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Optimal/Fair Taxation and Partial Control: Theory and Evidence

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Abstract

Since Mirrlees (1971), the modern income tax literature typically starts from a welfare maximizing social planner who wants to redistribute earnings on the basis of a distortionary income tax. The underlying reason for using distortionary taxes on earnings is the impossibility of using non-distortionary lump-sum taxes on ability, since the latter is not observed by the social planner. In practice however, tax-benefit schemes are based on much more information than earnings only. Different theoretical reasons, related to efficiency and equity, have been put forward. In this paper, we want to derive and test an optimal tax-benefit scheme based on several characteristics, which differ in terms of the degree of control, i.e., the extent to which a characteristic can be changed by exerting effort. Our theoretical model provides us with a testable relation between the tax rates on the two characteristics and their variance-covariance structure. We then set up an empirical model with a finite number of characteristics which allows us to estimate the tax (or subsidy) rates for the different characteristics in different European countries using the 2007 EU-SILC data. Partitioning the set of characteristics into characteristics with zero control -sex, age and disability- and characteristics with partial control- education and household composition-, output can be rewritten as a function of two "composite" characteristics, one with partial and one with no control as in the "Akerlof"-case. There is a clear tendency in all countries to compensate more for the "no control"-composite compared to the "partial control"-composite. In addition, when testing the empirical counterpart of the theoretical tax formula in the "Akerlof"-case, we cannot reject that some countries are fair.

1 Introduction

Since Mirrlees (1971), the modern optimal income tax literature typically starts from a welfare maximizing social planner who wants to redistribute earnings on the basis of a distortionary income tax; see, e.g., Salanié (2003) for an overview. The underlying reason for using distortionary taxes on earnings is the impossibility of using non-distortionary lump-sum taxes on ability, since the latter is not observed by the social planner. In practice however, tax-benefit schemes are based on much more information than earnings only. Different theoretical reasons, related to efficiency and equity, have been put forward.

There exist two efficiency arguments: externalities and tagging. First, if certain individual actions cause externalities, then there is a role for a government to subsidize or tax these activities in a Pigouvian way to restore efficiency; see, e.g., Friedman (1962) for educational externalities and Mirrlees (1972) for family size.¹ Second, Akerlof (1978) shows that tagging, i.e., differentiating the tax-benefit system on the basis of observable (usually exogenous) characteristics, the so-called tags, might be welfare improving, if these

¹Mirrlees only mentions externalities as a potential rationale for taxing/subsidizing family size.

characteristics correlate with the ability or the tastes distribution. The theoretical use of tags in optimal taxation schemes has been analyzed by Akerlof (1978), Immonen et al. (1998), Viard (2001a,b), Salanié (2002), Boadway and Pestieau (2006), and Cremer et al. (2010). While (most of) the previous authors do not focus on a specific tag, Diamond and Sheshinski (1995) and Parsons (1996) analyze disability to work in the context of optimal insurance design, Erosa and Gervais (2002), Lozachmeur (2006), Weinzierl (2007) and Blomquist and Micheletto (2008) consider age tags, Mankiw and Weinzierl (2008) study height, Alesina et al. (2008) and Cremer et al. (2010) focus on gender, and Logue and Slemrod (2008) discuss genetic tags.

Equity considerations can also provide a rationale to differentiate tax-benefit schemes on the basis of non-income characteristics which signal differences in needs.² One strand of the literature focusses on labour supply and *family size*; see, e.g., Lambert and Yitzhaki (1997) and Ebert and Moyes (2000) if both labour supply and family size are exogenous, Balcer and Sadka (1986), Bruno and Habib (1976), Atkinson and Bourguignon (2000), Cremer et al. (2002) for models with endogenous labour supply and exogenous fertility, Cigno (1986) for exogenous labour supply and endogenous family size, and Mirrlees (1972), Cigno (1983), Nerlove et al. (1984), Balestrino (2002) and Fleurbaey and Maniquet (2004) for endogenous labour supply and family size.³ Another strand of the literature deals with the optimal income tax treatment of *couples*, considering couple formation as given; see, e.g., Boskin & Sheshinsky (1983), Piggott and Whalley (1996), Apps and Rees (1999), Schroyen (2003), Brett (2007), Cremer et al. (2007) and Kleven et al. (2009).

In this paper, we want to *derive* and *test* a *fair* tax-benefit scheme based on several characteristics, which differ in terms of the *degree of control*, i.e., the extent to which a characteristic can be changed by exerting effort. Let us preview the four core ingredients of this paper:

1. **Fair taxation.** Unobservable differences in types and tastes (for effort) of individuals underly the observable differences in the different characteristics. Especially the taste differences make the normative exercise more complex. Following Atkinson and Stiglitz (1976) we normalize utility as follows: if everyone has the same type, then everyone has the same utility in the *laisser-faire*. We discuss the ethical implications of this normalization along the lines of Fleurbaey and Maniquet (2006).
2. **Partial control.** Individuals differ in several characteristics, which are each a weighted combination of type (drawn by nature) and effort (chosen by the individual). We refer to this weight as the degree of control. For some characteristics like sex, age and inborn handicaps the degree of control is zero (i.e., the characteristic is fully defined by the individual's type), while for other characteristics, think of education and family composition, the degree of control is strictly positive and thus partial control applies.
3. **Theory.** The complexity of multidimensional screening exercises, forces us to simplify some aspects of the model if one wants to keep analytical tractability. We assume (1) a linear production technology, (2) quasi-linear preferences, (3) independent and multivariate normal distributions for types

²Note that most of the papers that we mention here also care about efficiency, but, in contrast to the previous ones, they do not focus specifically on externalities or tagging.

³Note that Mirrlees (1972) and Cigno (1983) study a model in which the fertility and labour choice coincide: consumption is a function of the number of children.

and tastes, and (4) linear tax rates. Under these conditions, we characterize and discuss the optimal tax-benefit scheme. The ‘Akerlof’-case with two characteristics, one exogenous and one partially controlled, will be analyzed in greater detail, as it provides us with a testable relation between the tax rates on the two characteristics and their variance-covariance structure.

4. **Evidence.** We set up an empirical model with a finite number of characteristics which allows us to estimate the tax (or subsidy) rates for the different characteristics in different European countries using the 2007 EU-SILC data. Partitioning the set of characteristics into characteristics with zero control—sex, age and disability—and characteristics with partial control—education and household composition—, output can be rewritten as a function of two ‘composite’ characteristics, one with partial and one with no control as in the ‘Akerlof’-case. There is a clear tendency in all countries to compensate more for the ‘no control’-composite compared to the ‘partial control’-composite. In addition, we test the empirical counterpart of the theoretical tax formula in the ‘Akerlof’-case. We cannot reject that some countries are fair.

As far as we know, our work has three core ingredients (1, 3 & 4) in common with three papers only. Page and Roemer (2001) and Roemer et al. (2003) are based on Roemer’s (1993, 1998) ‘equality of opportunity’-concept. While Page and Roemer (2001) focus on the US, Roemer et al. (2003) estimate the optimal tax rate for 11 countries (including the US) and compare the optimal with the actual tax rates to judge the extent in which countries have equalized opportunities for income acquisition. Weinzierl (2009) introduces preference heterogeneity in a first-best model of optimal income taxation. He derives and tests the key implication that tax rates should decline with the degree of preference heterogeneity.

The setup of the paper is as follows. In Section 2, we setup a theoretical optimal tax model in which we introduce the notion of ‘partial control’. We describe and discuss the theoretical results and derive a testable hypothesis for a fair tax benefit scheme. In Section 3, we set-up an empirical model and describe the data used for the empirical analysis. Next we derive the implicit tax rates for different characteristics and test whether existing tax systems are actually fair. Section 4 concludes.

2 Theory

In the first part of this section, we describe the basic building blocks—production technology, preferences, and the social planner— of a model in which individuals differ in types (multidimensional productivity) and tastes (for multidimensional effort). In the second part, we describe and discuss the theoretical results, with a special focus on two cases of interest: the ‘Mirrlees’-case and the ‘Akerlof’-case. Especially the latter case turns out to provide us with a testable hypothesis which will return in the empirical section later on.

2.1 Model

The model is additively separable: output is linear in characteristics and characteristics are linear in effort and type, preferences are quasi-linear in (net) output, and the welfare function will be an exponential transform (such that adding a constant amount of ‘well-being’ to every individual does not change inequality). The additive specification is more convenient in terms of interpretation compared to the usual

multiplicative specification, but note that the same theoretical results can be obtained with a multiplicatively separable model, i.e., the model obtained by replacing the linear specification by a log-linear one in *each* of the basic building blocks.

Production technology. Individuals (or households) can be described by a vector $x \in \mathbb{R}^J$, with J a set of at most two characteristics. No additional theoretical insights are gained by adding characteristics. In addition, in the empirical part we will partition all characteristics into two groups, such that the theory applies to the corresponding composite characteristics. The pre-intervention or gross outcome is denoted y and is assumed to be a linear function of the different characteristics; formally:

$$y = \beta_0 + \sum_{j \in J} \beta_j x_j. \quad (1)$$

Without loss of generality, we assume $\beta = (\beta_j)_{j \in J} \gg 0$, and 0 denotes a vector of zeros of appropriate length. Characteristics are a combination of effort $e \in \mathbb{R}^J$ and type $\theta \in \mathbb{R}^J$, i.e., for each j in J we assume

$$x_j = \alpha_j e_j + (1 - \alpha_j) \theta_j. \quad (2)$$

The weights of effort in the different dimensions are collected in a vector $\alpha \in (0, 1)^J$ and they define the degree of control for each characteristic in between the extremes of no control ($\alpha_j \rightarrow 0$; the characteristic is pure type) and full control ($\alpha_j \rightarrow 1$; the characteristic is pure effort). In contrast to the characteristics, effort and type are not observable to the social planner (but the multivariate distribution is known).

Some special cases arise. First, if there is only one characteristic, say earnings x_1 , and assuming $\beta_0 = 0$ and $\beta_1 = 1$, then $y = x_1 = \alpha_1 e_1 + (1 - \alpha_1) \theta_1$, and we obtain an additive version of what we call the ‘Mirrlees’-case. Next, if there are two characteristics, individual earnings $x_1 = \alpha_1 e_1 + (1 - \alpha_1) \theta_1$ and a tag, an exogenous characteristic denoted $x_2 \rightarrow \theta_2$ (given $\alpha_2 \rightarrow 0$) and if $\beta_0 = 0$ and $\beta_1 = 1$, then $y = x_1 + \beta_2 x_2 \rightarrow (\alpha_1 e_1 + (1 - \alpha_1) \theta_1) + \beta_2 \theta_2$, and we arrive in the so-called ‘Akerlof’-case. Note that the tag $x_2 \rightarrow \theta_2$ can both correlate with the earnings ability θ_1 and affect well-being directly (via $\beta_2 > 0$). Finally, note that also the ‘Boskin and Sheshinsky’-case for the optimal taxation of couples can be considered as a special case: choosing $\beta_0 = 0$ and $\beta_1 = \beta_2 = 1$ we have $y = x_1 + x_2$ with $x_1 = \alpha_1 e_1 + (1 - \alpha_1) \theta_1$ and $x_2 = \alpha_2 e_2 + (1 - \alpha_2) \theta_2$ the earnings of the partners in a couple. We do not further discuss this case here.

Preference technology. Individual utility is equal to the net outcome c (to be defined later) minus the cost of effort; we assume:

$$U(c, e; \gamma, \delta) = c - \sum_{j \in J} \frac{\delta_j}{\exp(\gamma_j)} \exp\left(\frac{e_j}{\delta_j}\right), \quad (3)$$

with $\gamma \in \mathbb{R}^J$ a vector of taste parameters which defines the disutility of effort, and $\delta \in \mathbb{R}^J$, with $\delta \gg 0$, a vector controlling the degree of convexity of the cost of effort. This is a multidimensional version of the classical quasi-linear preferences which are often used in optimal tax theory to simplify the theoretical analysis by excluding income effects (see, e.g., Diamond, 1998). As usual, higher values for γ correspond with lower disutility of effort which can be thought of as more ambitious individuals; higher values for δ correspond with more elastic responses to effort and can be interpreted as the cost of taxation for the different characteristics.

Net outcomes and behaviour. The instruments of the social planner are restricted to ‘basic income-flat tax’ schemes. Although restrictive compared to non-linear tax instruments, linear schemes could be

close to optimal, at least for income taxation; see, e.g., Mankiw et al. (2009)'s lesson 3 for a discussion. Formally, the net outcome c satisfies

$$c \leq y - t_0 - \sum_{j \in J} t_j x_j, \quad (4)$$

with $t_0 \in \mathbb{R}$ controlling the overall level of the net outcome, and $t \in \mathbb{R}^J$ the tax rates applied to the different (observable) characteristics. One could object taxing or subsidizing non-income characteristics for horizontal equity reasons. Two brief remarks here. First, in the presence of preference heterogeneity, it is not obvious why taxing earnings as an imperfect signal of the underlying ability to earn would be different from taxing other signals. Second, even if objectionable, note that our empirical results show that many characteristics are taxed or subsidized implicitly (even though they are not explicitly used in the legal definition of taxes and benefits). For example, a higher education level is implicitly taxed in most countries since higher educated individuals (1) earn more and face a higher average tax rate (due to the progressivity of most tax systems), and (2) they have a lower probability to receive certain benefits (leading to a solidarity component in most social security systems).

Types and tastes are private information; in particular, we assume that individuals also know their type before choosing effort. However, the choice of effort would remain the same if individuals only know the distribution of types, and behaviour is modelled via expected utility maximization. Lemma 1 summarizes behaviour, i.e., choice and indirect utility.

LEMMA 1. Maximization of (3) with respect to (1), (2) & (4), leads to an effort choice⁴

$$e_j^* = \delta_j (\ln(\alpha_j (\beta_j - t_j) + \gamma_j)) \text{ for all } j \text{ in } J, \quad (5)$$

which results in characteristics

$$x_j^* = \alpha_j e_j^* + (1 - \alpha_j) \theta_j = \alpha_j \delta_j (\ln((\beta_j - t_j) \alpha_j) + \gamma_j) + (1 - \alpha_j) \theta_j, \quad (6)$$

and the corresponding indirect utility $V(t_0, t; \alpha, \beta_0, \beta, \delta; \gamma, \theta)$ equals

$$\kappa(t_0, t; \alpha, \beta_0, \beta, \delta) + \sum_{j \in J} (\beta_j - t_j) \alpha_j \delta_j \gamma_j + \sum_{j \in J} (\beta_j - t_j) (1 - \alpha_j) \theta_j, \quad (7)$$

with

$$\kappa(t_0, t; \alpha, \beta_0, \beta, \delta) = \beta_0 - t_0 + \sum_{j \in J} (\beta_j - t_j) \alpha_j \delta_j [\ln((\beta_j - t_j) \alpha_j) - 1]. \quad (8)$$

The social planner. The social planner does observe the multivariate type distribution F which is assumed to be independent from the multivariate taste distribution G . For analytical tractability, we assume multivariate normal distributions, or

$$\theta \sim N(\mu^\theta, \Sigma^\theta) \text{ and } \gamma \sim N(\mu^\gamma, \Sigma^\gamma), \quad (9)$$

with $\mu = (\mu_j)_{j \in J}$ a vector of means and $\Sigma = (\sigma_{ij})_{ij \in J^2}$ a variance-covariance matrix with $\sigma_{jj} > 0$ for all j in J and $(\sigma_{ij})^2 < \sigma_{ii} \sigma_{jj}$ for all i, j in J (excluding perfect correlation). The social planner sets taxes t_0 and t to maximize welfare—to be introduced next—subject to a budget constraint, denoted

$$t_0 + \int \int \left(\sum_{j \in J} t_j x_j^* \right) dF(\theta) dG(\gamma) \geq R_0, \quad (10)$$

⁴We define $e_j^* \rightarrow -\infty$ for all tax levels $t_j > \beta_j$.

with R_0 an exogenous (per-capita) revenue requirement, x_j^* defined in equation (6), and the distributions F and G defined in equation (9). In order to define aggregate welfare, we assume that the planner balances efficiency and equity. Efficiency is operationalized via the Pareto principle, while equity is defined as selective egalitarianism: individuals are held responsible for their tastes, but not for their type. We discuss efficiency and equity in an informal way here; see, e.g., Fleurbaey and Maniquet (2010) for a formal discussion.

A Pareto efficient planner defines welfare as an increasing function of individual well-being, and well-being is a specific cardinalization of the utility function. But which cardinalization is normatively interesting? Equity considerations will guide us. A selective egalitarian planner is egalitarian, but only with respect to a selection of outcome differences, more precisely, those outcome differences which are caused by differences in type for which individuals are not (held) responsible. We select two plausible principles, which are related to vertical and horizontal equity, respectively, since ‘responsibility for tastes’ means ‘taste differences are irrelevant for redistribution’, while ‘non-responsibility for types’ corresponds with ‘type differences are relevant for redistribution’. In line with vertical equity, if two individuals have the same tastes and make exactly the same effort choices, then any remaining outcome differences can be traced back to differences in type, which are deemed relevant for redistribution. (Only) in this case, the planner should approve of progressive Pigou-Dalton transfers, i.e., mean-preserving transfers from the richer to the poorer individual. In line with horizontal equity, if *all* individuals have the same type, then outcome differences in the *laissez-faire* allocation—i.e., the allocation which would be chosen by individuals in the absence of taxation—are only due to differences in tastes, which are deemed irrelevant for redistribution. So, if *all* individuals have the same type, then there is no reason to redistribute, and the *laissez-faire* allocation should result. This condition results in a specific cardinalization of preferences which also occurs in Atkinson and Stiglitz (1976) and Weinzierl (2009).

To combine Pareto efficiency and selective egalitarianism, we consider a social planner who maximizes a Kolm-Pollak welfare function, i.e., welfare is the sum of increasing and concave exponential functions of well-being. Well-being is defined as a specific cardinalization of utility, more precisely, the (direct) well-being in a given bundle (c, e) , denoted $u(c, e; \alpha, \beta_0, \beta, \delta; \gamma, \theta)$ is defined via indifference between (1) the received bundle (c, e) , and (2) the bundle the individual would choose—with here own tastes, but with a hypothetical type $\theta = (u(\cdot), u(\cdot), \dots, u(\cdot))$ —in the *laissez-faire*, i.e., $(t_0, t) = (R_0, 0)$. Figure 1 illustrates the construction of direct well-being for the Mirrlees-case ($y = x_1 = \alpha_1 e_1 + (1 - \alpha_1) \theta_1$), obtained by changing the intercept of the *laissez-faire* budget set (a budget line with intercept $(1 - \alpha_1) u - R_0$ and slope α_1) such that it is tangent to the indifference curve through the bundle (c, e_1) (to make the individual indifferent between the bundle (c, e_1) and choosing from the hypothetical budget set); Lemma 2 derives the corresponding direct well-being index formally.

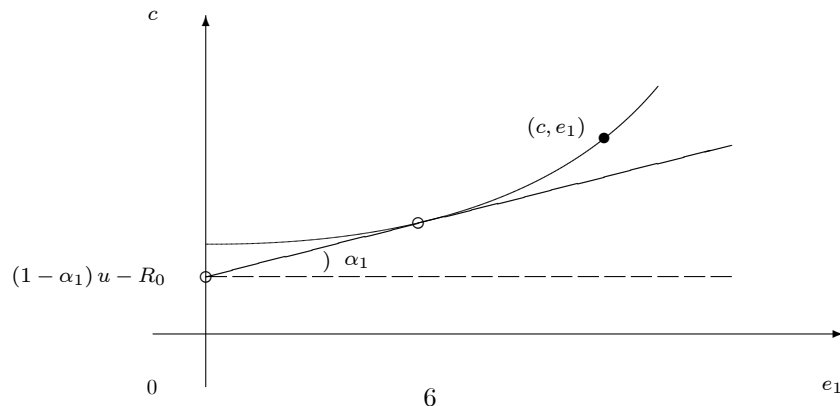


Figure 1: direct well-being $u(\cdot)$ in the additive ‘Mirrlees’-case

LEMMA 2. Given a bundle (c, e) , direct well-being $u(\cdot)$ is implicitly defined by

$$U(c, e; \gamma, \delta) = V(R_0, 0; \alpha, \beta_0, \beta, \delta; \gamma, (u(\cdot), u(\cdot), \dots, u(\cdot)))$$

with V the indirect utility function defined in Lemma 1. This results in $u(c, e; \alpha, \beta_0, \beta, \delta; \gamma)$ equal to

$$\frac{c - \sum_{j \in J} \frac{\delta_j}{\exp(\gamma_j)} \exp\left(\frac{e_j}{\delta_j}\right) - \kappa(R_0, 0; \alpha, \beta_0, \beta, \delta) - \sum_{j \in J} \alpha_j \beta_j \delta_j \gamma_j}{\sum_{j \in J} (1 - \alpha_j) \beta_j}. \quad (11)$$

A social planner who maximizes a sum of increasing and concave exponential functions of well-being—with well-being defined in Lemma 2—is both Pareto efficient and selective egalitarian. Pareto efficiency follows from the observation that welfare is increasing in well-being and well-being is a specific cardinalization of utility. For vertical equity, it suffices to note that welfare is a concave function of well-being and well-being in lemma 2 is linear in net outcome c . To see why horizontal equity holds, it is more convenient to work with the corresponding indirect well-being function, i.e., well-being measured at the bundle chosen by an individual for a given tax-benefit scheme (t_0, t) . Lemma 3 provides us with the indirect well-being formula.

LEMMA 3. Given a tax-benefit scheme (t_0, t) , indirect well-being $v(\cdot)$ is implicitly defined by

$$V(t_0, t; \alpha, \beta_0, \beta, \delta; \gamma, \theta) = V(R_0, 0; \alpha, \beta_0, \beta, \delta; \gamma, (v(\cdot), v(\cdot), \dots, v(\cdot)))$$

with V the indirect utility function defined in Lemma 1. This results in $v(t_0, t; \alpha, \beta, \delta; \gamma, \theta)$ equal to

$$\frac{\kappa(t_0, t; \alpha, \beta_0, \beta, \delta) - \kappa(R_0, 0; \alpha, \beta_0, \beta, \delta) - \sum_{j \in J} t_j \alpha_j \delta_j \gamma_j + \sum_{j \in J} (\beta_j - t_j) (1 - \alpha_j) \theta_j}{\sum_{j \in J} (1 - \alpha_j) \beta_j}. \quad (12)$$

From lemma 3 it follows that if all individuals have the same type, then they all obtain the same well-being level in the *laissez faire* with $(t_0, t) = (R_0, 0)$. As a consequence, introducing taxation via t would decrease welfare, since both average well-being would decrease and inequality would increase.

2.2 Results

The program of the social planner is to choose a tax-benefit scheme (t_0, t) in order to maximize welfare, a sum of increasing and concave exponential transformations of (indirect) well-beings, subject to a budget constraint; formally:

$$\max_{t_0, t} -\frac{1}{r} \ln \int \int \exp[-rv(t_0, t; \alpha, \beta, \delta; \gamma, \theta)] dF(\theta) dG(\gamma), \quad (13)$$

subject to the budget constraint (10), with $r > 0$ the inequality aversion parameter, R_0 the exogenous (per-capita) revenue requirement, indirect well-being $v(t_0, t; \alpha, \beta, \delta; \gamma, \theta)$ defined in lemma 3, and the distributions F and G defined in equation (9). Proposition 1 characterizes the general solution; note that the first-order conditions remain the same in case there are more than two characteristics.

PROPOSITION 1. The solution to the social planner’s problem is characterized as follows:

1. the budget constraint (and efficiency) leads to

$$t_0^* = R_0 - \sum_{j \in J} t_j \alpha_j \delta_j \ln((\beta_j - t_j) \alpha_j) - \sum_{j \in J} t_j \alpha_j \delta_j \mu_j^\gamma - \sum_{j \in J} t_j (1 - \alpha_j) \mu_j^\theta,$$

which can be plugged in in the welfare function to obtain some function $W(t; \alpha, \beta, \delta; r, R_0; \mu^\theta, \Sigma^\theta, \Sigma^\gamma)$ as defined in the appendix;

2. maximizing the previous welfare function leads to a system of first-order conditions (one for each j in J)

$$-\alpha_j \delta_j \frac{\zeta t_j}{\beta_j - t_j} - r \alpha_j \delta_j \sum_{k \in J} t_k \alpha_k \delta_k \sigma_{kj}^\gamma + r (1 - \alpha_j) \sum_{k \in J} (\beta_k - t_k) (1 - \alpha_k) \sigma_{kj}^\theta = 0,$$

with $\zeta = \sum_{j \in J} (1 - \alpha_j) \beta_j > 0$. The solution t^* satisfies $t^* \ll \beta$ and is a global maximum.

There is little we can say in general. If the planner does not care about vertical equity ($r \rightarrow 0$) or if vertical equity is empty due to type homogeneity ($\Sigma^\theta \rightarrow 0$), then the *laissez-faire* case results, i.e., $(t_0^*, t^*) = (R_0, 0)$ in the optimum. In the sequel, we discuss two specific cases: the ‘Mirrlees’-case, with outcome defined by one endogenous characteristic, income, and the ‘Akerlof’-case with an endogenous and an exogenous (uncontrollable) characteristic. Especially the second case will provide us with a hypothesis which will be tested in the empirical part.

The ‘Mirrlees’-case. To set the stage, we start with the simplest case possible. Suppose the outcome y is defined by one characteristic only, say earnings x_1 , with $y = x_1 = \alpha_1 e_1 + (1 - \alpha_1) \theta_1$. The system of first-order conditions in proposition 1 reduces to

$$-\alpha_1 (1 - \alpha_1) \delta_1 \frac{t_1}{1 - t_1} - r (\alpha_1 \delta_1)^2 t_1 \sigma_{11}^\gamma + r (1 - \alpha_1)^2 (1 - t_1) \sigma_{11}^\theta = 0.$$

We sum up the different theoretical results here; formal derivations can be found in the appendix. The first four results are standard; see, e.g., Mirrlees (1971) and Tuomala (1990). The fifth result appears in Judd and Su (2006) and Weinzierl (2009), while the sixth is new (as far as we know). The tax rate t_1^* on earnings x_1 :

1. lies in between the extremes of no taxation and complete taxation, i.e., $0 < t_1^* < 1$;
2. decreases with the elasticity δ_1 , ranging from complete taxation in case of perfect inelastic effort ($t_1^* \rightarrow 1$ if $\delta_1 \rightarrow 0$) to no taxation in case of perfect elastic effort ($t_1^* \rightarrow 0$ if $\delta_1 \rightarrow +\infty$);
3. increases with the inequality aversion r , ranging from no taxation if the planner is inequality neutral ($t_1^* \rightarrow 0$ if $r \rightarrow 0$) to partial taxation if the planner only cares about inequality ($t_1^* \rightarrow \frac{(1 - \alpha_1)^2 \sigma_{11}^\theta}{(\alpha_1 \delta_1)^2 \sigma_{11}^\gamma + (1 - \alpha_1)^2 \sigma_{11}^\theta}$ if $r \rightarrow +\infty$);
4. increases with type heterogeneity σ_{11}^θ , ranging from no taxation if everyone has the same type ($t_1^* \rightarrow 0$ if $\sigma_{11}^\theta \rightarrow 0$) to complete taxation if types become very heterogeneous ($t_1^* \rightarrow 1$ if $\sigma_{11}^\theta \rightarrow +\infty$);
5. decreases with taste heterogeneity σ_{11}^γ , ranging from partial taxation if everyone has the same taste ($0 < t_1^* < 1$ if $\sigma_{11}^\gamma \rightarrow 0$) to zero taxation if tastes become very heterogeneous ($t_1^* \rightarrow 0$ if $\sigma_{11}^\gamma \rightarrow +\infty$);

6. decreases with the degree of control α_1 , ranging from complete taxation if earnings cannot be controlled ($t_1^* \rightarrow 1$ if $\alpha_1 \rightarrow 0$) to no taxation if income is fully controlled ($t_1^* \rightarrow 0$ if $\alpha_1 \rightarrow 1$).

The Akerlof-case. Suppose there are two characteristics, earnings $x_1 = \alpha_1 e_1 + (1 - \alpha_1) \theta_1$ and an exogenous tag $x_2 = \theta_2$ and suppose output can be written as $y = x_1 + \beta_2 x_2$. The system of first-order conditions reduces to

$$\begin{aligned} -\alpha_1 \delta_1 \frac{\zeta t_1}{1 - t_1} - r (\alpha_1 \delta_1)^2 t_1 \sigma_{11}^\gamma + r (1 - \alpha_1) ((1 - t_1) (1 - \alpha_1) \sigma_{11}^\theta + (\beta_2 - t_2) \sigma_{21}^\theta) &= 0, \\ (1 - t_1) (1 - \alpha_1) \sigma_{12}^\theta + (\beta_2 - t_2) \sigma_{22}^\theta &= 0, \end{aligned}$$

with $\zeta = (1 - \alpha_1) + \beta_2$ here. The complete comparative statics results can be found in the appendix. Here, we highlight that the tax rate on earnings t_1^* also satisfies points 1-6 as described in the previous Mirrlees-case, except for a different limit if inequality aversion becomes large ($r \rightarrow +\infty$). In addition, in the limiting case of perfect type correlation ($(\sigma_{12}^\theta)^2 \rightarrow \sigma_{11}^\theta \sigma_{22}^\theta$), the tax rate on earnings t_1^* reduces to zero and all taxation can be done via the tax t_2^* on the tag, since the latter is a perfect signal of earnings ability and it can be taxed at no cost.

More interesting for our purposes, note that the second of the first-order conditions can be rewritten as

$$(\beta_2 - t_2) + (\sigma_{12}^\theta / \sigma_{22}^\theta) \times (1 - t_1) (1 - \alpha_1) = 0. \quad (14)$$

Note several things. First, two special cases are immediately clear from equation (14). In the absence of a needs effect of the tag ($\beta_2 \rightarrow 0$), the optimal tax on the tag reduces to $t_2 = (\sigma_{12}^\theta / \sigma_{22}^\theta) (1 - t_1) (1 - \alpha_1)$, which is positive (resp. negative) if the higher values for the tag signal a higher (resp. lower) ability to earn. In the absence of a signal ($\sigma_{12}^\theta = 0$), the optimal tax on the tag t_2 equals β_2 , i.e., the gross effect of the tag should be taxed away. More generally, equation (14) tells us that the total marginal effect of the tag θ_2 on the net outcome c should be equal to zero in a *fair* tax-benefit system. To see this, note that the total net marginal effect consists of two parts. The first part $(\beta_2 - t_2)$ is the direct marginal effect of θ_2 on the net outcome c . The second part can be interpreted as the indirect marginal effect of θ_2 on c : it is equal to $\sigma_{12}^\theta / \sigma_{22}^\theta$, the marginal effect of θ_2 on θ_1 , multiplied by $(1 - t_1) (1 - \alpha_1)$, the marginal effect of θ_1 on c . Third, to test equation (14), we must be able to rewrite it in terms of empirically observable quantities. Fortunately, we can use lemma 1 to obtain

$$\begin{aligned} x_1^* &= \alpha_1 \delta_1 (\ln((\beta_1 - t_1) \alpha_1) + \gamma_1) + (1 - \alpha_1) \theta_1, \\ x_2^* &= \theta_2, \end{aligned}$$

which implies that $\sigma_{12}^{x^*} = (1 - \alpha_1) \sigma_{12}^\theta$ and $\sigma_{22}^{x^*} = \sigma_{22}^\theta$. Using these formulas, we get the empirical counterpart of the theoretical formula (14):

$$(\beta_2 - t_2) + (\sigma_{12}^{x^*} / \sigma_{22}^{x^*}) \times (1 - t_1) = 0. \quad (15)$$

The next empirical section will test, among other things, this hypothesis.

3 Empirical evidence

This section consists of three parts. We start by setting up an empirical model, which consists of two steps: first, estimate the determinants of gross output y , and second, estimate the implicit tax rates for

these determinants on the basis of the observed taxes (or subsidies) $y - c$. Next, we describe the EU-SILC dataset, the selection of the sample, the gross and net output—income and equivalent income here—and the different covariates. Finally, we try to answer three questions: (1) how do countries compensate for the different determinants of gross output?; (2) do countries compensate more for uncontrolled characteristics compared to partially controlled ones?; (3) how fair is the tax-benefit system in the different countries, i.e., how close is the total effect of the uncontrolled characteristics, the left-hand side of equation (15), to zero?

3.1 Model

Implicit tax rates. To set up the empirical model, we start with two remarks. First, we make a distinction between covariates and characteristics: a characteristic can consist of several covariates, but not vice-versa. Two examples. The covariates for the characteristic ‘education’ will simply consist of the different education dummies. The covariates for the characteristic ‘no control’ will consist of the different covariates (of characteristics) which are thought to be beyond control. The last example makes clear how it is possible to get two ‘composite’ characteristics, ‘partial control’ and ‘no control’ as in the ‘Akerlof’-case, on the basis of a finite set of covariates. Second, an error term is inevitable in empirical work. It will play the role of an additional ‘unobserved’ characteristic later on.

Let z denote the vector of covariates, which can be decomposed as $z' = (z'_j)_{j \in J}$, with z'_j the covariates for characteristic j in J . The gross output regression becomes

$$\begin{aligned} y &= \beta_0 + w'z + \epsilon \\ &= \beta_0 + \sum_{j \in J} w'_j z_j + \epsilon \\ &= \beta_0 + \beta'x \end{aligned} \tag{16}$$

which brings us back to the theoretical model, with β being a vector of ones and $x = ((w'_j z_j)_{j \in J}, \epsilon)'$ the vector of characteristics, including the unobserved one. The tax (or subsidy, if negative) equals

$$\tau = y - c = t_0 + t'x. \tag{17}$$

Equations (16)-(17) directly suggest a simple two-step approach to estimate the tax rates t_0 and t . First, estimate equation (16) by OLS, which provides us with $\hat{x} = ((\hat{w}'_j z_j)_{j \in J}, \hat{\epsilon})'$. Second, estimate equation (17) by OLS, replacing x by the estimated \hat{x} (and adding another error term).

The Akerlof-case. Next, recall the ‘Akerlof’-case, with two characteristics, which we call partial control x_P and no control x_N in the sequel. For the empirical part, it means that we must classify each characteristic, including the unobserved ones, as either belonging to partial or no control, resulting in a new decomposition of x as $(x_P, x_N)'$. With this new notation, fair taxation requires that the total marginal effect of x_N on c is zero. Given the current notation and set-up, the total (marginal) effect (of x_N on c) equals

$$TE_N = (1 - t_N) + (\sigma_{P_N}^{x^*} / \sigma_{N_N}^{x^*}) \times (1 - t_P). \tag{18}$$

How can we estimate TE_N ? First, note that the OLS-estimate of b in the regression

$$x_P = a + bx_N + \eta \tag{19}$$

equals $\sigma_{P_N}^{x^*} / \sigma_{N_N}^{x^*}$. Second, the net outcome c equals

$$c = (\beta_0 - t_0) + (1 - t_P)x_P + (1 - t_N)x_N. \tag{20}$$

Plugging (19) into (20), we get

$$\begin{aligned}
c &= (\beta_0 - t_0) + (1 - t_P)(a + bx_N + \eta) + (1 - t_N)x_N \\
&= \underbrace{(\beta_0 - t_0) + (1 - t_P)a}_{\text{constant}} + \underbrace{[(1 - t_P)b + (1 - t_N)]x_N}_{TE_N} + (1 - t_P)\eta.
\end{aligned} \tag{21}$$

This suggests the following two-step procedure: first, estimate equation (16) as before by OLS, which provides us here with $\hat{x} = (\hat{x}_P, \hat{x}_N)'$; second, estimate equation (21) by plugging in the estimated \hat{x}_N , which provides us with an estimate \widehat{TE}_N as well as a confidence interval. The second step estimates the (composite) effect of ‘no control’-variables on the net outcome of interest, which is similar to the parametric application of Roemer’s equality of opportunity; see Bourguignon et al. (2007).

3.2 Data

We use the 2007 EU-SILC data (European Union - Statistics on Income and Living Conditions) whose aim is to collect harmonized and comparable multidimensional micro data on income poverty and social exclusion for 24 EU member states (all 2006 EU member states, except Malta) as well as Norway and Iceland. Our analysis is based on the 2007 EU-SILC wave which is the first to include gross income information for all countries. The sample size varies from 3,505 households in Cyprus to 20,982 households in Italy. The survey is representative for the whole population in each country due to the construction of population weights.

In the remainder, we refer to our total sample of countries as the "EU". To the fifteen old EU member states before the EU enlargement in 2004 we refer to as "EU-15"⁵ and to the nine included New Member States shortly as "NMS"⁶. When we refer to groups of countries, we use the geographical regions, such as Continental (AT, BE, DE, FR, LU, NL), Northern (DK, FI, SE), Southern (CY, ES, GR, IT, PT), Anglo-Saxon (IE, UK), Central Eastern (CZ, HU, PL, SI, SK) and Baltic (EE, LT, LV) countries. The subgroups of Central Eastern and Baltic countries add up to the group of Eastern European countries.

We extract pre-tax and post-tax household income as well as other variables; definitions and summary statistics can be found in the appendix. We use household equivalent income as our preferred measure of well-being to compensate for different household structures and possible economies of scales within households.⁷ In the remainder of the paper we always refer to equivalent measures of household income components unless explicitly noted otherwise. Note that our concept of pre-government income includes social insurance contributions paid by the employer as they can be very different across countries. To make incomes comparable across countries, we adjust national income amounts by the multilateral current purchasing power parities provided by Eurostat. The analysis only allocates those taxes and benefits that can be reasonably attributed to households. Therefore, corporate taxes as well as some types of government expenditures, such as expenditures on defense, are not considered. Due to data limitations, indirect taxes and in-kind benefits cannot be taken into account, either. Thus, in the remainder, we merely advert to

⁵Austria (AT), Belgium (BE), Denmark (DK), Germany (DE), Greece (GR), Spain (ES), Finland (FI), France (FR), Ireland (IE), Italy (IT), Luxembourg (LU), Netherlands (NL), Portugal (PT), Sweden (SE), United Kingdom (UK).

⁶Cyprus (CY), Czech Republic (CZ), Estonia (EE), Hungary (HU), Lithuania (LT), Latvia (LV), Poland (PL), Slovenia (SI), Slovak Republic (SK).

⁷For each person, the equivalent (per-capita) total net income is its household total net income divided by the equivalent household size according to the modified OECD scale, which assigns a weight of 1.0 to the head of household, 0.5 to every household member aged 14 or older and 0.3 to each child aged less than 14. Summing up the individual weights gives the household specific equivalence factor.

cash benefits when speaking of social benefits and to personal income taxes in the cases of taxes. We provide further information on the exact definition and computation of all income components in the data appendix (see also Fuest et al., 2010, for more details).

We select single and couple households with or without children that have earnings as the main source of income. In our preferred specification, we estimate a joint model on the pooled data. As a robustness check, we conduct separate estimations for singles and couples. We trim the top and bottom 1% of the income distribution in order to avoid estimation problems due to extreme outliers (Cowell and Victoria-Feser, 1996a, 1996b, 2002).

We use two composite characteristics: ‘no control’ and ‘partial control’. The ‘no control’ composite consists of the characteristics ‘sex’, ‘age’, ‘disability’, and ‘foreigner’, whereas the ‘partial control’ composite contains ‘education’, ‘number of children’, ‘marital status’, and ‘employment status’. Each characteristic consists of several covariates. The covariate ‘sex’ contains a gender dummy, ‘age’ contains several dummies for different age classes, ‘disability’ is constructed using information on disability status and the receipt of certain disability benefits, ‘foreigner’ contains two dummies for born outside of the country but within the EU and born outside the EU. The covariates for the characteristic ‘education’ simply consist of different education dummies (4 levels according to ISCED definition), ‘number of children’ contains information about the number of children according to three age groups, ‘marital status’ consists of dummies for living in a couple or not, and ‘employment status’ contains a dummy for not working. In our preferred specification, we use individual level covariates and characteristics. As a robustness check, we perform the estimations on household characteristics which are computed as averages of the individual covariates.

3.3 Results

This section consists of 5 parts. We start with the estimation of implicit tax rates for the two composite characteristics ‘no control’ and ‘partial control’. In the next step, we decompose the compensation rates into the contribution of taxes, social insurance contributions and benefits. We then test the ‘Akerlof’ hypothesis of a fair tax benefit scheme. In the fourth part, we apply several robustness checks. The last parts extends the analysis of implicit tax rates on single characteristics.

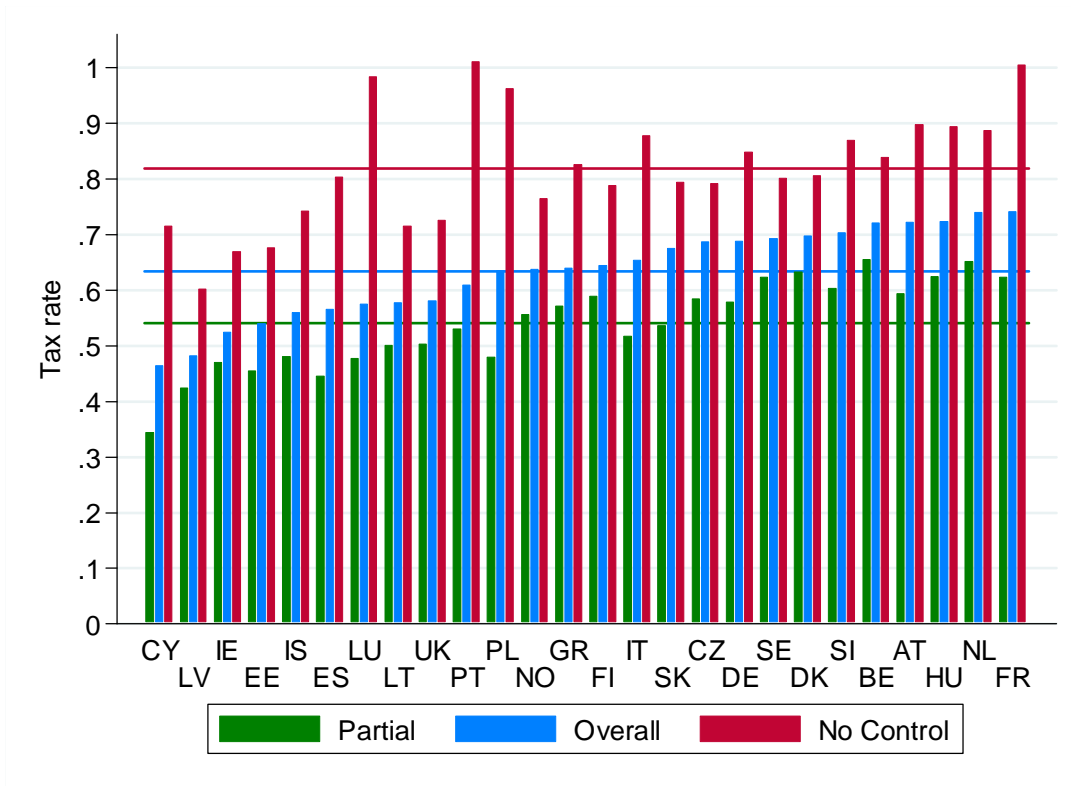
3.3.1 Partial control versus no control

In this section, we estimate implicit tax rates (see equation 17) for the two composite characteristics ‘no control’ and ‘partial control’ as well as on total income; this is basically the linear income tax schedule, consisting of a basic income (t_0) and a linear tax rate (t') calculated for equivalent incomes. Figure 1 shows compensation rates for all countries for both groups of characteristics; countries are sorted on the basis of the overall compensation rate. Notice that all countries compensate clearly differently (and, as expected from our theoretical predictions, more) for uncontrollable factors compared to controllable factors: on average, 0.82 for uncontrollable and 0.53 for (partially) controllable ones.

The overall compensation rate is rather low in the flat tax countries (Baltic states plus Iceland) as well as in some countries from Southern and Eastern Europe. Surprisingly, Norway and Finland have values around the average of 0.63. The highest compensation rates can be found in Continental countries.

The compensation rate for controllable factors is always below the overall compensation rate which is in turn lower than the compensation rate for uncontrollable factors in all countries. The pattern of

Figure 1: Implicit tax rates on composite characteristics



Source: Own calculations based on EU-SILC

implicit tax rates for partial controllable characteristics follows the pattern for the overall compensation rate, albeit not perfectly. Compensation is highest in Continental and Nordic countries and lowest in the Baltic, Anglo-Saxon and some Southern countries. The picture differs more for the ‘no control’ composite where the highest compensation rates can be found besides France in Portugal, Luxembourg and Poland which have below average overall compensation.

3.3.2 Role of income taxation versus social security benefits

In order to understand the role played by taxes, contributions and benefits separately, we decompose the total tax amount in equation (17) as

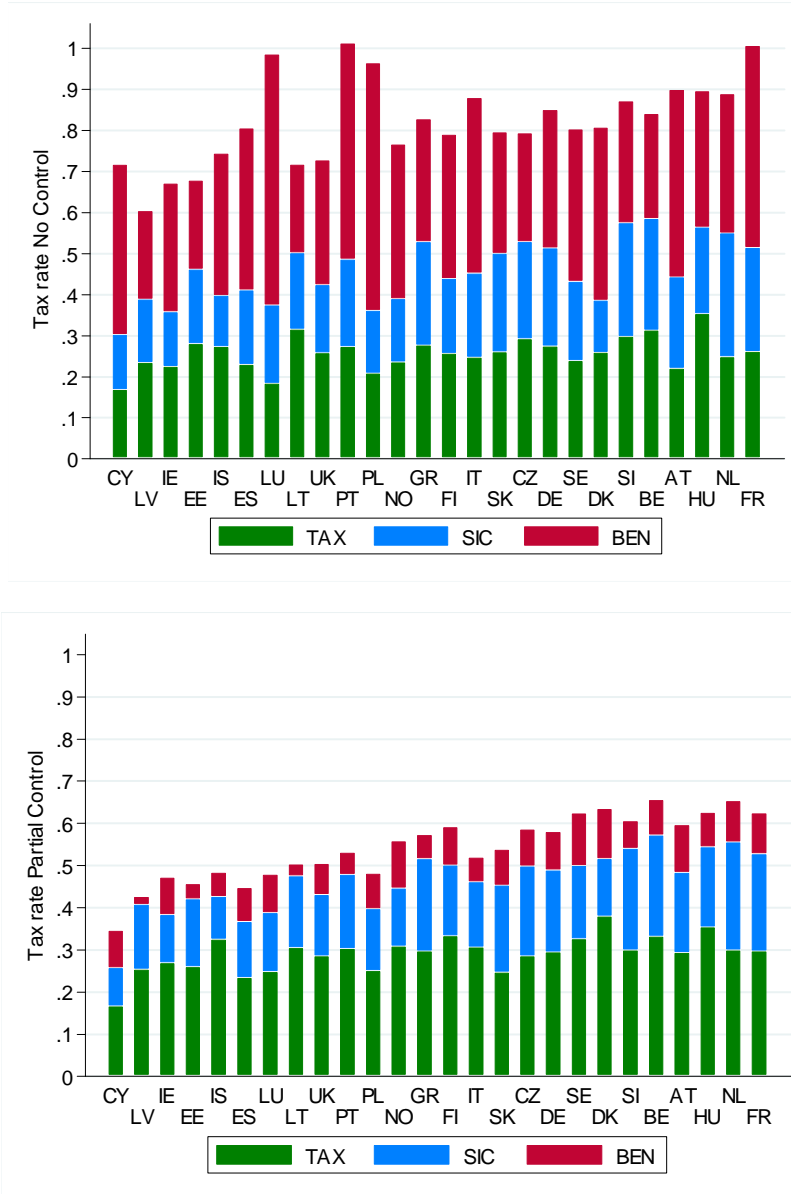
$$y - c = T_y + T_{ss} - B \quad (22)$$

with T_y (equivalized) income taxes, T_{ss} (equivalized) social security contributions and B (equivalized) social security benefits. We then can do the second step separately for each component, i.e.,

$$T_y = t_{y,0} + t'_y x \ \& \ T_{ss} = t_{ss,0} + t'_{ss} x \ \& \ -B = t_{B,0} + t'_B x, \ \text{with } t' = t'_y + t'_{ss} + t'_B. \quad (23)$$

The results (in terms of the respective instrument specific compensation rate) are reported in Figure 2 for ‘no control’ (upper panel) and ‘partial control’ (lower panel) as shares of the total compensation rate; countries are again sorted on the basis of the overall compensation rate.

Figure 2: Decomposition implicit tax rates on composite characteristics



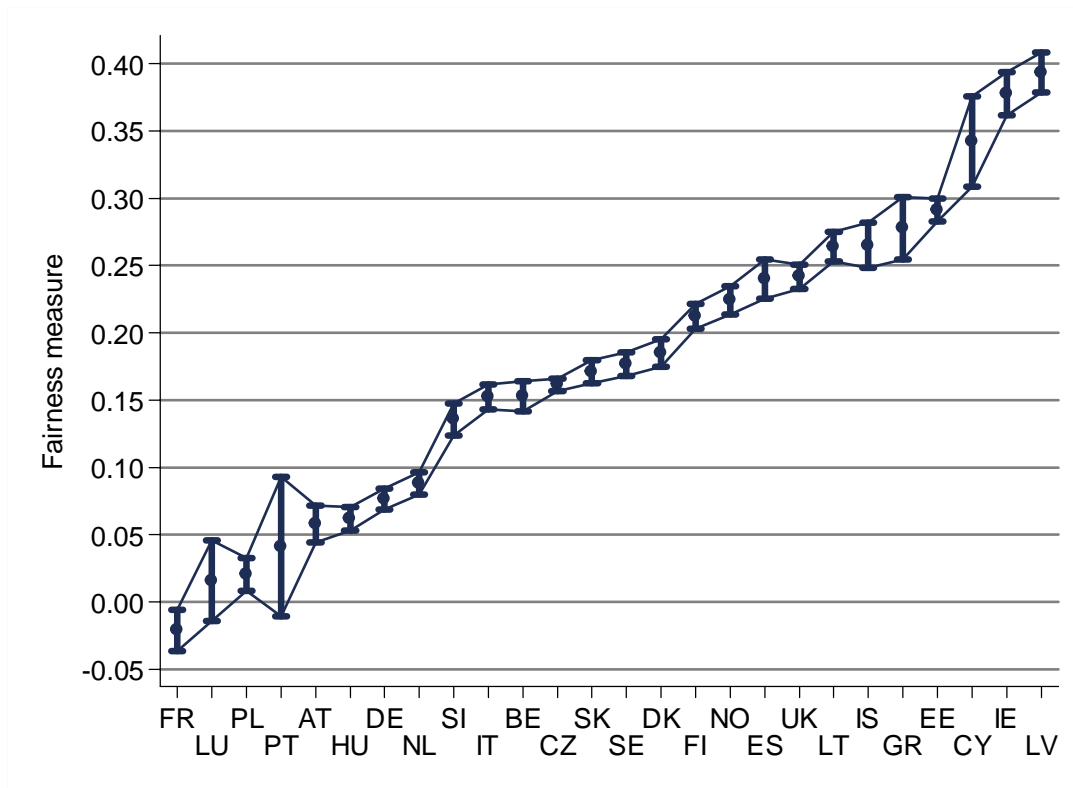
Source: Own calculations based on EU-SILC

In most cases, the different components contribute about the same share to the overall compensation rate. For the ‘no control’ composite, benefits are the most important contributor - both in absolute and relative terms, whereas they are only of minor importance for the ‘partial control’ characteristics. In fact, without benefits, the absolute compensation rates would be rather similar for both types of characteristics. In the ‘partial control’-case, taxes have the highest relative importance in all countries. This results implies that benefits are the main instrument to compensate for uncontrollable factors whereas taxes are mainly used for compensation of controllable characteristics. The differences between the size of the shares for taxes and contributions can be explained with the different importance of the instruments in the tax mix of the respective countries.

3.3.3 Testing the ‘Akerlof’-hypothesis

In the next step, we test the hypothesis from the ‘Akerlof’-case, i.e. that the the total marginal effect of the tag on the net outcome is equal to zero ($TE_N = 0$; see equation 18). The results are plotted in Figure 3; countries are ordered in ascending order of TE_N . A value of this ‘fairness measure’ greater than zero implies that the compensation for the ‘no control’ is too low relative to the ‘partial control’ composite and vice versa. Hence, a value below zero indicates ‘overcompensation’.

Figure 3: Fairness measure Akerlof case



Source: Own calculations based on EU-SILC.

The greater the distance from zero, the less fair is a country. In this specification, we cannot reject that France and Luxembourg have tax benefit systems that are actually fair - with Poland and Portugal

being close. High values of fairness are also achieved in other continental (AT, DE, NL) and Hungary. In general, fairness is lower in the Baltic, Anglo-Saxon and some Southern countries, and lowest in Cyprus, Ireland and Latvia.

In order to understand the ‘source of fairness’, recall the definition $TE_N = \underbrace{(1 - t_N)}_A + \underbrace{(\sigma_{PN}^*/\sigma_{NN}^*)}_B \times \underbrace{(1 - t_P)}_C \stackrel{!}{=} 0$ (equation 18). For ‘fairness’, the optimal relationship between the compensation rates depends on the correlation between the ‘no control’ and the ‘partial control’ composite (scaled with the variance of the uncontrollable characteristics). Hence, the optimal tax on the uncontrollable tag is higher (resp. lower) the higher (lower) the correlation of the tag x_N with x_P , i.e. x_N being a stronger (weaker) signal for a higher ability to earn. Note that in the absence of a signal ($\sigma_{PN}^* = 0$), the gross effect of the tag should be taxed away.

Table 1: Components of fairness measure

| | $(1 - t_N)$ | $(\sigma_{PN}^*/\sigma_{NN}^*)$ | $(1 - t_P)$ | TE_N | LCI | UCI |
|----|-------------|---------------------------------|-------------|--------|-------|-------|
| AT | 0.10 | -0.11 | 0.41 | 0.06 | 0.04 | 0.07 |
| BE | 0.16 | -0.02 | 0.35 | 0.15 | 0.14 | 0.16 |
| CY | 0.28 | 0.09 | 0.66 | 0.34 | 0.31 | 0.38 |
| CZ | 0.21 | -0.11 | 0.42 | 0.16 | 0.16 | 0.17 |
| DE | 0.15 | -0.18 | 0.42 | 0.08 | 0.07 | 0.08 |
| DK | 0.20 | -0.03 | 0.37 | 0.18 | 0.17 | 0.20 |
| EE | 0.32 | -0.06 | 0.55 | 0.29 | 0.28 | 0.30 |
| ES | 0.20 | 0.08 | 0.55 | 0.24 | 0.23 | 0.25 |
| FI | 0.21 | 0.00 | 0.41 | 0.21 | 0.20 | 0.22 |
| FR | -0.00 | -0.04 | 0.38 | -0.02 | -0.04 | -0.01 |
| GR | 0.17 | 0.24 | 0.43 | 0.28 | 0.25 | 0.30 |
| HU | 0.11 | -0.12 | 0.38 | 0.06 | 0.05 | 0.07 |
| IE | 0.33 | 0.09 | 0.53 | 0.38 | 0.36 | 0.39 |
| IS | 0.26 | 0.01 | 0.52 | 0.26 | 0.25 | 0.28 |
| IT | 0.12 | 0.06 | 0.48 | 0.15 | 0.14 | 0.16 |
| LT | 0.28 | -0.04 | 0.50 | 0.26 | 0.25 | 0.28 |
| LU | 0.02 | -0.00 | 0.52 | 0.02 | -0.01 | 0.05 |
| LV | 0.40 | -0.01 | 0.58 | 0.39 | 0.38 | 0.41 |
| NL | 0.11 | -0.07 | 0.35 | 0.09 | 0.08 | 0.10 |
| NO | 0.24 | -0.03 | 0.44 | 0.22 | 0.21 | 0.23 |
| PL | 0.04 | -0.03 | 0.52 | 0.02 | 0.01 | 0.03 |
| PT | -0.01 | 0.11 | 0.47 | 0.04 | -0.01 | 0.09 |
| SE | 0.20 | -0.06 | 0.38 | 0.18 | 0.17 | 0.19 |
| SI | 0.13 | 0.01 | 0.40 | 0.14 | 0.12 | 0.15 |
| SK | 0.21 | -0.08 | 0.46 | 0.17 | 0.16 | 0.18 |
| UK | 0.27 | -0.07 | 0.50 | 0.24 | 0.23 | 0.25 |

Source: Own calculations based on EU-SILC

Table 2 presents the different components. The correlation between x_N and x_P is usually weak and

around zero in most cases (the only exceptions are Germany and Greece). Hence, the value of the fairness measure is determined mostly by the value of the tax on the uncontrollable characteristics. Therefore, when the value of the fairness measure is high, i.e. fairness is low, $(1 - t_N)$ is too high implying that the compensation rate on the uncontrollable characteristics is too low. Hence, in order to improve the fairness of a tax benefit system, the compensation rate on uncontrollable characteristics has to increase.

3.3.4 Robustness checks

The results in the previous section were based on compensation rates for equivalent incomes jointly estimated for singles and couples on the individual level. In principle it is possible to use 8 different specifications for the estimation (our preferred specification is highlighted in italics):

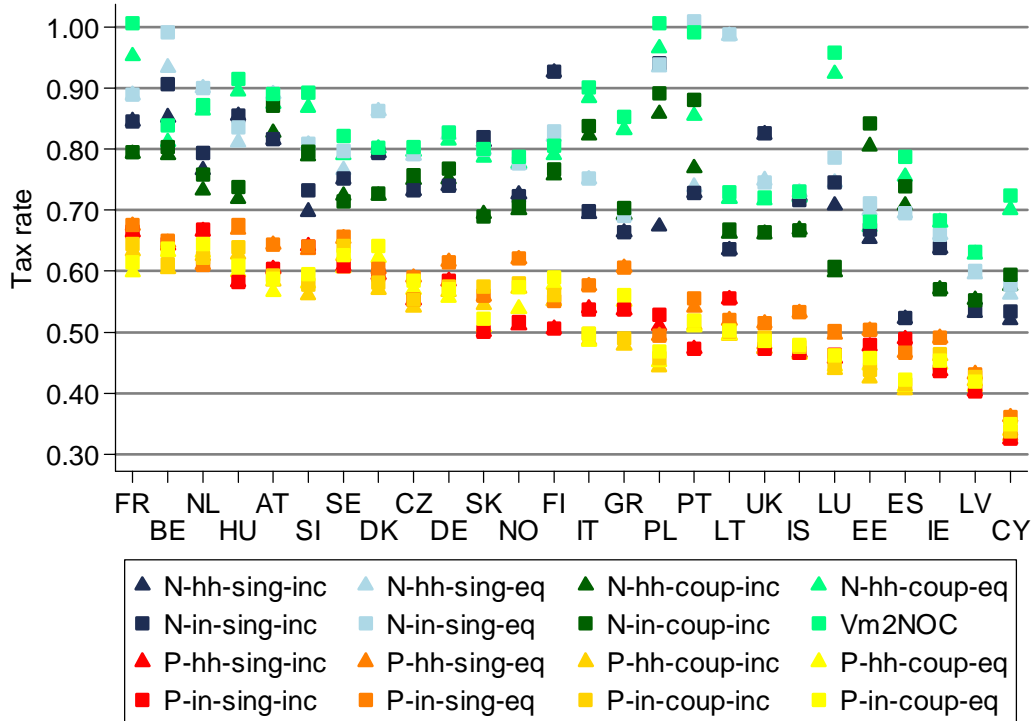
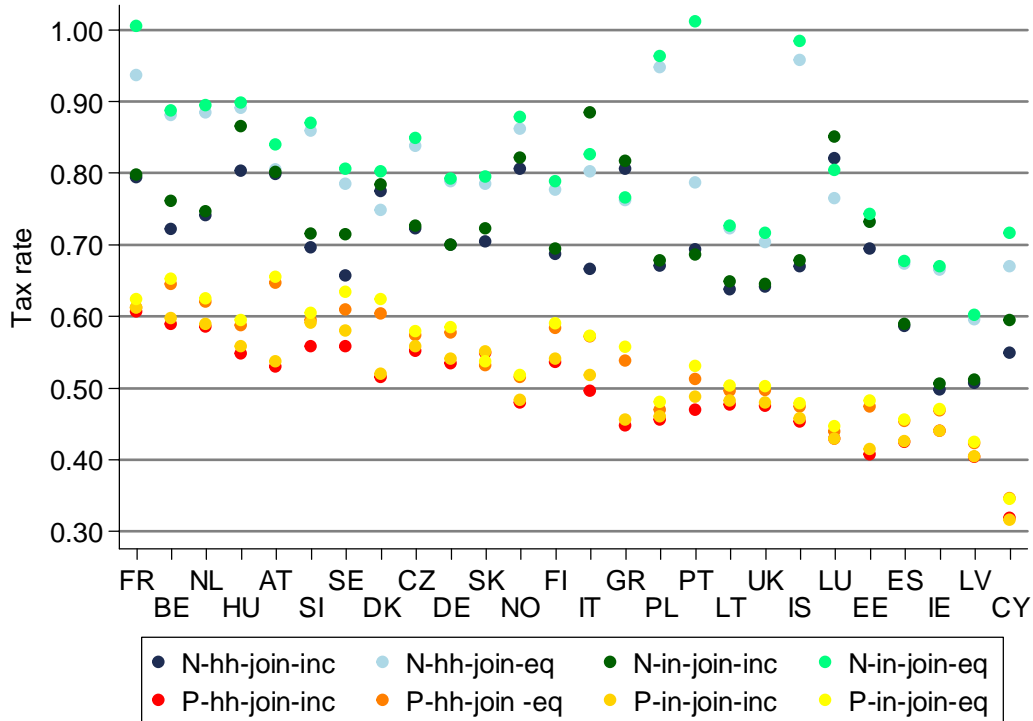
- 2 output definitions: income vs. *equivalent income* (i.e. adjusting for household size and economies of scale),
- 2 output (and characteristics) levels: *purely individual* vs. household (averages) level,
- 2 estimation methods: singles and couples separated vs. *joint (pooled)* estimation.

In order to assess the sensitivity of our results with respect to these choices, we provide estimates of the implicit tax rates on the composite characteristics (Figure 4) and of the fairness measure (Figure 5) for the different combinations. Note that the countries are ordered according to the fairness measure in our preferred specification (see Figure 3).

The results for the implicit tax rates in Figure 4 show that for all combinations the compensation rate on the ‘no control’ composite is always higher than for the ‘partial control’ characteristics. There is no clear pattern visible how the different choices affect the results. For the ‘partial control’ rate, in most cases the difference is less than or equal to 10 percentage points. This difference is bigger for the ‘no control’ composite. The choice of output definition (income vs. equivalent income) and especially the separate estimation for singles and couples affect the results. In general, for the ‘partial control’ composite, the compensation rate on singles (couples) is lower (higher) than the one derived in a pooled estimation, whereas it is the other way around for the ‘no control’ composite.

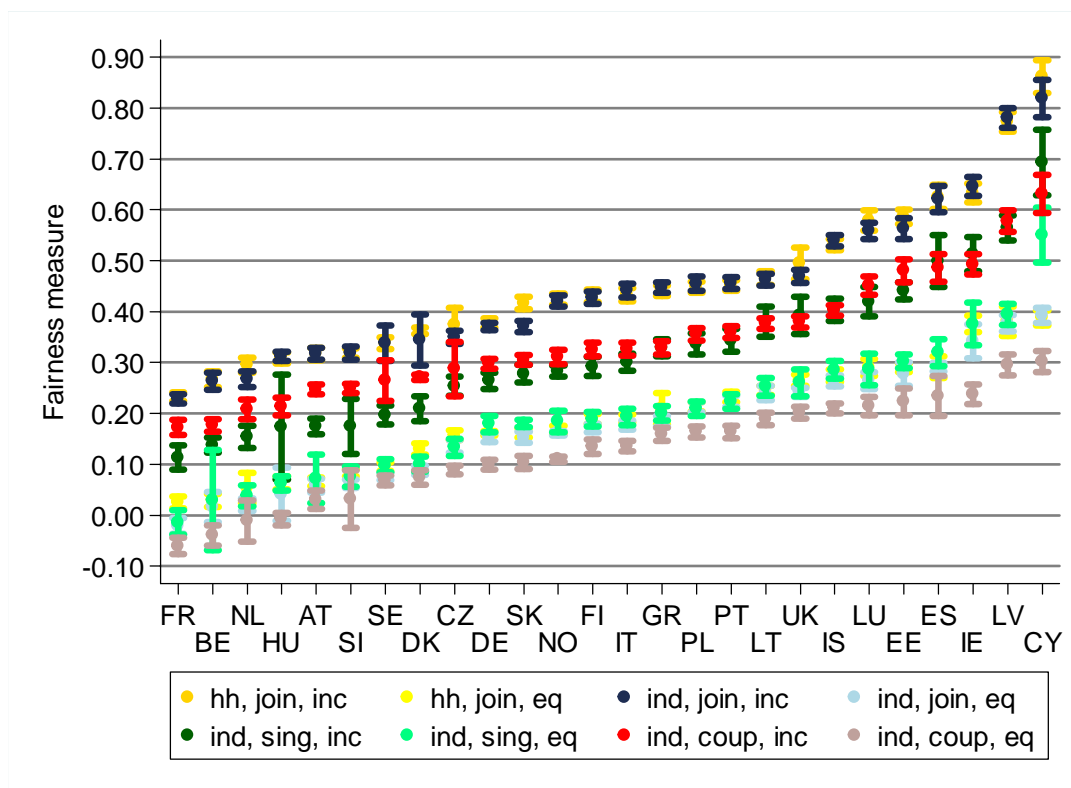
When looking at the fairness measure in Figure 5, the main difference is due to the choice of output definition. When using income instead of equivalent income, the value of the fairness measure is on average about 0.25 higher. In this case, i.e. when not accounting for economies of scale, we can reject the hypothesis that their tax benefit system is fair for all countries. However, the ranking of countries in terms of fairness remains robust - irrespective of the choices made.

Figure 4: Implicit tax rates: robustness checks



Source: Own calculations based on EU-SILC. Notes: ‘P’ (‘N’) indicates the implicit tax rate for no (partial) control, ‘hh’ (‘in’) the output level: household (individual), ‘join’ (‘sing’/ ‘coup’) the estimation method: joint (single/couple) and ‘inc’ (‘eq’) the output concept: (equivalent) income.

Figure 5: Fairness measures Akerlof: robustness checks



Source: Own calculations based on EU-SILC. Notes: 'hh' ('in') indicates the output level: household (individual), 'join' ('sing'/'coup') the estimation method: joint (single/couple) and 'inc' ('eq') the output concept: (equivalent) income.

3.3.5 Extension: order of deservingness

In addition to estimating the implicit tax rates on the composite characteristics ‘no control’ and ‘partial control’, it is also possible to directly estimate the compensation rates for each characteristic (i.e. ‘sex’, ‘age’, ‘disability’, and ‘foreigner’, ‘number of children’, ‘marital status’, and ‘employment status’) separately. Table 2 presents the ranking of the different tax rates for the different countries. The highest tax rates is assigned a value of 1, the second highest a 2 and so on.

Table 2: Order of deservingness

| | AGE | DIS | SIZE | UNEMP | SEX | COUPLE | IMMI |
|----|-----|-----|------|-------|-----|--------|------|
| AT | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| BE | 1 | 2 | 4 | 3 | 5 | 6 | 7 |
| CY | 1 | 3 | 2 | 6 | 5 | 7 | 4 |
| CZ | 1 | 2 | 3 | 5 | 4 | 7 | 6 |
| DE | 1 | 2 | 5 | 4 | 6 | 7 | 3 |
| DK | 2 | 1 | 5 | 3 | 4 | 7 | 6 |
| EE | 1 | 2 | 3 | 5 | 4 | 7 | 6 |
| ES | 1 | 2 | 3 | 4 | 7 | 5 | 6 |
| FI | 2 | 1 | 4 | 3 | 5 | 7 | 6 |
| FR | 1 | 2 | 4 | 5 | 3 | 6 | 7 |
| GR | 2 | 1 | 4 | 5 | 3 | 7 | 6 |
| HU | 1 | 3 | 2 | 4 | 5 | 7 | 6 |
| IE | 1 | 2 | 3 | 4 | 7 | 5 | 6 |
| IS | 3 | 1 | 4 | 2 | 5 | 6 | 7 |
| IT | 1 | 2 | 3 | 4 | 6 | 7 | 5 |
| LT | 1 | 2 | 3 | 6 | 4 | 7 | 5 |
| LU | 2 | 3 | 4 | 6 | 1 | 5 | 7 |
| LV | 1 | 2 | 3 | 5 | 6 | 7 | 4 |
| NL | 2 | 3 | 5 | 4 | 6 | 7 | 1 |
| NO | 2 | 1 | 3 | 4 | 5 | 7 | 6 |
| PL | 1 | 2 | 3 | 5 | 4 | 7 | 6 |
| PT | 1 | 4 | 3 | 2 | 5 | 6 | 7 |
| SE | 2 | 1 | 4 | 5 | 3 | 7 | 6 |
| SI | 1 | 2 | 3 | 6 | 5 | 7 | 4 |
| SK | 2 | 3 | 5 | 6 | 4 | 7 | 1 |
| UK | 1 | 2 | 5 | 4 | 3 | 6 | 7 |

Source: Own calculations based on EU-SILC

In general, we find the following order of solidarity: most support for elderly, followed by sick and disabled, families with children, and unemployed less towards gender and marriage and finally least to general assistance schemes and immigrants. Hence, countries implicitly have already implemented age-based tax benefit systems - as discussed in Erosa and Gervais (2002), Lozachmeur (2006), Weinzierl (2007) and Blomquist and Micheletto (2008), but in general no gender based taxation - as discussed in Alesina et al. (2008) and Cremer et al. (2010) - is recognizable. The only exceptions is Luxembourg which has the

highest compensation for gender based differences.

4 Conclusion

In this paper, we introduce the notion of ‘partial control’ to the literature on optimal taxation. Individuals differ in several observable characteristics, which are each a weighted combination of the unobservables type (drawn by nature) and effort (chosen by the individual). We refer to this weight as the degree of control which is zero for some characteristics (e.g. sex and age), but strictly positive for other characteristics (e.g. education and family composition). We then derive a fair tax-benefit schedule based on several characteristics, which differ in the degree of control. The general case does not generate much insight. Hence, we discuss two specific cases: the ‘Mirrlees’-case, with outcome defined by one endogenous characteristic, income, and the ‘Akerlof’-case with an endogenous and an exogenous (uncontrollable) characteristic. We show that in both cases the tax rate on earnings decreases with the degree of control, which has not been documented before in the literature.

In the next step, we take the optimal tax formulas to the data in order to estimate the compensation rates for the different characteristics in different European countries. Our analysis shows that there is a clear tendency in all countries to compensate more for the ‘no control’-composite compared to the ‘partial control’-composite which is in line with our theoretical predictions. In addition, we test the empirical counterpart of the theoretical tax formula in the ‘Akerlof’-case. We cannot reject that some countries (France, Luxembourg, The Netherlands and Austria) are actually fair in the sense that the total marginal effect of the tax on the net outcome is equal to zero.

There are different possible policy implications arising from our analysis. First, in a normative interpretation of our results, the deviation of the existing tax benefit systems from a fair system could be seen as a mission for policymakers to re-design the tax benefit scheme in line with our theory. We can reject for the majority of countries that their tax benefit systems are fair. In order to achieve fairness, these countries would have to increase the compensation rate for the uncontrollable factors. This, however, implies, of course, that the social preferences modeled here correspond with the ‘true’ preferences of voters / policymakers in a society. Second, in a positive analysis, it would be interesting to design an inverse optimum problem to derive the social preferences that result in the observed systems as being optimal. These preferences could then be compared to the actual preferences in order to verify or falsify our theoretical model. We will pursue these issues in future research.

Note that there are several limitations to our analysis. First, the complexity of the multidimensional screening exercise forces us to simplify several aspects of our theoretical model in order to keep it analytically tractable. Second, in the empirical analysis, type and effort are not directly observable. Hence, the identification comes from the differences in the degree of control of our covariates. Due to data availability, we cannot observe all potential ‘no control’ covariates and hence the analysis might be incomplete.

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Proof of proposition 1

The planner solves

$$\max_{t_0, t} W = -\frac{1}{r} \ln \int \int \exp[-rv(t_0, t; \alpha, \beta, \delta; \gamma, \theta)] dF(\theta) dG(\gamma),$$

subject to the budget constraint

$$t_0 + \int \int \left(\sum_{j \in J} t_j x_j^* \right) dF(\theta) dG(\gamma) \geq R_0,$$

and well-being of an individual $v(t_0, t; \alpha, \beta, \delta; \gamma, \theta)$ is defined as

$$\frac{\kappa(t_0, t; \alpha, \beta_0, \beta, \delta) - \kappa(R_0, 0; \alpha, \beta_0, \beta, \delta) - \sum_{j \in J} t_j \alpha_j \delta_j \gamma_j + \sum_{j \in J} (\beta_j - t_j) (1 - \alpha_j) \theta_j}{\sum_{j \in J} (1 - \alpha_j) \beta_j},$$

with $\kappa(t_0, t; \alpha, \beta_0, \beta, \delta) - \kappa(R_0, 0; \alpha, \beta_0, \beta, \delta)$ equal to

$$R_0 - t_0 + \sum_{j \in J} (\beta_j - t_j) \alpha_j \delta_j [\ln((\beta_j - t_j) \alpha_j) - 1] - \sum_{j \in J} \alpha_j \beta_j \delta_j [\ln(\alpha_j \beta_j) - 1],$$

while

$$x_j^* = \alpha_j \delta_j (\ln((\beta_j - t_j) \alpha_j) + \gamma_j) + (1 - \alpha_j) \theta_j.$$

Before analyzing the solution, notice that the optimal tax rates t^* must satisfy $t^* \ll \beta$. As defined before, x_j^* remains the same for all tax levels $t_j \geq \beta_j$, so it suffices for the planner to look at tax rates $t_j < \beta_j$ and $t_j = \beta_j$. In addition, a solution with $t_j^* = \beta_j$ (and $t_0 \rightarrow +\infty$) can never be efficient (the *laissez faire* is better for everyone), leaving us with $t_j < \beta_j$ for each j in J , as required.

First, efficiency requires that the budget constraint is satisfied with equality. Given independent (multivariate normal) distributions for θ and γ , we simply get

$$t_0 = R_0 - \sum_{j \in J} t_j \alpha_j \delta_j \ln((\beta_j - t_j) \alpha_j) - \sum_{j \in J} t_j \alpha_j \delta_j \mu_j^\gamma - \sum_{j \in J} t_j (1 - \alpha_j) \mu_j^\theta.$$

We can plug in this equation in the expression $\kappa(t_0, t; \alpha, \beta_0, \beta, \delta) - \kappa(R_0, 0; \alpha, \beta_0, \beta, \delta)$, to get

$$\sum_{j \in J} \alpha_j \beta_j \delta_j \ln\left(\frac{\beta_j - t_j}{\beta_j}\right) + \sum_{j \in J} t_j \alpha_j \delta_j (1 + \mu_j^\gamma) + \sum_{j \in J} t_j (1 - \alpha_j) \mu_j^\theta$$

and we can rewrite welfare $W = A + B + C$ with

$$\begin{aligned} A &= \frac{\sum_{j \in J} \alpha_j \beta_j \delta_j \ln\left(\frac{\beta_j - t_j}{\beta_j}\right) + \sum_{j \in J} t_j \alpha_j \delta_j (1 + \mu_j^\gamma) + \sum_{j \in J} t_j (1 - \alpha_j) \mu_j^\theta}{\sum_{k \in J} (1 - \alpha_k) \beta_k}, \\ B &= -\frac{1}{r} \ln \int \exp\left(\sum_{j \in J} \frac{rt_j \alpha_j \delta_j}{\sum_{k \in J} (1 - \alpha_k) \beta_k} \gamma_j\right) dG(\gamma), \\ C &= -\frac{1}{r} \ln \int \exp\left(\sum_{j \in J} \frac{-r(\beta_j - t_j)(1 - \alpha_j)}{\sum_{k \in J} (1 - \alpha_k) \beta_k} \theta_j\right) dF(\theta). \end{aligned}$$

Given a multivariate normal distribution for an arbitrary vector, say z with $z \sim N(\mu^z, \Sigma^z)$, we can use the following result

$$\ln \int \exp\left(\sum_{j \in J} a_j z_j\right) dF(z) = \sum_{j \in J} a_j \mu_j^z + \frac{1}{2} \sum_i \sum_j a_i a_j \sigma_{ij}^z,$$

to rewrite $W = A + B + C$ with

$$\begin{aligned}
A &= \frac{\sum_{j \in J} \alpha_j \beta_j \delta_j \ln \left(\frac{\beta_j - t_j}{\beta_j} \right) + \sum_{j \in J} t_j \alpha_j \delta_j (1 + \mu_j^\gamma) + \sum_{j \in J} t_j (1 - \alpha_j) \mu_j^\theta}{\sum_{k \in J} (1 - \alpha_k) \beta_k} \\
B &= -\sum_{j \in J} \frac{t_j \alpha_j \delta_j \mu_j^\gamma}{\sum_{k \in J} (1 - \alpha_k) \beta_k} - \frac{r \sum_i \sum_j t_i \alpha_i \delta_i t_j \alpha_j \delta_j \sigma_{ij}^\gamma}{2 \left(\sum_{k \in J} (1 - \alpha_k) \beta_k \right)^2} \\
C &= \sum_{j \in J} \frac{(\beta_j - t_j) (1 - \alpha_j) \mu_j^\theta}{\sum_{k \in J} (1 - \alpha_k) \beta_k} - \frac{r \sum_i \sum_j (\beta_i - t_i) (1 - \alpha_i) (\beta_j - t_j) (1 - \alpha_j) \sigma_{ij}^\theta}{2 \left(\sum_{k \in J} (1 - \alpha_k) \beta_k \right)^2}
\end{aligned}$$

Maximizing welfare leads to a system of equations, one for each j in J , defined as $\frac{\partial W}{\partial t_j} = \frac{\partial A}{\partial t_j} + \frac{\partial B}{\partial t_j} + \frac{\partial C}{\partial t_j} = 0$. Using the fact that

$$\frac{\partial}{\partial t_j} \left(\sum_i \sum_j \eta_i(t_i) \eta_j(t_j) \sigma_{ij}^\gamma \right) = 2 \frac{\partial \eta_j(t_j)}{\partial t_j} \sum_i \eta_i(t_i) \sigma_{ij}^\gamma,$$

we get

$$\begin{aligned}
\frac{\partial A}{\partial t_j} &= \frac{-\frac{\alpha_j \beta_j \delta_j}{\beta_j - t_j} + \alpha_j \delta_j (1 + \mu_j^\gamma) + (1 - \alpha_j) \mu_j^\theta}{\sum_{k \in J} (1 - \alpha_k) \beta_k}, \\
\frac{\partial B}{\partial t_j} &= -\frac{\alpha_j \delta_j \mu_j^\gamma}{\sum_{k \in J} (1 - \alpha_k) \beta_k} - \frac{r \alpha_j \delta_j \sum_k t_k \alpha_k \delta_k \sigma_{kj}^\gamma}{\left(\sum_{k \in J} (1 - \alpha_k) \beta_k \right)^2}, \\
\frac{\partial C}{\partial t_j} &= -\frac{(1 - \alpha_j) \mu_j^\theta}{\sum_{k \in J} (1 - \alpha_k) \beta_k} + \frac{r (1 - \alpha_j) \sum_k (\beta_k - t_k) (1 - \alpha_k) \sigma_{kj}^\theta}{\left(\sum_{k \in J} (1 - \alpha_k) \beta_k \right)^2}.
\end{aligned}$$

Putting everything together (and multiplying by $\zeta^2 := \left[\sum_{k \in J} (1 - \alpha_k) \beta_k \right]^2 > 0$), we get

$$-\alpha_j \delta_j \frac{t_j}{\beta_j - t_j} \zeta - r \alpha_j \delta_j \sum_k t_k \alpha_k \delta_k \sigma_{kj}^\gamma + r (1 - \alpha_j) \sum_k (\beta_k - t_k) (1 - \alpha_k) \sigma_{kj}^\theta = 0,$$

for each j in J .

Finally, to establish concavity, we directly focus on the case of two characteristics; the case of one characteristic can be seen from it as well:

$$\begin{aligned}
-\alpha_1 \delta_1 \frac{t_1}{\beta_1 - t_1} \zeta - r \alpha_1 \delta_1 \sum_k t_k \alpha_k \delta_k \sigma_{k1}^\gamma + r (1 - \alpha_1) \sum_k (\beta_k - t_k) (1 - \alpha_k) \sigma_{k1}^\theta &= 0 \\
-\alpha_2 \delta_2 \frac{t_2}{\beta_2 - t_2} \zeta - r \alpha_2 \delta_2 \sum_k t_k \alpha_k \delta_k \sigma_{k2}^\gamma + r (1 - \alpha_2) \sum_k (\beta_k - t_k) (1 - \alpha_k) \sigma_{k2}^\theta &= 0.
\end{aligned}$$

The Hessian matrix $H = (1/\zeta^2) \Upsilon$, and Υ has the following entries:

$$\begin{aligned}
\Upsilon_{11} &= -\alpha_1 \delta_1 \frac{\beta_1}{(\beta_1 - t_1)^2} \zeta - r (\alpha_1 \delta_1)^2 \sigma_{11}^\gamma - r (1 - \alpha_1)^2 \sigma_{11}^\theta, \\
\Upsilon_{12} &= \Upsilon_{21} = -r \alpha_1 \delta_1 \alpha_2 \delta_2 \sigma_{12}^\gamma - r (1 - \alpha_1) (1 - \alpha_2) \sigma_{12}^\theta, \\
\Upsilon_{22} &= -\alpha_2 \delta_2 \frac{\beta_2}{(\beta_2 - t_2)^2} \zeta - r (\alpha_2 \delta_2)^2 \sigma_{22}^\gamma - r (1 - \alpha_2)^2 \sigma_{22}^\theta.
\end{aligned}$$

To show that the Hessian matrix is negative semi-definite, we must have $\Upsilon_{11} \leq 0$, $\Upsilon_{22} \leq 0$ (which are true) and $|\Upsilon| = \Upsilon_{11} \Upsilon_{22} - (\Upsilon_{12})^2 \geq 0$. To show that $|\Upsilon| = \Upsilon_{11} \Upsilon_{22} - (\Upsilon_{12})^2 \geq 0$, note that the term $\Upsilon_{11} \Upsilon_{22}$ does not depend on the covariances and that Υ_{12} does not depend on β ; therefore the worst-case (read: smallest $|\Upsilon|$ possible) is obtained for $\sigma_{12}^\gamma = \sqrt{\sigma_{11}^\gamma \sigma_{22}^\gamma}$, $\sigma_{12}^\theta = \sqrt{\sigma_{11}^\theta \sigma_{22}^\theta}$ (maximal $(\Upsilon_{12})^2$) and $\beta \rightarrow 0$

(minimal $(Y_{11} Y_{22})$). Plugging in these values and manipulating the expression, we get a lower bound L for $|\Upsilon|$, with

$$\begin{aligned} L &= \left(r (\alpha_1 \delta_1)^2 \sigma_{11}^\gamma + r (1 - \alpha_1)^2 \sigma_{11}^\theta \right) \left(r (\alpha_2 \delta_2)^2 \sigma_{22}^\gamma + r (1 - \alpha_2)^2 \sigma_{22}^\theta \right) \\ &\quad - \left(r \alpha_1 \delta_1 \alpha_2 \delta_2 \sqrt{\sigma_{11}^\gamma \sigma_{22}^\gamma} + r (1 - \alpha_1) (1 - \alpha_2) \sqrt{\sigma_{11}^\theta \sigma_{22}^\theta} \right)^2 \\ &= r^2 \left(\alpha_1 \delta_1 (1 - \alpha_2) \sqrt{\sigma_{11}^\gamma \sigma_{22}^\theta} - \alpha_2 \delta_2 (1 - \alpha_1) \sqrt{\sigma_{11}^\theta \sigma_{22}^\gamma} \right)^2 \end{aligned}$$

which is non-negative, as required.

The Mirrlees-case

In case of one characteristic and $\beta_0 = 0$ and $\beta_1 = 1$, we get

$$-\alpha_1 \delta_1 \frac{t_1 (1 - \alpha_1)}{1 - t_1} - r (\alpha_1 \delta_1)^2 t_1 \sigma_{11}^\gamma + r (1 - \alpha_1)^2 (1 - t_1) \sigma_{11}^\theta = 0. \quad (24)$$

POINT 1. The optimal tax rate t_1^* on earnings x_1 lies in between the extremes of no taxation and complete taxation, i.e., $0 < t_1^* < 1$.

We know from proposition 1 that $t_1^* < 1$. In addition, also $t_1^* > 0$ must hold, since $t_1 \leq 0$ cannot satisfy the first-order condition.

POINT 2. The optimal tax rate t_1^* on earnings x_1 decreases with the elasticity δ_1 from complete taxation if the elasticity approaches zero ($t_1^* \rightarrow 1$ if $\delta_1 \rightarrow 0$) to no taxation if the elasticity becomes very high ($t_1^* \rightarrow 0$ if $\delta_1 \rightarrow +\infty$).

If $\delta_1 \rightarrow 0$, the first-order condition reduces to

$$r (1 - \alpha_1)^2 (1 - t_1) \sigma_{11}^\theta = 0,$$

which is satisfied for $t_1 \rightarrow 1$. If $\delta_1 \rightarrow +\infty$, the first-order condition reduces to (divide by $(\delta_1)^2 > 0$ and consider the limiting case $\delta_1 \rightarrow +\infty$)

$$-r (\alpha_1)^2 t_1 \sigma_{11}^\gamma = 0,$$

which is satisfied for $t_1 \rightarrow 0$. The comparative statics show that taxes decrease with δ_1 , since

$$\frac{dt_1}{d\delta_1} = -\frac{\frac{\partial(24)}{\partial\delta_1}}{\frac{\partial(24)}{\partial t_1}} = -\frac{-\alpha_1 \frac{t_1}{1-t_1} (1 - \alpha_1) - 2r\delta_1 (\alpha_1)^2 t_1 \sigma_{11}^\gamma}{-\alpha_1 \delta_1 (1 - \alpha_1) \left(\frac{1}{1-t_1} \right)^2 - r (\alpha_1 \delta_1)^2 \sigma_{11}^\gamma - r (1 - \alpha_1)^2 \sigma_{11}^\theta} < 0,$$

given $0 < t_1 < 1$.

POINT 3. The optimal tax rate t_1^* on earnings x_1 increases with the inequality aversion parameter r from no taxation if the planner is inequality neutral ($t_1^* \rightarrow 0$ if $r \rightarrow 0$) to partial taxation if income is fully controlled ($t_1^* \rightarrow \frac{(1-\alpha_1)^2 \sigma_{11}^\theta}{(\alpha_1 \delta_1)^2 \sigma_{11}^\gamma + (1-\alpha_1)^2 \sigma_{11}^\theta}$ if $r \rightarrow +\infty$).

If there is no inequality aversion ($r \rightarrow 0$), then the first-order condition equals

$$-\alpha_1 \delta_1 \frac{t_1}{1 - t_1} (1 - \alpha_1) = 0,$$

which is satisfied for $t_1 \rightarrow 0$. The other case ($r \rightarrow +\infty$) leads to (divide by $r > 0$ and take the limit)

$$-(\alpha_1 \delta_1)^2 t_1 \sigma_{11}^\gamma + (1 - \alpha_1)^2 (1 - t_1) \sigma_{11}^\theta = 0,$$

which can be solved to get

$$t_1^* = \frac{(1 - \alpha_1)^2 \sigma_{11}^\theta}{(\alpha_1 \delta_1)^2 \sigma_{11}^\gamma + (1 - \alpha_1)^2 \sigma_{11}^\theta}.$$

The comparative statics are

$$\frac{dt_1}{dr} = -\frac{\frac{\partial(24)}{\partial r}}{\frac{\partial(24)}{\partial t_1}} = -\frac{-(\alpha_1 \delta_1)^2 t_1 \sigma_{11}^\gamma + (1 - \alpha_1)^2 (1 - t_1) \sigma_{11}^\theta}{-\alpha_1 \delta_1 (1 - \alpha_1) \left(\frac{1}{1-t_1}\right)^2 - r (\alpha_1 \delta_1)^2 \sigma_{11}^\gamma - r (1 - \alpha_1)^2 \sigma_{11}^\theta}.$$

Using the first order condition, we can replace the numerator, to get

$$\frac{dt_1}{dr} = -\frac{\frac{1}{r} \alpha_1 \delta_1 \frac{t_1}{1-t_1} (1 - \alpha_1)}{-\alpha_1 \delta_1 (1 - \alpha_1) \left(\frac{1}{1-t_1}\right)^2 - r (\alpha_1 \delta_1)^2 \sigma_{11}^\gamma - r (1 - \alpha_1)^2 \sigma_{11}^\theta},$$

which is positive, given $0 < t_1 < 1$.

POINT 4. The optimal tax rate t_1^* on earnings x_1 increases with type heterogeneity σ_{11}^θ from no taxation if everyone has the same type ($t_1^* \rightarrow 0$ if $\sigma_{11}^\theta \rightarrow 0$) to complete taxation if types become very heterogeneous ($t_1^* \rightarrow 1$ if $\sigma_{11}^\theta \rightarrow +\infty$).

If $\sigma_{11}^\theta \rightarrow 0$, the first-order condition reduces to

$$-\alpha_1 \delta_1 \frac{t_1}{1-t_1} (1 - \alpha_1) - r (\alpha_1 \delta_1)^2 t_1 \sigma_{11}^\gamma = 0,$$

which is satisfied for $t_1 \rightarrow 0$. If $\sigma_{11}^\theta \rightarrow +\infty$, the first-order condition reduces to (divide by $\sigma_{11}^\theta > 0$ and consider the limiting case $\sigma_{11}^\theta \rightarrow +\infty$)

$$r (1 - \alpha_1)^2 (1 - t_1) = 0,$$

which is satisfied for $t_1 \rightarrow 1$. The comparative statics are

$$\frac{dt_1}{d\sigma_{11}^\theta} = -\frac{\frac{\partial(24)}{\partial \sigma_{11}^\theta}}{\frac{\partial(24)}{\partial t_1}} = -\frac{r (1 - \alpha_1)^2 (1 - t_1)}{-\alpha_1 \delta_1 (1 - \alpha_1) \left(\frac{1}{1-t_1}\right)^2 - r (\alpha_1 \delta_1)^2 \sigma_{11}^\gamma - r (1 - \alpha_1)^2 \sigma_{11}^\theta} > 0,$$

given $0 < t_1 < 1$.

POINT 5. The optimal tax rate t_1^* on earnings x_1 decreases with taste heterogeneity σ_{11}^γ from some taxation if everyone has the same taste ($0 < t_1^* < 1$ if $\sigma_{11}^\gamma \rightarrow 0$) to zero taxation if tastes become very heterogeneous ($t_1^* \rightarrow 0$ if $\sigma_{11}^\gamma \rightarrow +\infty$).

If there is no taste heterogeneity ($\sigma_{11}^\gamma \rightarrow 0$), then

$$-\alpha_1 \delta_1 \frac{t_1}{1-t_1} (1 - \alpha_1) + r (1 - \alpha_1)^2 (1 - t_1) \sigma_{11}^\theta = 0,$$

which can lead to any tax rate satisfying $0 < t_1^* < 1$. The other case ($\sigma_{11}^\gamma \rightarrow +\infty$) leads to (divide by $\sigma_{11}^\gamma > 0$ and consider the limiting case $\sigma_{11}^\gamma \rightarrow +\infty$)

$$-r (\alpha_1 \delta_1)^2 t_1 \sigma_{11}^\gamma = 0,$$

which holds for $t_1 \rightarrow 0$. Taxes decrease with σ_{11}^γ , since

$$\frac{dt_1}{d\sigma_{11}^\gamma} = -\frac{\frac{\partial(24)}{\partial\sigma_{11}^\gamma}}{\frac{\partial(24)}{\partial t_1}} = -\frac{-r(\alpha_1\delta_1)^2 t_1}{-\alpha_1\delta_1(1-\alpha_1)\left(\frac{1}{1-t_1}\right)^2 - r(\alpha_1\delta_1)^2\sigma_{11}^\gamma - r(1-\alpha_1)^2\sigma_{11}^\theta} < 0$$

given $0 < t_1 < 1$.

POINT 6. The optimal tax rate t_1^* on earnings x_1 decreases with the degree of control α_1 from complete taxation if earnings cannot be controlled ($t_1^* \rightarrow 1$ if $\alpha_1 \rightarrow 0$) to no taxation if income is fully controlled ($t_1^* \rightarrow 0$ if $\alpha_1 \rightarrow 1$).

If $\alpha_1 \rightarrow 0$, the first-order condition reduces to

$$r(1-\alpha_1)^2(1-t_1)\sigma_{11}^\theta = 0,$$

which is satisfied for $t_1 \rightarrow 1$. If $\alpha_1 \rightarrow 1$, the first-order condition reduces to

$$-r(\delta_1)^2 t_1 \sigma_{11}^\gamma = 0,$$

which is satisfied for $t_1 \rightarrow 0$. The comparative statics are

$$\frac{dt_1^*}{d\alpha_1} = -\frac{\frac{\partial(24)}{\partial\alpha_1}}{\frac{\partial(24)}{\partial t_1}} = -\frac{\delta_1 \frac{t_1}{1-t_1} (2\alpha_1 - 1) - 2r\alpha_1(\delta_1)^2 t_1 \sigma_{11}^\gamma - 2r(1-\alpha_1)(1-t_1)\sigma_{11}^\theta}{-\alpha_1\delta_1(1-\alpha_1)\left(\frac{1}{1-t_1}\right)^2 - r(\alpha_1\delta_1)^2\sigma_{11}^\gamma - r(1-\alpha_1)^2\sigma_{11}^\theta}.$$

Dividing both sides by $(1-\alpha_1)\alpha_1 > 0$ and using the first-order condition to replace $\delta_1 \frac{t_1}{1-t_1}$, we get

$$\begin{aligned} \frac{dt_1}{d\alpha_1} &= -\frac{(1-\alpha_1)\alpha_1 \left\{ \left[-\frac{r\alpha_1(\delta_1)^2 t_1 \sigma_{11}^\gamma}{1-\alpha_1} + \frac{r(1-\alpha_1)(1-t_1)\sigma_{11}^\theta}{\alpha_1} \right] (2\alpha_1 - 1) - 2r\alpha_1(\delta_1)^2 t_1 \sigma_{11}^\gamma - 2r(1-\alpha_1)(1-t_1)\sigma_{11}^\theta \right\}}{(1-\alpha_1)\alpha_1 \left(-\alpha_1\delta_1(1-\alpha_1)\left(\frac{1}{1-t_1}\right)^2 - r(\alpha_1\delta_1)^2\sigma_{11}^\gamma - r(1-\alpha_1)^2\sigma_{11}^\theta \right)} \\ &= -\frac{-\left(r(\alpha_1\delta_1)^2 t_1 \sigma_{11}^\gamma + r(1-\alpha_1)^2(1-t_1)\sigma_{11}^\theta \right)}{-\left((1-\alpha_1)\alpha_1 \left(\alpha_1\delta_1(1-\alpha_1)\left(\frac{1}{1-t_1}\right)^2 + r(\alpha_1\delta_1)^2\sigma_{11}^\gamma + r(1-\alpha_1)^2\sigma_{11}^\theta \right) \right)}, \end{aligned}$$

which is negative, given point 1 ($0 < t_1 < 1$).

The Akerlof-case

Suppose there are two variables, earnings x_1 and an exogenous tag x_2 (thus, $\alpha_2 \rightarrow 0$). The first-order conditions reduce to

$$\begin{aligned} -\alpha_1\delta_1 \frac{\zeta t_1}{1-t_1} - r(\alpha_1\delta_1)^2 t_1 \sigma_{11}^\gamma + r(1-\alpha_1) \left((1-t_1)(1-\alpha_1)\sigma_{11}^\theta + (\beta_2 - t_2)\sigma_{21}^\theta \right) &= 0, \\ (1-t_1)(1-\alpha_1)\sigma_{12}^\theta + (\beta_2 - t_2)\sigma_{22}^\theta &= 0, \end{aligned}$$

with $\zeta = (1-\alpha_1) + \beta_2$. The second of the first-order conditions can be rewritten as

$$\begin{aligned} t_2 &= \beta_2 + (1-t_1)(1-\alpha_1) \frac{\sigma_{12}^\theta}{\sigma_{22}^\theta} \\ &= \beta_2 + \rho_{12}^\theta \sqrt{\frac{\sigma_{11}^\theta}{\sigma_{22}^\theta}} (1-t_1)(1-\alpha_1), \end{aligned} \tag{25}$$

with $\rho_{12}^\theta = \frac{\sigma_{12}^\theta}{\sqrt{\sigma_{11}^\theta \sigma_{22}^\theta}}$ the type correlation. This can be plugged in in the other first-order condition to get

$$-\alpha_1 \delta_1 \frac{\zeta t_1}{1-t_1} - r (\alpha_1 \delta_1)^2 t_1 \sigma_{11}^\gamma + r (1-\alpha_1)^2 (1-t_1) \sigma_{11}^\theta \left(1 - (\rho_{12}^\theta)^2\right) = 0, \quad (26)$$

The latter equation does not depend on t_2 and therefore completely describes the solution for t_1 , which can afterwards be plugged in in (25) to obtain a solution for t_2 . Before proceeding, note that we consider $\sigma_{11}^\theta, \sigma_{22}^\theta$ and $\rho_{12}^\theta = \frac{\sigma_{12}^\theta}{\sqrt{\sigma_{11}^\theta \sigma_{22}^\theta}}$ as primitives of the model and $\sigma_{12}^\theta = \rho_{12}^\theta \sqrt{\sigma_{11}^\theta \sigma_{22}^\theta}$ adjusts.⁸

POINT 1. From proposition 1, we already know that $t_1 < 1$ in the optimum, and it is easy to verify that $t_1 < 0$ cannot satisfy equation (26). To summarize, we must have $0 < t_1 < 1$. As a consequence, we also have $t_2 \begin{smallmatrix} \geq \\ \leq \end{smallmatrix} \beta_2$ if $\rho_{12}^\theta \begin{smallmatrix} \geq \\ \leq \end{smallmatrix} 0$.

POINT 2. The tax rate on earnings t_1^* decreases with the degree of control α_1 , ranging from full taxation if earnings cannot be controlled ($t_1^* \rightarrow 1$ if $\alpha_1 \rightarrow 0$) to no taxation if income is fully controlled ($t_1^* \rightarrow 0$ if $\alpha_1 \rightarrow 1$); the tag is fully taxed, both if there is no control over earnings and if there is full control over earnings ($t_2^* \rightarrow \beta_2$, if either $\alpha_1 \rightarrow 0$ or $\alpha_1 \rightarrow 1$), but the change is undefined in general. We only know that, at $\alpha_1 \rightarrow 0$, the tax rate t_2^* increases (resp. decreases) with α_1 if the type correlation is positive (resp. negative) and vice-versa at $\alpha_1 \rightarrow 1$.

If $\alpha_1 \rightarrow 0$, condition (26) reduces to

$$(1-t_1) \left(1 - (\rho_{12}^\theta)^2\right) = 0,$$

which implies $t_1 \rightarrow 1$ and, using $t_1 \rightarrow 1$ in in (25), we get $t_2 \rightarrow \beta_2$. If $\alpha_1 \rightarrow 1$, condition (26) reduces to

$$-\delta_1 \frac{\zeta t_1}{1-t_1} - r (\delta_1)^2 t_1 \sigma_{11}^\gamma = 0,$$

which is satisfied for $t_1 \rightarrow 0$ and this leads to $t_2 \rightarrow \beta_2 + (1-\alpha_1) \frac{\sigma_{12}^\theta}{\sigma_{22}^\theta}$. The comparative statics for t_1 w.r.t. α_1 are

$$\frac{dt_1^*}{d\alpha_1} = -\frac{\frac{\partial eq(26)}{\partial \alpha_1}}{\frac{\partial eq(26)}{\partial t_1}} = -\frac{-\delta_1 \frac{t_1}{1-t_1} (1-2\alpha_1+\beta_2) - 2r\alpha_1 (\delta_1)^2 t_1 \sigma_{11}^\gamma - 2r(1-\alpha_1)(1-t_1) \sigma_{11}^\theta \left(1 - (\rho_{12}^\theta)^2\right)}{-\frac{\alpha_1 \delta_1 (1-\alpha_1+\beta_2)}{(1-t_1)^2} - r (\alpha_1 \delta_1)^2 \sigma_{11}^\gamma - r (1-\alpha_1)^2 \sigma_{11}^\theta \left(1 - (\rho_{12}^\theta)^2\right)}.$$

We can divide both sides by $\alpha_1 \zeta = \alpha_1 (1-\alpha_1+\beta_2) > 0$ and using the first-order condition to replace $-\alpha_1 \delta_1 \frac{\zeta t_1}{1-t_1}$, we get (after some manipulation) that

$$\frac{dt_1^*}{d\alpha_1} = -\frac{-(1+\beta_2)r(\alpha_1 \delta_1)^2 t_1 \sigma_{11}^\gamma - [(1-\alpha_1)(1+\beta_2) + 2\alpha_1 \beta_2] r (1-\alpha_1)(1-t_1) \sigma_{11}^\theta \left(1 - (\rho_{12}^\theta)^2\right)}{-\alpha_1 (1-\alpha_1+\beta_2) \left(\frac{\alpha_1 \delta_1 (1-\alpha_1+\beta_2)}{(1-t_1)^2} + r (\alpha_1 \delta_1)^2 \sigma_{11}^\gamma + r (1-\alpha_1)^2 \sigma_{11}^\theta \left(1 - (\rho_{12}^\theta)^2\right)\right)}$$

which is negative, given $0 < t_1 < 1$. The comparative statics for t_2 w.r.t. α_1 are

$$\frac{dt_2^*}{d\alpha_1} = \frac{\partial eq(25)}{\partial \alpha_1} + \frac{\partial eq(25)}{\partial t_1} \frac{dt_1}{d\alpha_1} = -\sigma_{12}^\theta \left(\frac{1-t_1}{\sigma_{22}^\theta} + \frac{1-\alpha_1}{\sigma_{22}^\theta} \frac{dt_1}{d\alpha_1}\right),$$

⁸The constraints on σ_{11}^θ ($-\sqrt{\sigma_{11}^\theta \sigma_{22}^\theta} \leq \sigma_{12}^\theta \leq \sqrt{\sigma_{11}^\theta \sigma_{22}^\theta}$) depend on (and thus move with) changes in σ_{11}^θ and σ_{22}^θ , which could complicate the comparative statics. This is not true for the constraints on ρ_{12}^θ (i.e., $-1 \leq \rho_{12}^\theta \leq 1$).

which is not defined in general. At $\alpha_1 \rightarrow 0$ (& thus, $t_1 \rightarrow \beta_1$), the derivative $\frac{dt_2}{d\alpha_1}$ equals $-\sigma_{12}^\theta \left(\frac{1}{\sigma_{22}^\theta} \frac{dt_1}{d\alpha_1} \right)$, so $\frac{dt_2}{d\alpha_1}$ is positive (resp. negative) if the type covariance/correlation is positive (resp. negative), while at $\alpha_1 \rightarrow 1$ (& thus, $t_1 \rightarrow 0$), $\frac{dt_2}{d\alpha_1}$ equals $-\sigma_{12}^\theta \left(\frac{1}{\sigma_{22}^\theta} \right)$ which leads to the opposite sign.

POINT 3. The tax rate on earnings t_1^* decreases with the cost of taxation δ_1 , ranging from $t_1 \rightarrow 1$ (if $\delta_1 \rightarrow 0$) to $t_1 \rightarrow 0$ (if $\delta_1 \rightarrow +\infty$). The tax rate t_2^* on the tag increases (resp. decreases) with the earnings elasticity δ_1 if the type correlation is positive (resp. negative), ranging from $t_2 = \beta_2$ (if $\delta_1 \rightarrow 0$) to $\beta_2 + \rho_{12}^\theta \sqrt{\frac{\sigma_{11}^\theta}{\sigma_{22}^\theta}} (1 - \alpha_1)$ (if $\delta_1 \rightarrow +\infty$).

If $\delta_1 \rightarrow 0$, condition (26) reduces to

$$(1 - t_1) \left(1 - (\rho_{12}^\theta)^2 \right) = 0,$$

which implies $t_1 \rightarrow 1$ and $t_2 \rightarrow t_2 = \beta_2$. If $\delta_1 \rightarrow +\infty$, we get

$$-r (\alpha_1)^2 t_1 \sigma_{11}^\gamma = 0,$$

which implies $t_1 \rightarrow 0$ and this leads to $t_2 = \beta_2 + \rho_{12}^\theta \sqrt{\frac{\sigma_{11}^\theta}{\sigma_{22}^\theta}} (1 - \alpha_1)$. The comparative statics for t_1 w.r.t. δ_1 are

$$\frac{dt_1^*}{d\delta_1} = - \frac{-\alpha_1 \frac{\zeta t_1}{1-t_1} - 2r\delta_1 (\alpha_1)^2 t_1 \sigma_{11}^\gamma}{-\frac{\alpha_1 \delta_1 (1-\alpha_1 + \beta_2)}{(1-t_1)^2} - r (\alpha_1 \delta_1)^2 \sigma_{11}^\gamma - r (1-\alpha_1)^2 \sigma_{11}^\theta \left(1 - (\rho_{12}^\theta)^2 \right)},$$

which is negative, i.e., the more elastic the lower the tax. The comparative statics for t_2 w.r.t. δ_1 are

$$\frac{dt_2^*}{d\delta_1} = \frac{\partial eq(25)}{\partial \delta_1} + \frac{\partial eq(25)}{\partial t_1} \frac{dt_1^*}{d\delta_1} = -\sigma_{12}^\theta \frac{1 - \alpha_1}{\sigma_{22}^\theta} \frac{dt_1}{d\delta_1},$$

the sign of which corresponds with the sign of the correlation.

POINT 4. The tax rate on earnings t_1^* increases with the type heterogeneity σ_{11}^θ for earnings, from $t_1 \rightarrow 0$ to $t_1 \rightarrow 1$; the tax rate on the tag t_2^* equals β_2 if there is no type heterogeneity σ_{11}^θ for earnings, while the comparative statics are undefined.

If $\sigma_{11}^\theta \rightarrow 0$ (and recall that $\sigma_{12}^\theta = \rho_{12}^\theta \sqrt{\sigma_{11}^\theta \sigma_{22}^\theta}$ adjusts to 0, leaving ρ_{12}^θ unchanged) then condition (26) reduces to

$$-\alpha_1 \delta_1 \frac{\zeta t_1}{1-t_1} - r (\alpha_1 \delta_1)^2 t_1 \sigma_{11}^\gamma = 0,$$

which leads to $t_1 \rightarrow 0$ and $t_2 \rightarrow \beta_2$. If $\sigma_{11}^\theta \rightarrow +\infty$, then condition (26) reduces to

$$r (1 - \alpha_1)^2 (1 - t_1) \left(1 - (\rho_{12}^\theta)^2 \right) = 0,$$

which implies $t_1 \rightarrow 1$ and t_2 undefined (since both $\sigma_{11}^\theta \rightarrow +\infty$ and $1 - t_1 \rightarrow 0$). The comparative statics for t_1^* w.r.t. σ_{11}^θ are equal to

$$\frac{dt_1^*}{d\sigma_{11}^\theta} = - \frac{r (1 - \alpha_1)^2 (1 - t_1) \left(1 - (\rho_{12}^\theta)^2 \right)}{-\frac{\alpha_1 \delta_1 (1-\alpha_1 + \beta_2)}{(1-t_1)^2} - r (\alpha_1 \delta_1)^2 \sigma_{11}^\gamma - r (1-\alpha_1)^2 \sigma_{11}^\theta \left(1 - (\rho_{12}^\theta)^2 \right)},$$

which is positive. The comparative statics for t_2 w.r.t. δ_1 are

$$\frac{dt_2^*}{d\sigma_{11}^\theta} = \frac{\partial eq(25)}{\partial \sigma_{11}^\theta} + \frac{\partial eq(25)}{\partial t_1} \frac{dt_1^*}{d\sigma_{11}^\theta} = \rho_{12}^\theta (1 - \alpha_1) \sqrt{\frac{\sigma_{11}^\theta}{\sigma_{22}^\theta}} \left(\frac{(1 - t_1)}{2\sigma_{11}^\theta} - \sqrt{\frac{\sigma_{11}^\theta}{\sigma_{22}^\theta}} \frac{dt_1^*}{d\sigma_{11}^\theta} \right),$$

the sign of which is not defined.

POINT 5. The tax rate on earnings t_1^* does not change with σ_{22}^θ . The tax rate on the tag t_2^* increases (resp. decreases) with σ_{22}^θ if the type correlation is negative (resp. positive).

Condition (26) does not change with σ_{22}^θ , indicating that t_1^* remains unchanged as well, thus $\frac{dt_1^*}{d\sigma_{22}^\theta} = 0$. The tax rate on the tag increases (resp. decreases) with σ_{22}^θ if the correlation is negative (resp. positive), which can be seen from

$$\frac{dt_2^*}{d\sigma_{22}^\theta} = \frac{\partial eq(25)}{\partial \sigma_{22}^\theta} + \frac{\partial eq(25)}{\partial t_1} \frac{dt_1^*}{d\sigma_{22}^\theta} = -\frac{\rho_{12}^\theta}{2\sigma_{22}^\theta} \sqrt{\frac{\sigma_{11}^\theta}{\sigma_{22}^\theta}} (1-t_1)(1-\alpha_1),$$

the sign of which is the opposite to the sign of the type correlation ρ_{12}^θ .

POINT 6. The tax rate on earnings t_1^* increases with ρ_{12}^θ if ρ_{12}^θ is negative, and t_1^* decreases with ρ_{12}^θ if ρ_{12}^θ is positive. At the extremes ($(\rho_{12}^\theta)^2 = 1$) the same tax rate $t_1^* = 0$ on earnings applies; the tax rate on the tag t_2^* increases from $\beta_2 - \sqrt{\frac{\sigma_{11}^\theta}{\sigma_{22}^\theta}} (1-\alpha_1)$ to $\beta_2 + \sqrt{\frac{\sigma_{11}^\theta}{\sigma_{22}^\theta}} (1-\alpha_1)$;

At the extremes ($\rho_{12}^\theta = \pm 1$), condition (26) reduces to

$$-\alpha_1 \delta_1 \frac{\zeta t_1}{1-t_1} - r (\alpha_1 \delta_1)^2 t_1 \sigma_{11}^\gamma = 0,$$

which implies $t_1 \rightarrow 0$. Note that

$$\frac{dt_1^*}{d\rho_{12}^\theta} = -\frac{-2r(1-\alpha_1)^2(1-t_1)\sigma_{11}^\theta\rho_{12}^\theta}{-\frac{\alpha_1\delta_1(1-\alpha_1+\beta_2)}{(1-t_1)^2} - r(\alpha_1\delta_1)^2\sigma_{11}^\gamma - r(1-\alpha_1)^2\sigma_{11}^\theta(1-(\rho_{12}^\theta)^2)},$$

the sign of which is inversely related to ρ_{12}^θ . The comparative statics for the tax rate on the tag equals

$$\frac{dt_2^*}{d\rho_{12}^\theta} = \frac{\partial eq(25)}{\partial \rho_{12}^\theta} + \frac{\partial eq(25)}{\partial t_1} \frac{dt_1^*}{d\rho_{12}^\theta} = \sqrt{\frac{\sigma_{11}^\theta}{\sigma_{22}^\theta}} (1-\alpha_1) \left((1-t_1) - \rho_{12}^\theta \frac{dt_1^*}{d\rho_{12}^\theta} \right),$$

which is positive (since $\rho_{12}^\theta \frac{dt_1^*}{d\rho_{12}^\theta} \leq 0$), increasing from $\beta_2 - \sqrt{\frac{\sigma_{11}^\theta}{\sigma_{22}^\theta}} (1-\alpha_1)$ to $\beta_2 + \sqrt{\frac{\sigma_{11}^\theta}{\sigma_{22}^\theta}} (1-\alpha_1)$.

POINT 7. The tax rate on earnings t_1^* and the tax rate on the tag t_2^* do not depend on σ_{22}^γ and ρ_{12}^γ , but decreases with taste heterogeneity for earnings σ_{11}^γ ; the tax rate for the tag t_2^* increases (resp. decreases in case $\rho_{12}^\theta < 0$) with σ_{11}^γ to reach $\beta_2 + \rho_{12}^\theta \sqrt{\frac{\sigma_{11}^\theta}{\sigma_{22}^\theta}} (1-\alpha_1)$ if $\sigma_{11}^\gamma \rightarrow +\infty$.

If $\sigma_{11}^\gamma \rightarrow 0$, then condition (26) reduces to

$$-\alpha_1 \delta_1 \frac{\zeta t_1}{1-t_1} + r(1-\alpha_1)^2(1-t_1)\sigma_{11}^\theta(1-(\rho_{12}^\theta)^2) = 0,$$

which does not give a clear prescription. If $\sigma_{11}^\gamma \rightarrow +\infty$, then condition (26) reduces to

$$-r(\alpha_1\delta_1)^2 t_1 = 0,$$

which implies $t_1 \rightarrow 0$ and $t_2 \rightarrow \beta_2 + \rho_{12}^\theta \sqrt{\frac{\sigma_{11}^\theta}{\sigma_{22}^\theta}} (1-\alpha_1)$. Comparative statics are

$$\frac{dt_1^*}{d\sigma_{11}^\gamma} = -\frac{-r(\alpha_1\delta_1)^2 t_1}{-\frac{\alpha_1\delta_1(1-\alpha_1+\beta_2)}{(1-t_1)^2} - r(\alpha_1\delta_1)^2\sigma_{11}^\gamma - r(1-\alpha_1)^2\sigma_{11}^\theta(1-(\rho_{12}^\theta)^2)},$$

which is negative, as required, and

$$\frac{dt_2^*}{d\sigma_{11}^\gamma} = \frac{\partial eq(25)}{\partial \sigma_{11}^\gamma} + \frac{\partial eq(25)}{\partial t_1} \frac{dt_1^*}{d\sigma_{11}^\gamma} = -\rho_{12}^\theta \sqrt{\frac{\sigma_{11}^\theta}{\sigma_{22}^\theta}} (1 - \alpha_1) \frac{dt_1^*}{d\sigma_{11}^\gamma}$$

the sign of which is the same as the sign of ρ_{12}^θ .

POINT 8. The tax rate on earnings t_1^* increases with the inequality aversion r , from $t_1 \rightarrow 0$ to $t_1 \rightarrow \frac{(1-\alpha_1)^2 \sigma_{11}^\theta (1-(\rho_{12}^\theta)^2)}{(\alpha_1 \delta_1)^2 \sigma_{11}^\gamma + (1-\alpha_1)^2 \sigma_{11}^\theta (1-(\rho_{12}^\theta)^2)}$; the tax rate on the tag increases (resp. decreases) with r from $\beta_2 + \rho_{12}^\theta \sqrt{\frac{\sigma_{11}^\theta}{\sigma_{22}^\theta}} (1 - \alpha_1)$ to $\beta_2 + \rho_{12}^\theta \sqrt{\frac{\sigma_{11}^\theta}{\sigma_{22}^\theta}} \frac{(\alpha_1 \delta_1)^2 \sigma_{11}^\gamma}{(\alpha_1 \delta_1)^2 \sigma_{11}^\gamma + (1-\alpha_1)^2 \sigma_{11}^\theta (1-(\rho_{12}^\theta)^2)}$ if the correlation is positive (resp. negative).

If $r \rightarrow 0$, then condition (26) reduces to

$$-\alpha_1 \delta_1 \frac{\zeta t_1}{1 - t_1} = 0,$$

which implies $t_1 \rightarrow 0$ and $t_2 \rightarrow \beta_2 + \rho_{12}^\theta \sqrt{\frac{\sigma_{11}^\theta}{\sigma_{22}^\theta}} (1 - \alpha_1)$. If $r \rightarrow +\infty$, then condition (26) directly implies

$$t_1 = \frac{(1 - \alpha_1)^2 \sigma_{11}^\theta (1 - (\rho_{12}^\theta)^2)}{(\alpha_1 \delta_1)^2 \sigma_{11}^\gamma + (1 - \alpha_1)^2 \sigma_{11}^\theta (1 - (\rho_{12}^\theta)^2)},$$

and t_2 equals

$$\beta_2 + \rho_{12}^\theta \sqrt{\frac{\sigma_{11}^\theta}{\sigma_{22}^\theta}} \frac{(\alpha_1 \delta_1)^2 \sigma_{11}^\gamma}{(\alpha_1 \delta_1)^2 \sigma_{11}^\gamma + (1 - \alpha_1)^2 \sigma_{11}^\theta (1 - (\rho_{12}^\theta)^2)}.$$

Comparative statics are

$$\frac{dt_1^*}{dr} = -\frac{-(\alpha_1 \delta_1)^2 t_1 \sigma_{11}^\gamma + (1 - \alpha_1)^2 (1 - t_1) \sigma_{11}^\theta (1 - (\rho_{12}^\theta)^2)}{-\frac{\alpha_1 \delta_1 (1 - \alpha_1 + \beta_2)}{(1 - t_1)^2} - r (\alpha_1 \delta_1)^2 \sigma_{11}^\gamma - r (1 - \alpha_1)^2 \sigma_{11}^\theta (1 - (\rho_{12}^\theta)^2)},$$

and using condition (), we get

$$\frac{dt_1^*}{dr} = -\frac{\alpha_1 \delta_1 \frac{\zeta t_1}{1 - t_1} \frac{1}{r}}{-\frac{\alpha_1 \delta_1 (1 - \alpha_1 + \beta_2)}{(1 - t_1)^2} - r (\alpha_1 \delta_1)^2 \sigma_{11}^\gamma - r (1 - \alpha_1)^2 \sigma_{11}^\theta (1 - (\rho_{12}^\theta)^2)},$$

which is positive, as required, and

$$\frac{dt_2^*}{dr} = \frac{\partial eq(25)}{\partial r} + \frac{\partial eq(25)}{\partial t_1} \frac{dt_1^*}{dr} = -\rho_{12}^\theta \sqrt{\frac{\sigma_{11}^\theta}{\sigma_{22}^\theta}} (1 - \alpha_1) \frac{dt_1^*}{dr}$$

the sign of which is the same as the sign of ρ_{12}^θ .

Data Appendix

4.1 A. Data construction

1. Pre-tax household income is the sum (at household level) of the remuneration of labour (earnings) and capital (rents), more precisely, the sum of

- (a) (gross) employee cash or near cash income,
 - (b) (gross) non-cash employee income,⁹
 - (c) employer's social insurance contributions,¹⁰
 - (d) (gross) cash benefits or losses from self-employment,
 - (e) (gross) rental income,
 - (f) (gross) interest, dividends and profit from capital investments in unincorporated business;
2. Post-tax household income is the pre-tax household income + the sum of (gross) benefits - taxes and social insurance contributions, more precisely, pre-tax household income

PLUS

- (a) (gross) unemployment benefits,
- (b) (gross) old-age and survivor benefits,
- (c) (gross) sickness and disability benefits,
- (d) (gross) education-related allowances,
- (e) (gross) child allowances,
- (f) (gross) other benefits (e.g., guaranteed minimum income),

MINUS

- (a) employer's social insurance contributions,
- (b) tax on income (including taxes on holdings and tax reimbursements) and (employee's) social security contributions.

3. To obtain equivalent (pre- or post-tax) income, we divide (pre- or post-tax) income by the (modified) OECD scale, i.e., $1 + 0.5 \times (\# \text{ of additional adults } (age \geq 14)) + 0.3 \times (\# \text{ of children } (age < 14))$.

4.2 B. summary statistics

⁹Imputed for the Netherlands on the basis of EU-SILC 2006 data.

¹⁰Imputed for Germany, Latvia and the UK.

| Concept | Definition / Imputation |
|--|--|
| Wages and Salaries | Gross employee cash or near cash income (including e.g. holiday payments, pay for overtime, bonuses etc.) plus non-cash employee income (e.g. company car, free or subsidized meals etc.). |
| Self-employment Income | Net operating profit or loss accruing to working owners of, or partners in, an unincorporated enterprise less interest on business loans; royalties earned on writing and inventions as well as rentals from business buildings, vehicles, equipment etc. |
| Capital Income | Imputed rent; income from rental of a property or land; interest, dividends, profits from capital investment in an unincorporated business; regular inter-household cash transfers received. |
| Social Insurance Contributions Employer | Payments made by the employers for the benefits of their employees to insurers (social security funds and private funded schemes) covering statutory, conventional or contractual contributions in respect of insurance against social risks. Information on the amount of social insurance contributions paid by the employer is not reported for DE, LT and the UK. In these cases, we use country-specific legal rules to impute the SIC paid by the employer based on the corresponding employee income. |
| Public Pensions | Old-age benefits (any replacement income when the aged person retires from the labor market, care allowances etc.) and survivor's benefits (such as survivor's pension and death grants). |
| Cash Benefits | Unemployment benefits, sickness benefits, disability benefits, education-related allowances; family/children related allowances, housing allowances, benefits for social exclusion not elsewhere classified (periodic income support for people with insufficient resources and other related cash benefits). |
| Income taxes | Taxes on income, profits and capital gains, assessed on the actual or presumed income of individuals, households or tax-units. EU-SILC only reports income taxes and employee SIC as an aggregated value. We subtract imputed SIC to isolate income tax payments as a single variable. |
| Total Social Insurance Contributions | Employer's SIC (see above) and employees' SIC (any contributions to either mandatory government or employer-based social insurance schemes. EU-SILC does not report SIC paid by the employee as a separate variable, therefore values are imputed (see above) applying the appropriate legal rules of each country. |

Table 3: Income concepts

| | N | g | g m | m | n | n m | t | t m |
|----|---------|---------|---------|-----|---------|---------|---------|---------|
| AT | 7576.0 | 33702.9 | 19728.6 | 1.6 | 31846.7 | 19806.5 | 1856.3 | -77.9 |
| BE | 7489.0 | 42850.8 | 24423.3 | 1.6 | 31613.5 | 18990.5 | 11237.3 | 5432.7 |
| CY | 3535.0 | 30558.1 | 15626.5 | 1.8 | 30899.2 | 16688.3 | -341.1 | -1061.8 |
| CZ | 11539.0 | 9535.9 | 5466.2 | 1.6 | 8960.6 | 5546.2 | 575.3 | -80.0 |
| DE | 18315.0 | 33299.0 | 19929.8 | 1.6 | 30973.2 | 19482.0 | 2325.8 | 447.8 |
| DK | 7891.0 | 66861.0 | 37421.4 | 1.7 | 47062.1 | 27301.3 | 19798.9 | 10120.1 |
| EE | 5625.0 | 9551.4 | 5218.5 | 1.7 | 8004.4 | 4591.2 | 1547.1 | 627.2 |
| ES | 11524.0 | 25148.7 | 13958.1 | 1.7 | 22531.1 | 13243.0 | 2617.5 | 715.1 |
| FI | 11768.0 | 48815.6 | 27380.5 | 1.7 | 37491.8 | 21823.0 | 11323.8 | 5557.5 |
| FR | 13335.0 | 36173.6 | 20352.0 | 1.7 | 31991.5 | 19128.7 | 4182.1 | 1223.3 |
| GR | 4832.0 | 22603.7 | 12388.7 | 1.7 | 18483.0 | 10964.9 | 4120.7 | 1423.9 |
| HU | 9483.0 | 6508.7 | 3698.1 | 1.6 | 6705.4 | 4129.6 | -196.7 | -431.6 |
| IE | 5589.0 | 36347.7 | 20440.8 | 1.6 | 39528.3 | 23984.9 | -3180.6 | -3544.0 |
| IS | 3276.0 | 82985.7 | 44608.4 | 1.8 | 60351.9 | 33227.5 | 22633.8 | 11380.9 |
| IT | 18564.0 | 30133.7 | 17255.5 | 1.6 | 27194.1 | 16756.5 | 2939.5 | 499.0 |
| LT | 5634.0 | 7054.4 | 4021.3 | 1.6 | 5989.4 | 3610.2 | 1065.1 | 411.1 |
| LU | 4939.0 | 60616.9 | 34840.5 | 1.7 | 56109.4 | 32851.6 | 4507.5 | 1988.9 |
| LV | 4528.0 | 5615.4 | 3302.9 | 1.6 | 5244.4 | 3248.3 | 371.0 | 54.6 |
| NL | 13960.0 | 52601.4 | 29943.4 | 1.7 | 36387.1 | 21734.8 | 16214.3 | 8208.6 |
| NO | 7868.0 | 66171.7 | 37402.5 | 1.7 | 55237.4 | 32217.4 | 10934.3 | 5185.1 |
| PL | 13310.0 | 6657.9 | 3560.9 | 1.7 | 6915.5 | 4084.0 | -257.5 | -523.1 |
| PT | 3838.0 | 14461.7 | 7933.2 | 1.7 | 15744.5 | 9364.2 | -1282.8 | -1431.0 |
| SE | 9304.0 | 44606.4 | 25656.4 | 1.7 | 34484.1 | 20514.8 | 10122.3 | 5141.6 |
| SI | 8095.0 | 23123.3 | 11486.3 | 1.8 | 20062.2 | 10776.7 | 3061.1 | 709.6 |
| SK | 4723.0 | 7441.7 | 3894.4 | 1.7 | 6965.6 | 4007.6 | 476.1 | -113.2 |
| UK | 10720.0 | 42760.1 | 25478.5 | 1.6 | 38099.1 | 23831.6 | 4660.9 | 1646.9 |

Table 4: Income all

| | sex | age2635 | age3645 | age4655 | age5665 | age6675 | age76plus | disabled | oth birth | educ2 | educ3 | educ4 | smallchild | mic |
|----|------|---------|---------|---------|---------|---------|-----------|----------|-----------|-------|-------|-------|------------|------|
| AT | 0.51 | 0.14 | 0.23 | 0.16 | 0.17 | 0.14 | 0.11 | 0.14 | 0.08 | 0.20 | 0.52 | 0.27 | 0.09 | 0.00 |
| BE | 0.32 | 0.17 | 0.22 | 0.18 | 0.16 | 0.14 | 0.09 | 0.11 | 0.05 | 0.15 | 0.34 | 0.37 | 0.10 | 0.00 |
| CY | 0.24 | 0.16 | 0.23 | 0.17 | 0.16 | 0.16 | 0.11 | 0.13 | 0.05 | 0.07 | 0.33 | 0.28 | 0.11 | 0.10 |
| CZ | 0.36 | 0.15 | 0.15 | 0.15 | 0.20 | 0.19 | 0.14 | 0.14 | 0.01 | 0.15 | 0.72 | 0.13 | 0.06 | 0.00 |
| DE | 0.42 | 0.11 | 0.22 | 0.18 | 0.19 | 0.21 | 0.08 | 0.11 | 0.10 | 0.08 | 0.43 | 0.48 | 0.05 | 0.00 |
| DK | 0.53 | 0.16 | 0.22 | 0.20 | 0.19 | 0.11 | 0.07 | 0.07 | 0.03 | 0.25 | 0.44 | 0.30 | 0.10 | 0.10 |
| EE | 0.53 | 0.12 | 0.20 | 0.18 | 0.17 | 0.17 | 0.11 | 0.17 | 0.18 | 0.17 | 0.46 | 0.31 | 0.07 | 0.00 |
| ES | 0.44 | 0.15 | 0.24 | 0.16 | 0.13 | 0.16 | 0.13 | 0.12 | 0.05 | 0.22 | 0.18 | 0.27 | 0.10 | 0.10 |
| FI | 0.52 | 0.15 | 0.19 | 0.21 | 0.19 | 0.10 | 0.06 | 0.14 | 0.01 | 0.08 | 0.41 | 0.34 | 0.09 | 0.00 |
| FR | 0.41 | 0.17 | 0.21 | 0.18 | 0.16 | 0.13 | 0.10 | 0.10 | 0.07 | 0.13 | 0.44 | 0.26 | 0.10 | 0.10 |
| GR | 0.28 | 0.13 | 0.21 | 0.15 | 0.12 | 0.18 | 0.16 | 0.11 | 0.06 | 0.10 | 0.28 | 0.21 | 0.09 | 0.00 |
| HU | 0.41 | 0.14 | 0.16 | 0.17 | 0.19 | 0.18 | 0.13 | 0.24 | 0.01 | 0.21 | 0.48 | 0.21 | 0.07 | 0.00 |
| IE | 0.60 | 0.10 | 0.18 | 0.16 | 0.17 | 0.19 | 0.18 | 0.18 | 0.03 | 0.17 | 0.16 | 0.30 | 0.08 | 0.00 |
| IS | 0.51 | 0.21 | 0.22 | 0.19 | 0.14 | 0.10 | 0.07 | 0.09 | 0.02 | 0.27 | 0.34 | 0.34 | 0.15 | 0.10 |
| IT | 0.35 | 0.12 | 0.21 | 0.16 | 0.14 | 0.18 | 0.17 | 0.12 | 0.04 | 0.25 | 0.28 | 0.16 | 0.09 | 0.00 |
| LT | 0.59 | 0.09 | 0.17 | 0.18 | 0.19 | 0.21 | 0.13 | 0.20 | 0.07 | 0.13 | 0.25 | 0.48 | 0.05 | 0.00 |
| LU | 0.35 | 0.26 | 0.26 | 0.18 | 0.13 | 0.09 | 0.05 | 0.10 | 0.08 | 0.09 | 0.31 | 0.30 | 0.17 | 0.10 |
| LV | 0.69 | 0.10 | 0.17 | 0.16 | 0.18 | 0.22 | 0.14 | 0.17 | 0.19 | 0.24 | 0.41 | 0.30 | 0.06 | 0.00 |
| NL | 0.50 | 0.16 | 0.25 | 0.20 | 0.18 | 0.11 | 0.07 | 0.11 | 0.03 | 0.19 | 0.36 | 0.36 | 0.12 | 0.10 |
| NO | 0.49 | 0.19 | 0.21 | 0.19 | 0.15 | 0.10 | 0.07 | 0.18 | 0.04 | 0.20 | 0.45 | 0.35 | 0.10 | 0.10 |
| PL | 0.43 | 0.14 | 0.18 | 0.20 | 0.17 | 0.17 | 0.11 | 0.14 | 0.01 | 0.00 | 0.56 | 0.20 | 0.09 | 0.00 |
| PT | 0.31 | 0.11 | 0.20 | 0.16 | 0.16 | 0.21 | 0.15 | 0.19 | 0.01 | 0.15 | 0.12 | 0.11 | 0.06 | 0.00 |
| SE | 0.53 | 0.18 | 0.19 | 0.17 | 0.17 | 0.11 | 0.09 | 0.14 | 0.07 | 0.08 | 0.44 | 0.38 | 0.13 | 0.00 |
| SI | 0.57 | 0.13 | 0.25 | 0.21 | 0.16 | 0.16 | 0.08 | 0.15 | 0.11 | 0.20 | 0.54 | 0.21 | 0.08 | 0.00 |
| SK | 0.42 | 0.10 | 0.20 | 0.19 | 0.19 | 0.19 | 0.11 | 0.20 | 0.00 | 0.14 | 0.65 | 0.19 | 0.04 | 0.00 |
| UK | 0.45 | 0.13 | 0.20 | 0.15 | 0.17 | 0.16 | 0.15 | 0.13 | 0.07 | 0.28 | 0.42 | 0.30 | 0.07 | 0.00 |

Table 5: Covariates V (individual) all