this problem with this method, which was the method used to link the regions in ICP 2005.

<sup>3</sup> If there is high inflation within the year for the country, then finding an appropriate annual price is more complicated; see Hill (1996).

<sup>4</sup> See the discussion in Chapter 7 on additive methods and substitution bias.

should be considered. In sections 3 and 4 below, alternative linking methods will be considered.

In section 3, the first alternative method for constructing interregional parities while respecting within region parities will be discussed. This method is due to Yuri Dikhanov (2007), who suggested that the GEKS method be used for all countries in the comparison, which will generate shares of world output (or a subaggregate) for each country. Now simply add up the shares of world output of the countries in each region in order to obtain regional shares. These between region shares plus the within region shares (determined independently by the regions) will generate an overall set of shares that respect within region parities.

In section 4, a variant of the method used in section 3 will be discussed: instead of using GEKS to generate individual country shares for each country in the comparison, one could use Robert Hill's spatial linking method<sup>5</sup> to form the shares of each country's output in the world aggregate. The basic idea behind this method is to link countries through a series of bilateral comparisons where the overall comparison rests on bilateral comparisons between countries which are most similar in their (relative) price structures.

In section 5, regional parities are computed using the methods suggested in sections 2-4 for a small artificial data set.

In section 6, another numerical example based on a subset of the 2005 ICP data is computed to again illustrate the fact that different methods can give quite different results.

Section 7 concludes.

## 2. Variants of Diewert's Suggested Method for Linking the Regions

This section begins by defining what basic data are needed in order to link the regions in a fashion that will respect the parities and relative volumes that have been determined within each region. It is assumed that there are  $R$  regions in the comparison and region  $r$  has  $C(r)$  countries in it for  $r = 1, 2, \dots, R$ . In ICP 2005,  $R$  was equal to 5 with varying numbers of countries in each region. Assume that there are  $N$  commodity groups and that each country in the comparison has collected expenditure data on these  $N$  commodity groups in their own currencies. In ICP 2005,  $N$  was equal to 155. Let  $E_{rcn}$  be the expenditure (in the currency of country  $c$  in region  $r$ ) on commodity class  $n$  for the reference year for  $r = 1, \dots, R$ ;  $c = 1, \dots, C(r)$ ;  $n = 1, \dots, N$ . Assume that these country expenditure data have been collected.

Each region  $r$  has constructed a *Purchasing Power Parity* (PPP) for each commodity group  $n$  and each country  $c$  in the region,  $\alpha_{rcn}$  say, for  $r = 1, \dots, R$ ;  $c = 1, \dots, C(r)$  and  $n = 1, \dots, N$ . Assume that country  $c = 1$  is the numeraire country in each region and so the PPP for this numeraire country is set equal to unity so that:

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<sup>5</sup> See Robert Hill (1999a) (1999b) (2001) (2004). Fisher (1922; 271-274) is a precursor to the work of Hill.

$$(1) \alpha_{r1n} = 1 ; \quad r = 1, \dots, R; n = 1, \dots, N.$$

Thus for  $c \neq 1$ ,  $\alpha_{rcn}$  is the price in the currency of country  $c$  in region  $r$  of a bundle of commodity  $n$  which costs one currency unit in the currency of the numeraire country (country 1) for region  $r$ .

The next assumption is that the central office has constructed a set of *interregional linking PPPs*,  $\beta_{rn}$ , that link the PPPs of the numeraire country in each region with the numeraire country in the numeraire region, is assumed to be region 1. Thus assume that

$$(2) \beta_{1n} = 1 ; \quad n = 1, \dots, N.$$

For  $r \neq 1$ ,  $\beta_{rn}$  is the price in the currency of country 1 in region  $r$  of a bundle of commodity  $n$  which costs one currency unit in the currency of the numeraire country 1 in region 1, the numeraire region.

Now the two sets of PPPs can be multiplied together to form a consistent set of *world PPPs*,  $\gamma_{rcn}$ , defined as follows:

$$(3) \gamma_{rcn} \equiv \beta_{rn} \alpha_{rcn} ; \quad r = 1, \dots, R; c = 1, \dots, C(r); n = 1, \dots, N.$$

Thus  $\gamma_{rcn}$  is the price in the currency of country  $c$  in region  $r$  of a bundle of commodity  $n$  which costs one currency unit in the currency of the numeraire country (country 1) in the numeraire region (region 1).

If the numeraire region is changed, what happens to the world PPPs  $\gamma_{rcn}$  defined by (3)? Thus suppose region 2 becomes the numeraire region in place of region 1. In this case, the original linking PPPs (the  $\beta_{rn}$ ) should be replaced by the new linking PPPs defined as

$$(4) \beta_{rn}^* \equiv \beta_{rn} / \beta_{2n} ; \quad r = 1, \dots, R; n = 1, \dots, N$$

and the original set of world PPPs, the  $\gamma_{rcn}$  defined by (3) should be replaced by the following new set of world PPPs:

$$(5) \gamma_{rcn}^* \equiv \beta_{rn}^* \alpha_{rcn} = \gamma_{rcn} / \beta_{2n} ; \quad r = 1, \dots, R; c = 1, \dots, C(r); n = 1, \dots, N.$$

Thus the new set of world PPPs are equal to the old set of world PPPs except for a divisor that depends on the commodity  $n$  under consideration. Hence the net effect of switching the numeraire region is to leave the PPPs unchanged except that the new set of PPPs are measured in a *new system of units*; instead of measuring commodity units in terms of a dollar's worth of purchases of commodity  $n$  in the numeraire country of region 1, now commodity units are measured in terms of a peso's worth of purchases of commodity  $n$  in the numeraire country of region 2.

Now suppose that the numeraire country in a region is changed. If the change of numeraire country is outside region 1, (say in region 2 where there is a switch from country 1 to country 2 as the numeraire country), then it can be seen that nothing will happen to the world PPPs  $\gamma_{rcn}$ : the old  $\gamma_{rcn} = \beta_{rn}\alpha_{rcn}$  will remain the same for all regions  $r$  except for  $r = 2$  and for the second region,  $\alpha_{2cn}$  will be replaced by  $\alpha_{2cn}/\alpha_{22n}$  for  $n = 1, \dots, N$  and  $c = 1, \dots, C(2)$  and  $\beta_{2n}$  will be replaced by  $\beta_{2n}\alpha_{22n}$  for  $n = 1, \dots, N$ . Thus the new  $\gamma_{rcn}^*$  will still equal the old  $\gamma_{rcn}$  for all  $r$  including  $r = 2$ .

If the change of numeraire country is within region 1 so that the new numeraire country in region 1 is country 2 in place of country 1, then the within region parities for region 1 become  $\alpha_{1cn}/\alpha_{12n}$  for  $n = 1, \dots, N$  and the new set of interregional linking PPPs become  $\beta_{2n}/\alpha_{12n}$  for  $n = 1, \dots, N$ . Thus in this case, the new set of world PPPs becomes:

$$(6) \gamma_{rcn}^* = \gamma_{rcn}/\alpha_{12n}; \quad r = 1, \dots, R; c = 1, \dots, C(r); n = 1, \dots, N.$$

Thus in this case of a change of the numeraire country within the numeraire region, the new set of world PPPs are equal to the old set of world PPPs except for a divisor that depends on the commodity  $n$  under consideration. Hence as was the case with a change in the numeraire region, the net effect of switching the numeraire country within the numeraire region is to leave the PPPs unchanged except that the new set of PPPs are measured in a *new system of units*.

Recall a knowledge of the expenditures (in the local currencies) of country  $c$  in region  $r$  by commodity class  $n$ ,  $E_{rcn}$ , is assumed for  $r = 1, \dots, R; c = 1, \dots, C(r); n = 1, \dots, N$ . This expenditure information is used along with the consistent set of world PPPs, the  $\gamma_{rcn}$  defined by (3) above, in order to define consistent (across countries) *imputed quantities or volumes*,  $Q_{rcn}$ , for each commodity  $n$  and each country  $c$  in each region  $r$  as follows:

$$(7) Q_{rcn} \equiv E_{rcn}/\gamma_{rcn}; \quad r = 1, \dots, R; c = 1, \dots, C(r); n = 1, \dots, N.$$

These imputed quantities are measured in units which are comparable across countries and regions. Thus these quantities can be added across countries within a region and the resulting regional totals,  $Q_{rn}$ , are also comparable across regions; i.e., define *regional total volumes or quantities by commodity class* as follows:

$$(8) Q_{rn} \equiv \sum_{c=1}^{C(r)} Q_{rcn}; \quad r = 1, \dots, R; n = 1, \dots, N.$$

Now form *regional quantity or volume vectors* from the  $Q_{rn}$ :

$$(9) Q^r \equiv [Q_{r1}, \dots, Q_{rN}]; \quad r = 1, \dots, R.$$

Note that if the numeraire region or the numeraire countries within a region are changed, then provided that the world PPPs are changed in a consistent manner, the regional quantity vectors will be identical to the initial regional quantity vectors defined by (7)-(9) except that the units of measurement for these vectors may have been changed by the change in numeraires; i.e., a change in the numeraire region or numeraire country within

a region will lead to identical regional quantity vectors  $Q^r$  or to a new set of  $Q^{r*}$  which are equal to  $\hat{\delta} Q^r$  where  $\hat{\delta}$  is a diagonal matrix where the diagonal elements reflect changes in the units of measurement of the N commodity groups.

Regional price vectors  $P^r$  are now needed to match up with the above regional quantity vectors  $Q^r$  defined by (9). There are at least three possible strategies at this point and each will be discussed in turn.

### **Option 1: The Conversion to Regional Currency Unit Values Option:**

The strategy using this option will be to convert country expenditures in each commodity classification category into common regional expenditures (in a numeraire country's currency) using market exchange rates for the reference year and then deflate these regional commodity expenditures by the corresponding regional quantities defined by (8) in order to form regional unit value prices, which will be used as the regional prices. As mentioned in the introduction, this is the same type of strategy that is used by national income accountants in forming annual price and quantity vectors from subannual information.

Suppose the *reference year exchange rate* for country c in region r is  $\varepsilon_{rc}$  for  $r = 1, \dots, R$  and  $c = 1, \dots, C(r)$ . Assume that country 1 in each region is the numeraire region as usual so that

$$(10) \varepsilon_{r1} = 1 ; \quad r = 1, \dots, R.$$

Thus  $\varepsilon_{rc}$  for  $c \neq 1$  tells us how many units of the numeraire (country 1) currency in region r is equal to one unit of the currency unit of country c in region r. These market exchange rates can be used to convert country expenditures (in the country's currency) on commodity class n within region r into region r numeraire currency units,  $V_{rcn} \equiv \varepsilon_{rc} E_{rcn}$ , and then these *country expenditures on commodity class n in a common regional currency* can be summed to regional totals  $V_m$  defined as follows:

$$(11) V_m \equiv \sum_{c=1}^{C(r)} \varepsilon_{rc} E_{rcn} ; \quad r = 1, \dots, R ; n = 1, \dots, N.$$

Now *regional unit value prices*  $P_m$  can be formed by taking the regional values defined by (11) and dividing them by the corresponding regional quantity totals  $Q_m$  defined by (8):

$$(12) P_m \equiv V_m / Q_m ; \quad r = 1, \dots, R ; n = 1, \dots, N.$$

Finally, form *regional unit value price vectors* from the components,  $P_m$ :

$$(13) P^r \equiv [P_{r1}, \dots, P_{rN}] ; \quad r = 1, \dots, R.$$

Now consider what happens to the regional total expenditures on commodity class n,  $V_m$ , if a numeraire country within a region is changed or there is a change the numeraire

region. If the numeraire region is changed, nothing happens to regional expenditures defined by (11) since it is not necessary to relate the regional exchange rates across regions. However, the numeraire country within a region  $r$  is changed, then it can be seen that that all of the regional values for this region will change by a scalar factor; i.e., if in region  $r$ , the numeraire country is changed from country 1 to country 2, then the new market exchange rates will be  $\varepsilon_{rc}/\varepsilon_{r2}$  for  $c = 1, \dots, C(r)$  and hence the new regional totals for region  $r$  will be

$$(14) V_m^* \equiv \sum_{c=1}^{C(r)} [\varepsilon_{rc}/\varepsilon_{r2}] E_{rcn} = V_m/\varepsilon_{r2}; \quad n = 1, \dots, N.$$

Thus the new regional expenditure totals on the  $N$  commodity groups in region  $r$ ,  $V_m^*$ , will be equal to a scalar multiple ( $1/\varepsilon_{r2}$ ) times the old regional expenditure totals in region  $r$ ,  $V_m$ .

The above is a careful exposition of the regional unit value method for forming regional price and quantity vectors that could be used to link the regions by using a multilateral index number method with the above regional price and quantity vectors as the input vectors to the method.

Now consider linking the  $R$  regions using a *multilateral index number method*. Diewert (1999) defined such methods in terms of share functions for the  $R$  regions; i.e., he looked at a system of *shares of world output*,  $s_1(P^1, \dots, P^R; Q^1, \dots, Q^R), \dots, s_R(P^1, \dots, P^R; Q^1, \dots, Q^R)$ , where  $s_r(P^1, \dots, P^R; Q^1, \dots, Q^R)$  is region  $r$ 's share of world output and considered the axiomatic properties of such multilateral systems. The next question that needs to be addressed is: what properties does the multilateral method have to satisfy in order to ensure that the regional shares are independent of the choice of the numeraire region and the numeraire countries within the regions?

From the above discussion, it is clear that the multilateral method must satisfy the following two properties:

*Property 1: Invariance to changes in the units of measurement.*

Thus let  $\delta_n > 0$  for  $n = 1, \dots, N$  and define  $\hat{\delta}$  as the  $N$  by  $N$  matrix with the elements  $\delta_n$  running down the main diagonal. Then this property requires that the multilateral share system satisfy the following equations:

$$(15) s_r(\hat{\delta} P^1, \dots, \hat{\delta} P^R; \hat{\delta}^{-1} Q^1, \dots, \hat{\delta}^{-1} Q^R) = s_r(P^1, \dots, P^R; Q^1, \dots, Q^R); \quad r = 1, \dots, R.$$

*Property 2: Homogeneity of degree zero in the regional price vectors.*

Let  $\lambda_1 > 0, \dots, \lambda_R > 0$ . Then this property requires that the multilateral share system satisfy the following equations:

$$(16) s_r(\lambda_1 P^1, \dots, \lambda_R P^R; Q^1, \dots, Q^R) = s_r(P^1, \dots, P^R; Q^1, \dots, Q^R); \quad r = 1, \dots, R.$$

This homogeneity property means that it is relative regional prices that affect the interregional volume comparisons and not the absolute level of regional prices.

It can be seen that the above two properties are also sufficient to imply that a multilateral method using the unit value regional price and quantity vectors defined above will generate regional shares that are independent of the choice of the numeraire region and the numeraire countries within the regions. It should be noted that the GEKS multilateral method satisfies these two properties; see Diewert (1999; 33).

A drawback of this method for linking the regions is that it brings market exchange rates into the picture. Since market exchange rates are often far removed from their corresponding PPPs, it is desirable to avoid their use in constructing the interregional PPPs and relative volumes. The following two methods make use of the regional quantity vectors  $Q^r$  defined by (8) but the corresponding regional price vectors  $P^r$  do not make use of exchange rates.

### **Option 2: The Use of Regional Numeraires as Regional Price Weights**

It is not necessary to bring in market exchange rates to convert regional expenditures into a common currency. Rather than doing this, one can simply use the PPPs for each numeraire country 1 in each region  $r$ , relative to the numeraire country in the numeraire region, the  $\beta_m$  described above, as the price for commodity  $n$  in region  $r$ ; i.e., define the regional price for commodity  $n$  in region  $r$  as follows:

$$(17) P_m \equiv \beta_m ; \quad r = 1, \dots, R ; n = 1, \dots, N.$$

Then use (13) to form the *regional price vectors*  $P^r$  in the usual way. The regional quantity vectors,  $Q^r$ , are defined as before by (7)-(9) and these equations do not involve exchange rates. Finally, use these regional price and quantity vectors,  $P^r$  and  $Q^r$ , in order to calculate the share functions for the  $R$  regions using a favoured multilateral method,  $s_1(P^1, \dots, P^R; Q^1, \dots, Q^R), \dots, s_R(P^1, \dots, P^R; Q^1, \dots, Q^R)$ , where  $s_r(P^1, \dots, P^R; Q^1, \dots, Q^R)$  is region  $r$ 's share of world output.

Unfortunately, as Sergeev (2009b) pointed out, this method is not invariant to the choice of the numeraire countries within the regions. Thus this method should not be used in ICP 2011.

### **Option 3: The Use of the Geometric Average of the Regional Numeraires as Regional Price Weights**

Sergeev (2009b) suggested a way to avoid the lack of numeraire invariance in Option 2: within each region, take the *geometric mean of the country parities* over all countries in the region. Thus equations (17) are replaced by the following equations:<sup>6</sup>

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<sup>6</sup> This method of aggregation within a region is related to the geometric average price multilateral method originally suggested by Walsh (1901; 381 and 398) which was noted by Gini (1924; 106) and implemented by Gerardi (1982; 387). These authors used reference world prices that were the geometric mean over all

$$(18) P_{rn} \equiv \prod_{c=1}^{C(r)} \gamma_{rcn}^{1/C(r)} ; \quad r = 1, \dots, R ; n = 1, \dots, N.$$

Then use (13) to form the *regional price vectors*  $P^r$  in the usual way. Finally, use these regional price and quantity vectors defined by (9),  $P^r$  and  $Q^r$ , in order to calculate the share functions for the  $R$  regions using a favoured multilateral method,  $s_1(P^1, \dots, P^R; Q^1, \dots, Q^R), \dots, s_R(P^1, \dots, P^R; Q^1, \dots, Q^R)$ , where  $s_r(P^1, \dots, P^R; Q^1, \dots, Q^R)$  is region  $r$ 's share of world output. If the multilateral method satisfies Property 1 above (Invariance to changes in the units of measurement), then the resulting regional shares will be invariant to the choices of both numeraire countries and the numeraire region.

As mentioned above, Option 2 can be ruled out as a method for linking the regions for ICP 2011 due to its lack of country numeraire invariance within the regions. However, there is a good case for ruling out Options 1 and 3 as well. This is due to the fact that all of the methods in this section *impose a set of common prices to add up the quantities within a region*; i.e., the methods impose a form of *additivity*. Additive multilateral methods are subject to *substitution bias* if there are three or more countries in the comparison.<sup>7</sup>

Thus in the following two sections, *nonadditive methods* for linking the regions will be discussed which avoid the substitution bias that is inherent in the methods discussed in the present section.

### 3. The Global Comparison GEKS Method

Recall definitions (7) which defined the quantity of commodity  $n$ ,  $Q_{rcn}$ , that was finally demanded by purchasers in country  $c$  in region  $r$ . Define the country  $c$  in region  $r$  quantity or volume vector,  $Q^{rc}$ , in the usual way, using definitions (7) to define the components,  $Q_{rcn}$ :

$$(19) Q^{rc} \equiv [Q_{rc1}, \dots, Q_{rcN}] ; \quad r = 1, \dots, R ; c = 1, \dots, C(r).$$

The country price vectors  $P^{rc}$  that correspond to the country quantity vectors  $Q^{rc}$  defined by (19) are defined using the global set of PPPs,  $\gamma_{rcn}$ , defined by equations (3) above. Recall that  $\gamma_{rcn}$  is the price in the currency of country  $c$  in region  $r$  of a bundle of commodity  $n$  which costs one currency unit in the currency of the numeraire country (country 1) in the numeraire region (region 1). The basic heading price vector for country  $c$  in region  $r$ ,  $P^{rc}$ , is defined as follows:

$$(20) P^{rc} \equiv [\gamma_{rc1}, \dots, \gamma_{rcN}] ; \quad r = 1, \dots, R ; c = 1, \dots, C(r).$$

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countries in the world that were applied to all countries so that the resulting volume estimates are additive over all countries and all regions. In section 6 when the Sergeev method parities are computed using a subset of the ICP 2005 data, the regional parities price parities  $P_{rn}$  are equal to a regional constant times the parities defined by (18). This will not affect the final regional parities.

<sup>7</sup> For an explanation of the problem, see Marris (1984; 52), Diewert (1999; 48-50) or Chapter 7.

Thus there are country price and quantity vectors  $(P^{rc}, Q^{rc})$  for all  $C(1) + C(2) + \dots + C(R)$  countries in the ICP.

At this stage, any multilateral method could be used in order to form price comparisons between each individual country participating in the International Comparisons Project.

One such multilateral method which has good axiomatic and economic properties is the GEKS method.<sup>8</sup> The algebra for this method works as follows. First define the *Fisher* (1922) *quantity index* for country  $c$  in region  $r$  relative to country  $d$  in region  $s$  as follows:<sup>9</sup>

$$(21) Q_F(rc/sd) \equiv [P^{rc} \cdot Q^{rc} P^{sd} \cdot Q^{rc} / P^{rc} \cdot Q^{sd} P^{sd} \cdot Q^{sd}]^{1/2};$$

$$r = 1, \dots, R; c = 1, \dots, C(r); s = 1, \dots, R; d = 1, \dots, C(s)$$

where  $P^{rc} \cdot Q^{rc} \equiv \sum_{n=1}^N P_{rcn} Q_{rcn}$  denotes the inner product between the vectors  $P^{rc}$  and  $Q^{rc}$ . If the base country is fixed (i.e., fix the region  $s$  and the country  $d$  in region  $s$ ) and let  $r = 1, \dots, R$  and  $c = 1, \dots, C(r)$ , then the Fisher indexes defined by (21) can be interpreted as the volume of each country  $rc$  in the comparison relative to the base country  $sd$  and then these relative volumes can be normalized into a set of shares of world product using country  $sd$  as the base country; i.e., a set of country “star” shares are obtained for each  $rc$  with country  $sd$  as the “star” country.<sup>10</sup> The GEKS method then takes the geometric mean of all of these country parities over all possible “star” bases. Thus define these geometric mean relative parities as follows:

$$(22) Q(rc) \equiv [\prod_{s=1}^R \prod_{d=1}^{C(s)} Q_F(rc/sd)]^{1/[C(1)+\dots+C(R)]};$$

$$r = 1, \dots, R; c = 1, \dots, C(r).$$

Now normalize the relative parities given by (22) into country shares of world product. Thus define the sum of the parities defined by (22) as  $\sigma$ :

$$(23) \sigma \equiv \sum_{r=1}^R \sum_{c=1}^{C(r)} Q(rc).$$

The *GEKS share of world product for country  $c$  in region  $r$*  can now be defined as  $Q(rc)$  divided by  $\sigma$ :

$$(24) s_{rc} \equiv Q(rc)/\sigma;$$

$$r = 1, \dots, R; c = 1, \dots, C(r).$$

Following Dikhanov (2007), one can aggregate over the individual country shares of world product defined by (24) within each region in order to obtain the following *GEKS regional shares of world output*:

<sup>8</sup> See Gini (1924) (1931), Eltetö and Köves (1964) and Szulc (1964) and Diewert (1999; 31-37) for the properties of this method.

<sup>9</sup> Instead of using the Fisher ideal quantity index as the basic building block for this method, any other superlative quantity index could be used in this multilateral method. However, the Fisher index is generally preferred due to its strong axiomatic and economic properties; see Diewert (1976) (1992).

<sup>10</sup> Kravis (1984; 10) introduced this “star” terminology.

$$(25) S_r \equiv \sum_{c=1}^{C(r)} s_{rc} ; \quad r = 1, \dots, R.$$

The between region shares  $S_1, S_2, \dots, S_R$  defined by (25) can be used in conjunction with the within region shares in each region in order to obtain a system of world product shares for each country and the resulting parities respect the regional parities that are independently determined by the regions.

Why are the GEKS regional shares defined by (25) to be preferred over the various regional shares defined in the previous section? The reason for this preference is that the GEKS shares are consistent with broader patterns of substitutability between commodities; i.e., if the preferences of each country can be represented by certain homothetic preferences (that can approximate arbitrary homothetic preferences to the second order), then the GEKS country shares will give exactly the “right” relative volumes across countries.<sup>11</sup>

In the following section, a variant of the method used in this section will be studied.

#### 4. Spatial Comparisons Based on Similar Price Structures

The GEKS multilateral method treats each country “star” parity as being equally valid and hence an averaging of the parities is appropriate under this hypothesis. However, is it really the case that all bilateral comparisons of volume between two countries are equally accurate? One could argue that the answer to this question is that the answer is no: if the relative prices in countries A and B are very similar, then the Laspeyres and Paasche quantity indexes will be very close to each other and hence it is likely that the “true” volume comparison between these two countries (using the economic approach to index number theory) will be very close to the Fisher volume comparison. On the other hand, if the structure of relative prices in the two countries is very different, then it is likely that the structure of relative quantities in the two countries will also be different and hence the Laspeyres and Paasche quantity indexes will likely differ considerably and we can no longer be certain that the Fisher quantity index will be close to the “true” volume comparison. The above considerations suggest that a more accurate set of world product shares could be constructed if an initial bilateral comparison was made between the two countries which had the most similar relative price structures. Then look for a third country which had the most similar relative price structure to the first two countries and link in this third country to the comparisons of volume between the first two countries and so on. At the end of this procedure, a *minimum spanning tree* would be constructed: a path between all countries that minimized the sum of the relative price similarity measures. This linking methodology has been developed by Robert Hill (1999a) (1999b) (2001) (2004) (2009).

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<sup>11</sup> On the other hand, the additive methods discussed in the previous section are consistent with homothetic preferences that can provide only a first order approximation to arbitrary homothetic preferences. For further explanation of this point, see Diewert (1999; 31) who introduced the concept of a superlative multilateral system. The GEKS system is a superlative method, whereas the additive methods are not. Balk (2009;82) provided a recent overview of various multilateral methods and endorsed the GEKS-Fisher method as a centre stage method, particularly from the economic approach to international comparisons.

A key aspect of this methodology is the choice of the measure of similarity (or dissimilarity) of the relative price structures of two countries. Various measures of the similarity or dissimilarity of relative price structures have been proposed by Aten and Heston (2009), Diewert (2009), Hill (2009) and Sergeev (2001) (2009a). The following *weighted log quadratic measure of relative price dissimilarity*,  $\Delta_{PLQ}(p^1, p^2, q^1, q^2)$ , was suggested by Diewert (2009; 207) (the smaller the measure, the more similar is the structure of relative prices between the two countries):

$$(26) \Delta_{PLQ}(p^1, p^2, q^1, q^2) \equiv \sum_{n=1}^N (1/2)(s_n^1 + s_n^2) [\ln(p_n^2/p_n^1 P_F(p^1, p^2, q^1, q^2))]^2$$

where  $P_F(p^1, p^2, q^1, q^2) \equiv [(p^2 \cdot q^1 \cdot p^2 \cdot q^2)/(p^1 \cdot q^1 \cdot p^1 \cdot q^2)]^{1/2}$  is the Fisher ideal price index between countries 2 and 1 and  $s_n^c \equiv p_n^c q_n^c / p^c \cdot q^c$  is the country  $c$  expenditure share on Basic Heading category  $n$  for  $c = 1, 2$  and  $n = 1, \dots, N$ .

It can be seen that if prices are proportional for the two countries so that  $p^2 = \lambda p^1$  for some positive scalar  $\lambda$ , then  $P_F(p^1, p^2, q^1, q^2) = \lambda$  and the measure of relative price dissimilarity  $\Delta_{PLQ}(p^1, p^2, q^1, q^2)$  defined by (26) will equal its minimum of 0. Thus the smaller is  $\Delta_{PLQ}(p^1, p^2, q^1, q^2)$ , the more similar is the structure of relative prices in the two countries.

Hill's method of spatial linking using the relative price dissimilarity measure defined by (26) will be illustrated in the following two sections using small numerical examples. Basically, instead of using the GEKS country shares  $s_{rc}$  defined by (24) in the previous section, the shares generated by the minimum spanning tree are used to link all of the countries in the International Comparison Project. Once these country shares  $s_{rc}$  have been defined, again use equations (25) are used in order to form the regional shares  $S_r$ .

The narrowing of Paasche and Laspeyres spreads by the use of a spatial linking method is not the only advantage of this method of linking countries. There are advantages at *lower levels of aggregation* in that if we compare similar in structure countries, we will find that product overlaps are maximized:

“Many differences in quality and proportion of high tech items discussed above are likely to be more pronounced between countries with very different economic structures. If criteria can be developed to identify countries with similar economic structure and they are compared only with each other, then it may overcome many of the issues of quality and lowest common denominator item comparisons. Economically similar countries are likely to have outlet types in similar proportions carrying the same types of goods and services. So direct comparisons between such countries will do a better job of holding constant the quality of the items than comparisons across more diverse countries.” Bettina Aten and Alan Heston (2009; 251).

The above quotation suggests that perhaps the similarity criterion should not be based only on the similarity of the structure of relative prices across the two countries being compared. In addition, we should look at the degree of similarity in the structure of absolute per capita quantity vectors and take a sum of the two measures of similarity as our overall measure of similarity in structure.

There are some disadvantages to the spatial linking method. The two most important disadvantages are:

- The path of bilateral links between countries generated by the method tends to be unstable; i.e., the most similar tree linking the countries tends to change when we move from one cross sectional comparison between countries to another cross sectional comparison.
- Some countries in the comparison will inevitably have lower quality data than other countries and if these poorer data quality countries end up having many bilateral links with many countries in the minimum spanning tree, then the quality of the entire comparison may be low.

Hill (2009) discusses both of these problems and offers “reasonable” solutions to these difficulties. The first difficulty is not really a difficulty if the overall volume comparisons remain more or less the same even if the particular bilateral links change. In particular, it may be the case that countries break up into two or more relatively homogeneous groups. Within each group, the bilateral dissimilarity measures are all low so even if the links within each group change, the relative volume indexes within each group remain roughly unchanged. The key problem then boils down to the bilateral links between the various groupings. In order to get more stability between these groupings, it may be advisable to have more than one link between the groupings and this constraint can readily be imposed. The second difficulty can be dealt with by specifying that countries with lower quality data are not allowed to have more than one link in the overall tree of comparisons.

Of course, a problem with the above “solutions” to the problems associated with spatial linking is that the solutions appear to have an ad hoc character and this may lead to charges by outside observers that the ICP is being manipulated. This potential problem could be mitigated by experimentation with the ICP 2005 data set so a firm a priori strategy could be put in place before the results of ICP 2011 were calculated.

In order to get an impression as to how the different methods suggested in this section and the previous 2 sections perform in practice, two small numerical examples will be computed in the following two sections. The first example uses an artificial data set and the second example uses a small subset of the ICP 2005 data.

## 5. An Artificial Data Set Numerical Example

In this example, there are only 4 countries and only 2 commodities. There are two regions where region 1 consists of the first two countries and region 2 consists of the second two countries. The basic data for the countries is the within region  $r$  Basic Heading PPP for commodity  $n$  for country  $c$  in region  $r$ ,  $\alpha_{rcn}$ , the expenditure on commodity class  $n$  for country  $c$  in region  $r$  in domestic currency,  $E_{rcn}$ , and the market exchange rate for country  $c$  in region  $r$ ,  $\varepsilon_{rc}$  (which does not depend on the commodity  $n$ ), for  $r = 1,2$ ,  $c = 1,2$  and  $n = 1,2$ . The Basic Heading interregional PPPs for the numeraire countries in each region are also needed, the  $\beta_m$ , which are the parities for commodity  $n$  for region  $r$  relative to region 1. See Table 1 below for a listing of the data.

**Table 1: Data for Four Countries in Two Regions**

	Region 1						Region 2						Regional BH Parities	
	Country 1			Country 2			Country 1			Country 2				
<b>n</b>	$\alpha_{11n}$	$E_{11n}$	$\varepsilon_{11}$	$\alpha_{12n}$	$E_{12n}$	$\varepsilon_{12}$	$\alpha_{21n}$	$E_{21n}$	$\varepsilon_{21}$	$\alpha_{22n}$	$E_{22n}$	$\varepsilon_{22}$	$\beta_{1n}$	$\beta_{2n}$
1	1	10	1	20	2	5	1	20	3	10	20	18	1	2
2	1	10	1	4	8	5	1	20	3	4	160	18	1	4

There are 8 world Basic Heading PPPs that are obtained using equations (3),  $\gamma_{rcn} \equiv \beta_{rn}\alpha_{rcn}$ . Thus for commodity class 1, the following world BH PPPs are obtained:  $\gamma_{111} = 1$ ;  $\gamma_{121} = 20$ ;  $\gamma_{211} = 2$ ;  $\gamma_{221} = 20$ . For commodity class 2, the following world BH PPPs are obtained:  $\gamma_{112} = 1$ ;  $\gamma_{122} = 4$ ;  $\gamma_{212} = 4$ ;  $\gamma_{222} = 16$ .

Working through the algebra in section 2, the Option 1 shares of world output for Regions 1 and 2 turn out to equal 0.45134 for Region 1 and 0.54866 for Region 2. As expected, the same regional shares are obtained no matter which region is chosen as the numeraire region and no matter which country is chosen as the numeraire country within a region.

The Option 2 regional shares turn out to depend on the choice of the regional numeraire countries as expected. Letting Country 1 in each region be the numeraire country, 0.45676 and 0.54324 are obtained as the two regional shares of world product. Letting Country 2 in Region 1 be the numeraire country and maintaining Country 1 in Region 2 as the numeraire country leads to 0.46287 and 0.53713 as the new regional shares of world product. Letting Country 1 in Region 1 be the numeraire country and letting Country 2 in Region 2 be the numeraire country leads to 0.46041 and 0.53959 as the new regional shares of world product. The bottom line is that the Option 2 regional shares are not invariant to the choice of the numeraire countries in the regions.

The Option 3 regional shares<sup>12</sup> (Sergeev Option) turn out to be  $S_1$  equal to 0.46186 and  $S_2$  equal to 0.53814. Recall that the Option 1 regional shares were 0.45134 for Region 1 and 0.54866 for Region 2. Thus the Option 1 and 3 regional shares differ by about 2.3%.

The regional shares for the methods explained in section 3 above will now be calculated. The individual country shares of world output using each of the four countries as the “star” in bilateral Fisher index number comparisons are listed in the first four rows of Table 2. It can be seen that there are some relatively large differences between these world shares, particularly for the small country, Country 2 in Region 1. The GEKS country shares of world product are listed in the last line of Table 2.

**Table 2: Star and GEKS Country Shares of World Product**

<sup>12</sup> The Sergeev regional price and quantity vectors  $P^r$  and  $Q^r$  for  $r = 1, 2$  turn out to be  $P^1 = [4.472, 2.000]$ ,  $P^2 = [6.325, 8.000]$ ,  $Q^1 = [10.1, 12.0]$  and  $Q^2 = [11.0, 15.0]$ .

	Region 1		Region 2	
	Country 1	Country 2	Country 1	Country 2
Region 1, Country 1 Star	0.43523	0.02879	0.30775	0.22823
Region 1, Country 2 Star	0.47078	0.03114	0.32258	0.17550
Region 2, Country 1 Star	0.42439	0.02897	0.30009	0.24655
Region 2, Country 2 Star	0.44331	0.04125	0.28296	0.23248
GEKS	0.44436	0.03226	0.30387	0.21951

The GEKS volume shares in the last row of the above table are defined by equations (24); i.e., the entries for this last row are the GEKS shares  $s_{11}$ ,  $s_{12}$ ,  $s_{21}$  and  $s_{22}$  respectively. Using these country shares, the *regional GEKS shares*  $S_r$  are defined by (25) so that using the entries in the last line of Table 2,  $S_1$  and  $S_2$  are defined as follows:

$$(27) S_1 = s_{11} + s_{12} = 0.47662 ; S_2 = s_{21} + s_{22} = 0.52338.$$

Thus the GEKS share of world output for Region 1 is 0.47662 which is higher than the corresponding Region 1 shares for the Option 1 method (0.45134) and for the Option 3 method (0.46479).

Finally, the spatial linking method regional shares will be calculated. Recall that the weighted log quadratic measure of relative price dissimilarity between countries 1 and 2,  $\Delta_{PLQ}(p^1, p^2, q^1, q^2)$ , was defined by (26) above. The 4 by 4 matrix of relative price dissimilarity measures is listed in Table 3 below. For convenience in labeling the countries, set country 1 equal to Country 1 in Region 1, country 2 to Country 2 in region 1, country 3 to Country 1 in Region 2 and country 4 to Country 2 in Region 2.

**Table 4: Weighted Log Quadratic Relative Price Dissimilarities between Countries i and j**

	Country 1	Country 2	Country 3	Country 4
Country 1	0.00000	0.59465	0.12011	0.01057
Country 2	0.59465	0.00000	1.22741	0.25253
Country 3	0.12011	1.22741	0.00000	0.17933
Country 4	0.01057	0.25253	0.17933	0.00000

From the above Table, it can be seen that countries 1 and 4 have the most similar structures of relative prices with a dissimilarity measure equal to 0.01057. The next pair of countries that have most similar structure of relative price is countries 1 and 3 with a dissimilarity measure equal to 0.12011. Thus countries 3 and 4 can be linked to country 1 using the bilateral Fisher quantity index between 4 and 1 and between 3 and 1. The next most similar in structure pair of countries is 3 and 4 with a dissimilarity measure equal to 0.17933. But these two countries are already linked the next lowest measure of relative price dissimilarity is considered. The next most similar pair of countries is 2 and 4 with a dissimilarity measure equal to 0.25253. The bilateral Fisher quantity index is used to link country 2 to country 4 and now all of the countries in the “world” have been linked using

bilateral links. The volumes of all 4 countries relative to country 1 turn out to be 1.00000, 0.09305, 0.70711 and 0.52440. This leads to the following vector of country shares of world output: 0.43019, 0.04003, 0.30419, 0.22559. Adding up the shares of the countries in each region leads to the following *spatially linked regional shares of world output*:

$$(28) S_1 = s_{11} + s_{12} = 0.47022 ; S_2 = s_{21} + s_{22} = 0.52978.$$

Thus the spatial linking method leads to 0.47022 as the estimate of Region 1's share of world output compared to the GEKS estimate of 0.47662.

A summary of the Region 1 share of world output using the various methods can be found in Table 5 below.

**Table 5: The Share of World Output for Region 1 using Various Methods**

	<b>Region 1 Share of World Output</b>
Option 1 (Regional Unit Values Method)	0.45134
Option 2: Base Countries: 1 in Region 1; 1 in Region 2	0.45676
Option 2: Base Countries: 2 in Region 1; 1 in Region 2	0.46287
Option 2: Base Countries: 1 in Region 1; 2 in Region 2	0.46041
Option 3: Geometric Mean Average Prices in each Region	0.46186
GEKS	0.47662
Spatial Linking	0.47022

The differences between the various methods are fairly substantial: a 4.2 % difference in the share of Region 1 for the highest share (Spatial Linking) versus the lowest share (Regional Unit Values Method). In the following section, another numerical example based on actual ICP 2005 data is considered that lead to even bigger numerical differences between the various methods.

## 6. A Numerical Example Based on ICP 2005 Data

Yuri Dikhanov generated some highly aggregated data (across Basic Heading groups) from ICP 2005 on 5 consumption components for 8 countries.

The 8 countries are:

- 1 = Hong Kong;
- 2 = Bangladesh;
- 3 = India;
- 4 = Indonesia;
- 5 = Brazil;
- 6 = Japan;
- 7 = Canada and
- 8 = US.

The 5 commodity groups were:

- 1 = durables;
- 2 = food, alcohol and tobacco;
- 3 = other nondurables excluding food, alcohol, tobacco and energy;
- 4 = energy and
- 5 = services.

The expenditure data (converted to US dollars) and the quantity data for the 8 countries are listed in the following Tables 6 and 7.

**Table 6: Expenditures in US Dollars for Eight Countries and Five Consumption Categories**

	Country 1	Country 2	Country 3	Country 4	Country 5	Country 6	Country 7	Country 8
1	14320	1963	23207	8234	52722	307547	94121	967374
2	10562	24835	176782	83882	105527	448995	82056	778665
3	14951	5100	60748	15158	60798	272875	69461	992761
4	2619	3094	42126	17573	39933	125835	43342	524288
5	62124	11627	166826	61248	273669	1736977	379629	5559458

**Table 7: Quantities (Volumes) in Comparable Units for Eight Countries and Five Consumption Categories**

	Country 1	Country 2	Country 3	Country 4	Country 5	Country 6	Country 7	Country 8
1	15523	2312	30189	9781	46146	280001	81021	967374
2	9164	47509	356756	138273	163868	251846	63689	778665
3	317564	10588	180964	29879	65274	200614	58261	992761
4	1095	3033	38377	22084	23963	59439	35714	524288
5	81148	47611	786182	223588	541236	1695136	417210	5559458

If the entries in Table 6 (expenditures converted to US dollars at market exchange rates) are divided by the entries in Table 7 (quantities in comparable units), the Basic Heading prices (converted into US dollars at market exchange rates) for each commodity class for each country are obtained. These prices are listed in Table 8 below.

**Table 8: Prices of Consumption Components in US Dollars for Eight Countries and Five Consumption Categories**

	Country 1	Country 2	Country 3	Country 4	Country 5	Country 6	Country 7	Country 8
1	0.92250	0.84905	0.76872	0.84184	1.14250	1.09838	1.16169	1.0
2	1.15255	0.52274	0.49553	0.60664	0.64398	1.78282	1.28839	1.0
3	0.85123	0.48168	0.33569	0.50731	0.93143	1.36020	1.19224	1.0
4	2.39178	1.02011	1.09769	0.79573	1.66644	2.11704	1.21359	1.0
5	0.76556	0.24421	0.21220	0.27393	0.50564	1.02468	0.90992	1.0

Thus the US price level for each commodity group is set equal to 1 and the other prices are the average domestic prices for the commodity group converted into US dollars at the 2005 market exchange rates. Note that for durables, Bangladesh has the lowest price level at 0.84 and Canada has the highest level at 1.16; for food, Bangladesh has the lowest prices at 0.52 while Japan has the highest at 1.78; for other nondurables, India has the lowest price level at 0.33 while Japan has the highest level at 1.36; for energy, Indonesia has the lowest price level at 0.79 while Japan has the highest at 2.12 and for services, India has the lowest price level at 0.21 while Japan has the highest level at 1.02. Thus the amount of price level variation across countries ranges from 38% for durables to 500% for services.

The above data are used to compute relative consumption volumes for the 8 countries using various multilateral methods. Instead of normalizing the relative volumes into shares of “world” consumption, the consumption of each country relative to the consumption of the US will be calculated. This is simply an alternative normalization of the country relative volumes.

The GEKS method for comparing relative volumes was explained in section 3<sup>13</sup> and the Hill spatial linking method was explained in section 4 above. We also computed relative consumption volumes for the 8 countries using two other (additive) methods: the Geary-Khamis (GK) method<sup>14</sup> and the Iklé Dikhanov Balk (IDB) method.<sup>15</sup> The country consumption volumes relative to the US for the four multilateral methods are listed in Table 9 below.

**Table 9: Country Consumption Volumes Relative to the US Using Four Multilateral Methods**

Method	HK	BGD	INDIA	INDO	BRA	JPN	CAN	US
<b>GEKS</b>	0.01315	0.01332	0.15317	0.04966	0.09128	0.26556	0.07357	1.0
<b>Spatial Linking</b>	0.01349	0.01310	0.14720	0.04779	0.09214	0.27596	0.07429	1.0
<b>GK</b>	0.01386	0.01357	0.16258	0.05057	0.09613	0.27814	0.07431	1.0
<b>IDB</b>	0.01346	0.01392	0.16187	0.05143	0.09441	0.27076	0.07417	1.0

It can be seen that the relative consumption volumes generated by the two superlative methods (the GEKS and Spatial Linking Methods) are fairly close to each other and the relative consumption volumes generated by the two additive methods (GK and IDB) are also fairly close to each other but the additive methods tend to overstate the consumption

<sup>13</sup> The Fisher star parities for the 7 countries relative to the US had the following relative volume ranges: Hong Kong: 0.012570 to 0.013550; Bangladesh: 0.012770 to 0.014370; India: 0.14351 to 0.16439; Indonesia: 0.046600 to 0.053220; Brazil: 0.089840 to 0.095040; Japan: 0.24629 to 0.27724 and Canada: 0.071290 to 0.074640.

<sup>14</sup> The GK method was suggested by Geary (1958) and Khamis (1972) provided an existence proof.

<sup>15</sup> Iklé (1972; 202-204) suggested the method in a very indirect way, Dikhanov (1994) (1997) suggested a much clearer system of equations which define the method and Balk (1996; 207-208) provided the first existence proof. The GK and IDB methods are extensively discussed in Chapter 7.

levels of the poorer countries (Bangladesh, India, Indonesia and Brazil) relative to the US.<sup>16</sup>

It is of some interest to determine which countries have the most similar structure of relative prices (and of course, this information is required in order to implement the Hill type spatial linking method). Recall that the weighted log quadratic measure of relative price dissimilarity between countries 1 and 2,  $\Delta_{PLQ}(p^1, p^2, q^1, q^2)$ , was defined by (26) above. The 8 by 8 matrix of these relative price dissimilarity measures is listed in Table 10 below.

**Table 10: Weighted Log Quadratic Relative Price Dissimilarities between Eight Countries**

	<b>HK</b>	<b>BGD</b>	<b>IND</b>	<b>INDO</b>	<b>BRA</b>	<b>JPN</b>	<b>CAN</b>	<b>US</b>
<b>HK</b>	0.00000	0.10056	0.11017	0.09067	0.07011	0.01381	0.03660	0.06143
<b>BGD</b>	0.10056	0.00000	0.01188	0.01165	0.05632	0.10506	0.13237	0.23223
<b>IND</b>	0.11017	0.01188	0.00000	0.03133	0.08980	0.13429	0.18955	0.29841
<b>INDO</b>	0.09067	0.01165	0.03133	0.00000	0.07084	0.07726	0.09610	0.19600
<b>BRA</b>	0.07011	0.05632	0.08980	0.07084	0.00000	0.09146	0.08770	0.14328
<b>JPN</b>	0.01381	0.10506	0.13429	0.07726	0.09146	0.00000	0.01904	0.05322
<b>CAN</b>	0.03660	0.13237	0.18955	0.09610	0.08770	0.01904	0.00000	0.02020
<b>US</b>	0.06143	0.23223	0.29841	0.19600	0.14328	0.05322	0.02020	0.00000

Looking at the above Table, it can be seen that the 8 countries group themselves into two groups that have similar price structures: the rich countries HK, JPN, CAN and US form one group and the poorer countries BGD, INDIA, INDO and BRA form the other group. The linking between the two groups took place via Hong Kong and Brazil.<sup>17</sup> The details of the spatial linking process are as follows. The rich countries are 1,6,7 and 8. Country 7 (CAN) is linked to 8 (US) ( the dissimilarity measure  $\Delta$  equals 0.0202) and 7 (CAN) is linked to 6 (JPN) as well ( $\Delta = 0.019$ ). Then country 6 (JPN) is linked to 1 (HK) ( $\Delta = 0.0138$ ) and this completes the linking of the rich countries. Country 2 acts as a star country for the poorer countries: 2 (BGD) is linked to 4 (INDO) ( $\Delta = 0.0116$ ); 2 (BGD) is linked to 3 (INDIA) ( $\Delta = 0.0118$ ) and 2 (BGD) is linked to 5 (BRAZIL) ( $\Delta = 0.056$ ). Finally, the two groups of countries are linked via countries 1 (HK) and 5 (BRAZIL) ( $\Delta = 0.070$ ).

Now consider the problems associated with forming regional shares of “world” consumption where the “world” is simply the 8 countries listed above. Suppose that the first four countries form an “Asian” region, Region 1 and the remaining four countries form a “Rest of the World” region, Region 2. Obviously, the Dikhanov method for forming regional shares can be applied to the data listed in Tables 8 and 9 above. The

<sup>16</sup> The GK volume relative to the GEKS volume for India was 6.1% higher and the IDB volume relative to the GEKS volume for India was 5.7% higher.

<sup>17</sup> Note that another possible bilateral link between the two regions would be via Indonesia and Japan which have a dissimilarity measure equal to 0.07726 which is a bit higher than the Hong Kong and Brazil dissimilarity measure which was equal to 0.07011.

consumption volumes (relative to the US) listed there for the GEKS, Spatial Linking, GK and IDB methods can be converted into shares of “world” consumption and then the first four country shares can be summed to form the Region 1 shares, S1 and S2. The resulting Region 1 shares for the four methods are listed below in Table 14.

It is also useful to calculate the Region 1 shares of “world” consumption for the Option 1 (Diewert) and Option 3 (Sergeev) methods. Equations (7) and (8) are first used to calculate the country and regional quantity vectors where the country expenditures  $E_{rcn}$  are listed in Table 6 above and the country Basic Heading PPPs (which can be set equal to the  $\gamma_{rcn}$ ) are listed in Table 8 above. The resulting matrix of regional consumption quantities,  $Q_{rn}$ , are listed in Table 11 below.

**Table 11: Regional Consumption Quantities or Volumes  $Q_{rn}$  by Commodity**

<b>Commodity</b>	<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>	<b>5</b>
<b>Region 1</b>	57805	551702	238995	64589	1138529
<b>Region 2</b>	1374542	1258068.	1316910	643404	8213040

The regional unit value commodity prices  $n$  for each region  $r$ ,  $P_{rn}$ , defined by (12) are listed in Table 12 below.

**Table 12: Regional Unit Value Consumption Prices  $P_{rn}$  by Commodity**

<b>Commodity</b>	<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>	<b>5</b>
<b>Region 1</b>	0.82560	0.53663	0.40150	1.01274	0.26510
<b>Region 2</b>	1.03435	1.12493	1.05998	1.13987	0.96794

As could be expected, the Region 2 unit value commodity prices are all relatively close to the US prices (which are all equal to unity) since 3 out of 4 of the Region 2 countries are “rich” and hence have price structures similar to the US structure. The Region 1 unit value prices are all lower than the corresponding Region 2 prices and for Commodity 5, Services, the Region 1 unit value price is considerably lower. Now the Fisher quantity index for Region 2 relative to Region 1 can be calculated using the regional price and quantity data listed in Tables 12 and 11 and the resulting index is equal to 6.26177. Thus the relative regional consumption volumes are 1 and 6.17739 and when these volumes are converted into shares, it is found that the Region 1 share of world consumption is 0.13771 and the Region 2 share is 0.86229.

Finally, the Option 3 additive method which was suggested by Sergeev (2009) is evaluated. In order to implement this method, it is necessary to compute the geometric mean of the regional Basic Heading prices that are listed in Table 8; recall equations (18) above. These regional geometric mean prices  $P_{rn}^*$  are listed in Table 13 below.

**Table 13: Regional Geometric Mean Prices  $P_{rn}^*$  by Commodity**

<b>Commodity</b>	<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>	<b>5</b>
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<b>Region 1</b>	0.84377	0.65236	0.51405	1.20824	0.32287
<b>Region 2</b>	1.09882	1.10282	1.10861	1.43846	0.82863

The pattern of regional commodity prices is fairly similar in Tables 12 and 13. To complete the analysis for this case, the Fisher quantity index for Region 2 relative to Region 1 is calculated using the regional price and quantity data listed in Tables 13 and 11 and the resulting index is equal to 6.17739, which is very similar to the corresponding Option 1 Fisher index which was equal to 6.26177. Thus the relative regional consumption volumes for the Sergeev method are 1 and 6.17739 and when these volumes are converted into shares, then the Region 1 share of world consumption is 0.13933 and the Region 2 share is 0.86067.

The resulting Region 1 shares of “world” consumption that have been generated by the various methods discussed above are summarized in Table 14 below.

**Table 14: The Share of World Output for Region 1 using Various Methods**

	<b>Region 1 Share of Consumption</b>
<b>Option 1 (Regional Unit Values Method)</b>	0.13771 (3.4 %)
<b>Option 3: Geometric Mean Average Prices in each Region</b>	0.13933 (4.6 %)
<b>GEKS</b>	0.13815 (3.7 %)
<b>GK</b>	0.14243 (7.0 %)
<b>IDB</b>	0.14326 (7.6 %)
<b>Spatial Linking</b>	0.13316

While the variations in the Region 1 shares that the various methods generate are not huge, they are certainly not negligible. The percentage differences between the various estimated shares and the Spatial Linking share are listed in brackets in the above Table.

More experimentation using the 2005 data needs to be done before a definitive decision can be made on which method should be used to link the regions in ICP 2011.

## 7. Conclusion

Some tentative conclusions that can be drawn from the analysis and examples presented above are as follows:

- The Option 2 method should be ruled out for ICP 2011 since as Sergeev has pointed out, it is not invariant to the choice of the numeraire countries in the regions.
- The Option 1 method should also be ruled out for ICP 2011 since it depends on market exchange rates which are not reliable and hence their use should be avoided if possible.
- The Option 3 method is not fully consistent with the economic approach to index number theory because the method is inherently an additive method (when

- constructing regional volume aggregates) and additive methods are subject to substitution bias.
- The use of GK or IDB should perhaps be ruled out for “headline” estimates for country and interregional parities due to their inherent substitution biases. These methods however could be used to provide users with analytical tables where the users demand an additive method.<sup>18</sup>
  - GEKS remains a viable method for constructing regional shares in a consistent manner.
  - Various forms of spatial linking should also be considered for ICP 2011 but the use of this method should await more experimental results using the 2005 data base.<sup>19</sup>

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<sup>18</sup> Based on some empirical evidence developed by Deaton and Heston (2010) using ICP 2005 data, if it desired to have the additive method parities approximate the GEKS parities, then IDB appears to be better than GK. The axiomatic properties of IDB may be more attractive to users than the axiomatic properties of GK; see Chapter 7 for a discussion of the axiomatic properties of the GK and IDB methods.

<sup>19</sup> Some discussion of the dissimilarity measure that should be used to measure the degree of relative price similarity across countries is also required.

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