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Multidimensional Measures of Well-being with Dimensional Hierarchy

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Abstract

In this paper we propose an axiomatic framework to measure multidimensional well-being and deprivation indices. When multiple dimensions are considered, it is often the case, that there is a need to determine the relative importance attached to the different dimensions. The current literature has largely focused on a two-tier partition of dimensions, for instance basic and non-basic dimensions. Our proposed framework allows us to differentiate dimensions in more than two hierarchical orders. The resulting well-being and deprivation indices can be estimated using minimal data that is binary in nature. We illustrate the multidimensional measures by using data from the American Community Survey, the largest household level survey in the U.S. We compile information on 9 dimensions of well-being for 6 years. Our results show that during the Great recession, between 2008 and 2011, there was a rise in the index measuring social deprivation and a decline in the values of the social well-being index. The trend is reversed in the recovery period following the recession, between 2011 and 2013. We also test the robustness of our estimates to different values of the parameters and thresholds.

Introduction

In the last two decades or so, a wide variety of multidimensional indices have been proposed in the literature. They differ in the dimensions and the thresholds applied, the assumed substitutability between dimensions, the relative weights given to them, the aggregation method used and so on. Most of the existing measures tend to aggregate deprivation or achievement in the various dimensions, thus essentially assuming that each of these dimensions are separate from and substitutable by one another. However, when multiple dimensions are considered, it is often the case, that there is a need to determine the relative importance attached to the different dimensions. Choosing a criterion for how deprivation in different dimensions should contribute to overall poverty carries significance for a convincing multidimensional evaluation. Indeed, different criteria may lead to contrasting evaluation results, with important consequences in terms of policy implications (Brandolini, 2007). Given a multiplicity of dimensions, policy makers often prefer to prioritize by focusing primarily on the removal of deprivation in terms of some basic attributes, and relegating to a second place the objective of removing deprivation in non-basic attributes.

Though most of the standard multidimensional measures do not include a hierarchical order on dimensions, there is recent growth in literature proposing such measures. Esposito and Chiappero-Martinetti (2010) propose multidimensional measures based on the notion of restricted and unrestricted hierarchy. The hierarchy is unrestricted when even a very small deprivation in the dominating dimension is deemed more important than a large deprivation in the dominated dimension. If this ranking only holds for the same deprivation level in the two dimensions, then the hierarchy is said to be restricted. Anderson et al (2014) enhances the Alkire and Foster (2011) multi-dimensional poverty index based on the counting approach, by admitting the possibility of sub-component substitutability/complementarity in the index. Their proposed index retains the ability to measure the

impact of improvement or worsening of sub-components within each category. Dhongde et al (2016) propose a framework similar to Esposito and Chiappero-Martinetti (2010)'s unrestricted hierarchy. They propose a setting in which the multiple dimensions are grouped as basic attributes that are of fundamental importance for an individual's quality of life and non-basic attributes which are at a much lower level of importance. Thus there is implicit substitutability among the basic attributes and substitutability among the non-basic attributes but no such substitutability among attributes belonging to the two distinct classes. Both the previous studies (Esposito and Chiappero-Martinetti, 2010 and Dhongde et al 2016) use binary hierarchical orders: basic and non-basic. However we believe that this framework needs to be extended to include more than two criteria. In this paper we formulate a more flexible framework of multidimensional indices and propose both well-being and deprivation measures where dimensions can be classified into two or more hierarchical orders.[1]

Furthermore, the indices we propose can be used with sparse data with only "yes and no" type of answers. Although, existing literature largely assumes that each of the multiple attributes or dimensions under consideration is cardinally measurable along real intervals, many times survey data is of ordinal nature. Ordinal data can be classified in two types: (1) information is available on only two levels of achievement, namely, 1 (satisfactory/not deprived) and 0 (unsatisfactory/deprived); for example, the American Community Survey asks whether a household has a flush toilet or not, and (2) information is available on more than two ordered levels; for example, the Survey of Household Economics and Decision-making, used by the Federal Reserve Bank to measure well-being, asks respondents about their financial management by giving them options such as a. living comfortably, b. doing ok, c. just getting by, d. finding it difficult to get by. In this paper, we assume that the only data available is binary ordinal data with yes and no kind of responses.

[1] In a recent paper, Permanyer (2016) raise a related question: will improvements in shortfalls and improvements in achievements mirror each other or not? They present the conditions under which both classes of measures rank alternative states of affairs in a consistent way.

We apply the proposed indices to measure well-being and deprivation among the population of the United States (U.S.). Several studies compare well-being measures between countries (e.g. Berenger and Verdier-Couchane, 2007, Deaton, 2008, OECD, 2015), or between a subset of countries, for example with the European Union (e.g. Anand et al, 2015, Pittau et al, 2010). However the literature on well-being measures in the U.S. has grown only recently. The Gallup-Healthways Well-Being Index uses the Gallup daily data to measure the state of wellbeing in the U.S.[2] The index tracks the well-being of approximately 1,000 U.S. residents, aged 18 and older, 350 days per year. Reports on the index are published annually at state, community and congressional district level. Oswald and Wu (2011) estimated life satisfaction equations by controlling for people's personal characteristics and found no correlation between states' regression-adjusted well-being and their GDP per capita. Deaton (2012) used Gallup daily data on self-reported well-being to examine how the Great Recession affected the emotional and evaluative lives of the population in the U.S. Since 2013, the Federal Reserve Bank (2016) publishes an annual report on economic well-being of the U.S. households. The report is based on the Survey of Household Economics and Decision-making and contains information on the percent of population satisfied with their level of income, savings, access to banking and credit and so on.

Compared to the literature measuring well-being, the literature measuring deprivation in the U.S. is relatively sparse. A recent paper by Dhongde and Haveman (2016) is perhaps the only comprehensive study which systematically measures trends in multidimensional deprivation in the U.S. during the Great Recession. The authors estimate the Alkire and Foster (2011) index to measure multidimensional deprivation in the U.S. during the Great Recession. They find that about 15% of the population was multidimensional deprived. They also find that unlike the official income poverty measure, the proportion of multidimensional deprived population steadily declined during the recovery period.

[2] http://www.gallup.com/topic/well_being_index.aspx

Our empirical application is distinct from the previous studies. As far as we are aware, this is the only study which measures both multidimensional well-being and deprivation in the U.S. over a period of 6 years during the Great Recession. We use data from the American Community Survey (ACS), which is one of the largest household surveys in the U.S. Our sample comprises of more than 2 million individuals from the ACS for each year; we compile data for 6 years, from 2008 to 2014. Recommendations made by the Commission on the Measurement of Economic Performance and Social Progress (Stiglitz et al. 2009) are used as guidance to choose 9 different dimensions of well-being. We illustrate the proposed framework by estimating both well-being and deprivation indices and testing their robustness to different parametric values.

The rest of the paper is organized as follows. In Section 2 we formulate the axiomatic framework and in Section 3 we propose a multi-level hierarchical structure for the attributes. In Section 4 we estimate the well-being index and in Section 5 we estimate the deprivation index by using U.S. data. Trends in these indices are analyzed over time in Section 6. Results are summarized in Section 6.

II. Axiomatic Framework with Binary Ordinal Data

Let there be m attributes: $f_1, \dots, f_m$, and $M = \{1, \dots, m\}$. Suppose the measurement of each attribute is binarily ordinal, so that, for each attribute, there are only two levels: adequate (1) and inadequate (0). Let the society have n individuals. The society is denoted by $N = \{1, \dots, n\}$.

Let $\mathcal{A}$ be the set of all $n \times m$ matrices with 0 or 1 entries. Each $A \in \mathcal{A}$ is interpreted as the society's achievement matrix: the (ij)th entry of the matrix records individual i's achievement in attribute f_j.

For each individual $i \in N$ in the society and an achievement matrix $A \in \mathcal{A}$, let $M_0^i(A)$ denote the set of all $j \in M$, such that i's achievement in terms of f_j is 0, and let $M_1^i(A)$ denote the set of all $j \in M$, such that i's achievement in terms of f_j is 1.

A. Well-being measures for the individual and for the society:

A well-being measure of the society is a function from $\mathcal{A}$ to $[0,1]$ with the interpretation that, for all $A, B \in \mathcal{A}$, $g(A) \geq g(B)$ indicates that the society's well-being level under A is at least as high as the society's well-being level under B, $g(A) > g(B)$ indicates that the society's well-being level under A is higher than the society's well-being level under B, and $g(A) = g(B)$ indicates that the society's well-being level under A is the same as the society's well-being level under B.

For a well-being measure of the society, we consider the following axiomatic properties:

Normalization: For all $A = (a_{ij}) \in \mathcal{A}$ and for all $\delta \in \{0,1\}$, if [$\delta_{ij} = \delta$ for all $i \in N$ and all $j \in M$], then $g(A) = \delta$.

Anonymity: Let σ be a bijection from N to N. Then, for all $A, B \in \mathcal{A}$, if [$a_{ij} = b_{\sigma(i)j}$ for all $i \in N$ and all $j \in M$], then $g(A) = g(B)$.

Monotonicity: For all $A, B \in \mathcal{A}$, if [$a_{ij} \geq b_{ij}$ for all $i \in N$ and all $j \in M$ and $A \neq B$], then $g(A) > g(B)$.

Independence: For all $A, B, A', B' \in \mathcal{A}$, and for all $i' \in N$, if [(for all $i \in N \backslash \{i'\}$: $a_{ij} = b_{ij}$ and $a'_{ij} = b'_{ij}$ for all $j \in M$), and ($a_{i'j} = a'_{i'j}$ and $b_{i'j} = b'_{i'j}$ for all $j \in M$)], then $g(A) - g(B) = g(A') - g(B')$.

Additivity: For each $j \in M$, there exists a function φ_j such that, for all $A, B \in \mathcal{A}$ with, for some $i' \in N$, [$a_{ij} = b_{ij} = 0$ for all $j \in M$ and all $i \in N \backslash \{i'\}$], we have $g(A) \geq g(B) \Leftrightarrow \sum_{j=1}^{m} \varphi_j(a_{i'j}) \geq \sum_{j=1}^{m} \varphi_j(b)$.

Proposition 1. Let g be a well-being measure of the society. Then, g satisfies Normalization, Anonymity, Monotonicity, Independence and Additivity if and only if g is given below:

for some increasing function $\rho: [0,1] \to [0,1]$ with $\rho(0) = 0, \rho(1) = 1$, and some positive constants $w_1, \dots, w_m$ with $w_1 + \cdots + w_m = 1$, we have $g(A) = \frac{1}{n}\sum_{i=1}^{n} \rho(\sum_{j=1}^{m} w_j\, a_{ij})$ for all $A \in \mathcal{A}$.

For a given achievement matrix $A \in \mathcal{A}$, in our subsequent empirical section, we shall take a ρ function figured in the above propositions to be a power function, $r_i^{\alpha}(A)$, where $0 < \alpha \leq 1$, to measure the well-being of an individual i, and subsequently, the society's well-being under an achievement matrix A will be measured by $\frac{\sum_{i \in N} r_i^{\alpha}(A)}{n}$. Note that, given our notation of $M_1^i(A)$, for any achievement matrix $A \in \mathcal{A}$, $r_i(A) = \sum_{j \in M_1^i(A)} w_j = \sum_{j=1}^{m} w_j\, a_{ij}$.

B. Deprivation measures for the individual and for the society:

From the discussions above, we see that there are m positive weights, $w_1, \dots, w_m$, for the m attributes such that $\sum_{j \in M} w_j = 1$, for each individual $i \in N$ and a given achievement vector $a_i = (a_{i1}, \dots, a_{im})$ of i, i's well-being level can be measured by $\sum_{j=1}^{m} w_j a_{ij}$.

Suppose there is a positive real number $t \in [0,1]$ such that, for an achievement matrix $A \in \mathcal{A}$, a person i is deprived iff $r_i(A) \equiv \sum_{j \in M_1^i(A)} w_j < t$. Let $N^d(A)$ be the set of deprived individuals. For a given achievement matrix $A \in \mathcal{A}$, we shall use $(\frac{t - r_i(A)}{t})^{\beta}$, where $\beta \geq 1$, to measure the deprivation of an individual $i \in N^d(A)$, and the society's deprivation will be measured by $\sum_{i \in N^d(A)} (\frac{t - r_i(A)}{t})^{\beta} / n$.

We now introduce another property, the Stiglitz-Sen-Fitoussi condition, which is more suitable when one considers measuring deprivation. The intuition of the original Sen-Stiglitz-Fitoussi condition for measures of deprivation is that the harm caused by k different dimensional deprivations occurring

simultaneously is greater than the sum of the harms caused by the k separate dimensional deprivations occurring one at a time (Stiglitz et al. 2009).

The Stiglitz-Sen-Fitoussi property: For all $A, B, C \in \mathcal{A}$, all $i' \in N$, and all $k, k' \in M$, if (for all $i \in N \backslash \{i'\}$: $a_{ij} = b_{ij} = c_{ij}$ for all $j \in M$) and ($a_{i'j} = b_{i'j} = c_{i'j}$ for all $j \in M \backslash \{k, k'\}$, and $a_{i'k} = a_{i'k'} = 1, (b_{i'k} = 1, c_{i'k} = 0), (b_{i'k'} = 0, c_{i'k'} = 1)$), then $g(A) > g(B) + g(C)$.

Proposition 2. Let g be a deprivation measure of the society. Then, g satisfies Normalization, Anonymity, Monotonicity, Independence, Additivity and the Stiglitz-Sen-Fitoussi property if and only if g is given below:

for some increasing function $\rho: [0,1] \to [0,1]$ with $\rho(0) = 0, \rho(1) = 1$, and some positive constants $w_1, \dots, w_m$ with $w_1 + \cdots + w_m = 1$, we have $g(A) = \frac{1}{n} \sum_{i=1}^{n} \rho(\sum_{j=1}^{m} w_j \, a_{ij})$ for all $A \in \mathcal{A}$; and, in addition, $\rho\left(\sum_{j \in M'} w_j\right) < \sum_{j \in M'} \rho\left(w_j\right)$ for all non-empty $M' \subseteq M$.

III. Hierarchical importance of attributes and the robustness of the measures

Suppose F is partitioned into $F^1, \dots, F^K$ with the interpretation that the attributes in F^1 are the most important attributes, …, and the attributes in F^K are the least important ones. For each partition F^k, there are m_k attributes in it. Suppose the same weight is attached to each attribute in the same partition. For each attribute in F^k, $(k = 1, \dots K)$, let the weight be $\gamma^{k-1} a$ where $a > 0$ and $\gamma \in (0,1)$. Then

$$a m_1 + \gamma a m_2 + \cdots + \gamma^{K-1} a m_K = 1$$

So that

$$a = \frac{1}{m_1 + \gamma m_2 \ldots + \gamma^{k-1} m_k + \cdots + \gamma^{K-1} m_K}$$

Consequently, we have that for each attribute in the partition F^k, the weight is given by

$$\frac{\gamma^{k-1}}{m_1 + \gamma m_2 \ldots + \gamma^{k-1} m_k + \cdots + \gamma^{K-1} m_K}$$

For a given achievement matrix A and a given $\gamma \in (0,1)$, let $r_i(A,\gamma) \equiv \sum_{j \in M_1^i(A)} w_j$ where the weights $w_j s$ are defined above, and $N^d(A,\gamma)$ be the set of deprived individual. Consequently, the society's deprivation is given by $h(A,\gamma) = \sum_{i \in N^d(A,\gamma)} (\frac{t - r_i(A,\gamma)}{t})^\beta / n$.

Let $W \subseteq (0,1)$ and be non-empty. We shall interpret W as the set of reasoned γs that will be used for checking the robustness of the measures. If, for two achievement matrices $A, B \in \mathcal{A}$, we have $[h(A,\gamma) \geq h(B,\gamma)$ for all $\gamma \in W]$, then it is unambiguous to say that the social deprivation under A is at least as great as the social deprivation under B.

IV. Empirical Application for the U.S.

A. Data

We use the American Community Survey (ACS), which is the largest U.S. household survey. The survey is conducted annually for a sample of more than 2 million individuals. It collects information on the demographic, social, economic, and housing characteristics of the sample population. The ACS randomly selects samples in all counties across the nation (and all municipios in Puerto Rico) every month. We use ACS records on individuals, aged 18 and above, from the Public Use Microdata Sample (PUMS) files.[3] Individual records are replicated using person weights; data on individual's household characteristics is

[3] PUMS files provide data from areas with population of 65,000 or more.

used as well.[4] The ACS has consistent data since 2008; we compile data for six years during the Great Recession, from 2008 to 2013.

B. Choosing Multiple Dimensions of Well-being

Although the choice of well-being dimensions is largely dictated by the availability of relevant data in the ACS, we use as guidance, the recommendations made by the Commission on the Measurement of Economic Performance and Social Progress (Stiglitz et al. 2009) on the multiple dimensions of well-being. Previous studies measuring multidimensional deprivation in the U.S. (Dhongde and Haveman, 2016, Mitra and Brucker, 2017) too have used the Commission's recommendations listed in its report (pg. 14-15). Table 1 lists the different variables in ACS that we choose to reflect an individual's achievement and deprivation in a particular dimension.[5] Detailed definition of each attribute is provided in the Appendix to the paper.

Table 1: Multiple Dimensions of Well-Being

Dimensions Recommended by the Commission	**Well-being Attributes**
Std. of Living	1. Family income above poverty threshold 2. Housing costs are less than 50% of household income
Health	3. Individual has fewer than 2 of 6 disabilities
Education	4. Individual has at least a high-school diploma
Personal activities including work	5. Individual is employed
Economic Security	6. Individual has health insurance coverage
Social Connections and Relationships	7. Individual speaks English fluently
Environment	8. Household has less than one occupant per room 9. Household has kitchen and plumbing facilities

Source: Stiglitz et al (2009) and American Community Survey

[4] Individuals living in group quarters such as college residence halls, residential treatment centers, skilled nursing facilities, group homes, military barracks, correctional facilities, and workers' dormitories are not included.
[5] Only one dimension in the commission's report is not included, that is "political voice and governance", since we did not find any suitable data in the ACS.

V. Estimates of Well-being Measures for different parametric values

A. No Hierarchy of Dimensions

For a given achievement matrix $A \in \mathcal{A}$, we take a ρ function to be a power function, $r_i^{\alpha}(A)$, where $0 < \alpha \leq 1$, to measure the well-being of an individual i, and subsequently, the society's well-being under an achievement matrix A will be measured by $\frac{\sum_{i \in N} r_i^{\alpha}(A)}{n}$. It may be noted that, for each achievement matrix A and each individual i, $r_i(A) \equiv \sum_{j \in M} w_j$ may be interpreted as this individual's `nominal overall achievement', and consequently, for a given parameter $0 < \alpha < 1$, $r_i^{\alpha}(A)$ may be interpreted as $i$'s `real overall achievement'. In Table 2 (column 1), we assign equal weights to all dimensions ($w_1 = \cdots = w_9 = \frac{1}{9}$) and estimate society's real overall well-being index. We do so using ACS data for 2013 and by varying the values of α ($\alpha = \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1$). As seen in the table, the value of the social well-being index decreases as the value of α increases. On average, the weighted sum of achievements for individuals was equal to about 0.9 (for $\alpha = 1$), which means that most individuals had achievements in 8 out of 9 attributes.

Table 2: Estimates of Well-being Measures

	(1)	(2)	(3)
α	No Hierarchy	2-Tier Hierarchy	3-Tier Hierarchy
1/4	0.976	0.973	0.971
1/2	0.955	0.949	0.944
3/4	0.935	0.927	0.920
1	0.916	0.907	0.899

Source: Authors' calculations based on ACS data from 2013

B. Hierarchy of Dimensions

Suppose we introduce a hierarchy among dimensions by partitioning F into $F^1, \ldots, F^K$ with the interpretation that the attributes in F^1 are the most important attributes, …, and the attributes in F^K are the least important ones. Consider a relatively simple structure where we have exactly two categories $(k = 2)$: basic attributes $(F^1)$and non-basic attributes (F^2). From Table 1, attributes 1. income, 2. housing costs, 3. disabilities and 4. high-school education are treated as basic attributes $(m_1 = 4)$ and 5. employment, 6. health insurance, 7. English fluency, 8. occupancy per room, and 9. housing facilities are treated as non-basic attributes $(m_2 = 5)$. Suppose the same weight is attached to each attribute in the same partition. We calculate the weights using the formula $\frac{\gamma^{k-1}}{m_1+\gamma m_2 \ldots+\gamma^{k-1}m_k+\cdots+\gamma^{K-1}m_K}$. Let $\gamma = \frac{1}{2}$ so that the weight attached to each basic attribute is equal to 2/13 and that attached to each non-basic attribute is 1/13. In Table 2 (column 2), we list estimates of the social well-being index $\frac{\sum_{i\in N} r_i^{\alpha}(A)}{n}$ for different values of α.

Next, suppose we partition the dimensions in three categories as high (F^1), moderate (F^2) and less (F^3) important. Let attributes- 1. income, 2. housing costs, 3. disabilities and 4. high-school education- be highly important, then attributes 5. employment and 6. health insurance- be moderately important and finally, attributes- 7. English fluency, 8. occupancy per room, and 9. housing facilities-be less important. Thus we have: $m_1 = 4, m_2 = 2, m_3 = 3$. Keeping $\gamma = \frac{1}{2}$, the weight attached to each highly important attribute is equal to 4/23, to each moderately important attribute is 2/23 and to each less important attribute is 1/23. The last column (3) in Table 2 shows estimates of the well-being index with this type of hierarchy among dimensions. It is seen that for every value of the power function $(\alpha = \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1)$, the well-being index estimates are higher when all dimensions are weighed equally. Once we introduce a hierarchical structure, the value of the estimated well-being index lowers, as we

increase the number of partitions. This follows our intuition; for instance compared to no partition where all attributes are equally weighted, in a 3-tier partition achievement in some attributes carries greater weight than in others. As a result, the well-being index decreases in value.

V. Estimates of Deprivation Measures for different Parametric Values

A. No Hierarchy of Dimensions

Suppose there is a positive real number $t \in [0,1]$ such that, for an achievement matrix $A \in \mathcal{A}$, a person i is deprived iff $r_i(A) \equiv \sum_{j \in M_1^i(A)} w_j < t$. For a given t, and for a given achievement matrix $A \in \mathcal{A}$, $t - r_i(A) \equiv t - \sum_{j \in M_1^i(A)} w_j$ may be interpreted as individual i's `nominal overall deprivation' and $(\frac{t-r_i(A)}{t})$ may be interpreted as $i$'s `normalized nominal overall deprivation'. The deprivation benchmark t is specified with reference to the nominal overall achievement of an individual. When the individual's nominal overall achievement falls short of t, the individual is considered to be deprived. For example, when all dimensions are equally weighted, and $t = 1/3$, then individuals with fewer than one-third achievements are considered as deprived. The 'real overall deprivation' of individual i is given by a power function of i's normalized nominal overall deprivation, $\left(\frac{t-r_i(A)}{t}\right)^{\beta}$. The society's overall deprivation is just the sum of all the deprived individuals' real overall deprivations. Let $N^d(A)$ be the set of deprived individuals, then the society's deprivation is measured by $\sum_{i \in N^d(A)} (\frac{t-r_i(A)}{t})^{\beta}/n$. Note that, depending on the value of t and the weights w_j, it is possible for an individual to be non-deprived even when she has some dimensional deprivations.

Table 3: Estimates of Deprivation Measures

	(1)	(2)	(3)		(4)	(5)	(6)
$\beta = 1$	No Hierarchy	2-Tier Hierarchy	3-Tier Hierarchy	$\beta = 2$	No Hierarchy	2-Tier Hierarchy	3-Tier Hierarchy
$t = \frac{1}{3}$	0.00002309	0.00016724	0.00057777	$t = \frac{1}{3}$	0.000007952	0.00002872	0.00012649
$t = \frac{1}{2}$	0.0010	0.0025	0.0046	$t = \frac{1}{2}$	0.0002	0.0006	0.0013
$t = \frac{2}{3}$	0.0062	0.0122	0.0175	$t = \frac{2}{3}$	0.0015	0.0032	0.0052

Source: Authors' calculations based on ACS data from 2013

In Table 3 (columns 1 and 4), we assign equal weights to all dimensions and vary the value of t. Thus we alternately consider the following: i) all those individuals who have achievements in less than 3 out of the 9, $\left(t = \frac{1}{3}\right)$ attributes are considered deprived, ii) all those individuals who have less than 50% of achievements $\left(t = \frac{1}{2}\right)$ are considered deprived and finally, iii) all those individuals who have less than 6 out of 9 $\left(t = \frac{2}{3}\right)$ are considered deprived. As expected, for a given value of β, the deprivation index increases as the benchmark t increases. On the other hand, for a given value of t, the deprivation index decreases as the value of β increases ($\beta = 1, 2$).

B. Hierarchy of Dimensions

Now suppose we introduce a similar hierarchical structure as we did in the context of the well-being index (Section IV. B). First, we consider a two-fold partition with attributes divided as basic and non-basic (Table 3, columns 2 and 4) and then we use a three-fold partition of the attributes (Table 3, columns 3 and 6). For a given value of t and β, we notice that the value of the deprivation index increases as the partitions of the attributes increase.

VI. Estimates of Well-Being and Deprivation Measures over Time

In Sections 4 and 5, we estimated the sensitivity of the well-being and deprivation indices to different parametric values and different thresholds. In this Section, we fix our parametric values and estimate these indices over a period of 6 years, from 2008 to 2013. For $\alpha = 1$, $\gamma = \frac{1}{2}$ we estimate social well-being indices over time in Table 4. Recall that the social well-being index is calculated as $\frac{\sum_{i \in N} r_i(A)}{n}$ where for each achievement matrix A and each individual i, $r_i(A) \equiv \sum_{j \in M} w_j$. We use the same three weighting schemes used in the previous section. When all dimensions were weighed equally, we find that on average, the social well-being index was 0.91, with a two tier structure it was about 0.90 and with a 3 tier-structure it was 0.89. Values of the well-being index declined between 2008 to 2010, there were the least and almost equal during the peak of the recession in 2010-2011 and improved since then.

Table 4: Estimates of Well-being Measures over Time

	(1)	(2)	(3)
Years	No Hierarchy	2-Tier Hierarchy	3-Tier Hierarchy
2008	0.919	0.910	0.901
2009	0.914	0.906	0.897
2010	0.911	0.903	0.894
2011	0.912	0.903	0.894
2012	0.915	0.906	0.897
2013	0.916	0.907	0.899

Source: Authors' calculations based on ACS data; $\alpha = 1$, $\gamma = \frac{1}{2}$

The deprivation index is estimated by letting $\beta = 1$ and the benchmark $t = 2/3$. That is, we treat all individuals with less than 2/3 rd achievements as deprived $N^d(A)$. With $\beta = 1$, the social deprivation index is the average of individual's deprivation measured as a proportional shortfall from the benchmark and is given by $\sum_{i \in N^d(A)} \left(\frac{2/3 - r_i(A)}{2/3}\right)/n$. As expected, we find a reverse trend in the deprivation index values listed in Table 5. The trend is robust for all 3 types of hierarchical structures.

Table 5: Estimates of Deprivation Measures over Time

	(1)	(2)	(3)
	No Hierarchy	2-Tier Hierarchy	3-Tier Hierarchy
2009	0.0071	0.0131	0.0184
2011	0.0074	0.0138	0.0195
2013	0.0062	0.0122	0.0175

Source: Authors' calculations based on ACS data; $\beta = 1, t = \frac{2}{3}, \gamma = \frac{1}{2}$

VII. Summary

In this paper we proposed an axiomatic framework to measure multidimensional well-being and deprivation indices using binary data. The proposed framework allowed us to differentiate dimensions in more than two hierarchical orders. We estimated the multidimensional measures by using data from the American Community Survey. We found that during the Great recession, there was a rise in the index measuring social deprivation and a decline in the values of the social well-being index. The trend reversed in the recovery period. We also tested the robustness of our estimates to different values of the parameters and thresholds and found these to be consistent.

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Appendix

The American Community Survey (ACS) defines and measures variables in a specific way. Furthermore, we made certain adjustments to the available data, all of which are detailed below.

1. Family income above poverty threshold: The income-poverty ratio in the ACS is estimated by comparing the person's total family income in the last 12 months with the poverty threshold appropriate for that person's family size and composition, according to the standards specified by the Office of Management and Budget in Statistical Policy Directive.

2. Housing costs are less than 50% of household income: The ACS reports monthly owner costs and gross rent as a percentage of household income. It reports selected monthly owner costs such as mortgage payments, taxes, insurance, utilities, fuel costs and gross rent as a percentage of household income. Typical housing burden categories are: No housing burden (under 30 % of income spent on housing costs), moderate burden (between 30 and 49.9 %), and severe burden (over 50 %).

3. Individual has less than 2 out of 6 disabilities: Disability in the ACS is identified as serious difficulty with four basic areas of functioning—hearing, vision, cognition, and ambulation—supplemented by questions about difficulties with self-care such as difficulty in bathing and dressing, and difficulty performing independent errands such as shopping

4. Individual has at least High-school diploma: Adults 18 years or older with at least high school education.

5. Individual is employed: The employment status recode in the ACS, asks whether 1. Civilian employed, at work, 2.Civilian employed, with a job but not at work, 3.Unemployed, 4. Armed forces, at work, 5. Armed forces, with a job but not at work, 6.Not in labor force. We treat an individual as deprived in this dimension if she is unemployed, and not deprived otherwise.

6. Individual has health insurance coverage: ACS asks whether an individual has any health insurance, private or public.

7. Individual speaks English fluently: Individual's ability to speak English is very well, well or not well. Deprivation is measured as individual's ability to speak English: not at all.

8. Household has less than one occupant per room: The number of persons per room in a housing unit is a common measure of overcrowding. The ACS reports data on occupants per room by dividing the number of people in each occupied housing unit by the number of rooms in the unit. We use the ACS threshold that a crowded unit as one which has more than one occupant per room.

9. Household has kitchen and plumbing facilities: The ACS reports data on housing facilities such as plumbing (hot and cold running water, a flush toilet, a bathtub or shower) and kitchen facilities (a sink with a faucet, a stove, range, or a refrigerator)