Estimating Capital Services in the U.S.: An Empirical Assessment of Implementation Differences

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Abstract

1 Introduction

International guidelines for measuring the sources of growth emphasize the importance of using the flow of capital services as a component of total inputs in estimates of MFP growth. Unfortunately, this flow of capital services is difficult to measure because producers typically own the capital stock that they use for both current and future production. The standard approach taken by statistical agencies and researchers following the guidelines is to estimate the capital stock using the perpetual inventory method, estimate rental prices, and then combine the two pieces to arrive at an estimate of capital input, also known as capital services. The importance of using capital services measures versus capital stock decomposing growth is labeled as "capital quality" by (Jorgenson, Ho, & Stiroh, 2005), and they estimate that substitution towards relatively more productive capital accounts for a significant share of economic growth in the U.S.

Even within the "standard approach", however, there can be differences in methodology and how those methodologies are implemented. The guidelines themselves note broad issues affecting capital measurement such as choice of depreciation formula and its relation to the age-efficiency profile, aggregation of assets, rates of return, and the treatment of negative user costs as implementation issues for which there is no international consensus. It is an open question how much these methodological and implementation choices, which are all consistent with the guidelines, affect empirical estimates of the sources of growth in an integrated framework.

The purpose of this paper is to review the methodologies used to estimate capital services for the U.S. and to examine the extent to which different assumptions affect estimates of capital's contribution to U.S. economic growth. We frame our analysis within the BEA-BLS integrated industry-level production account. The Bureau of Labor Statistics (BLS) is the statistical agency charged with producing and publishing official productivity statistics. The official MFP statistics have included capital services measures since the original work of (Harper, 1982). These measures are consistent with the basic guidelines of the OECD manuals on capital measurement and include estimates of investment, the productive capital stock, the wealth stock, capital services and MFP at the aggregate and industry level for the U.S. The official productivity statistics are separate from the U.S. national accounts produced by the Bureau of Economic Analysis (BEA) and differ in scope, with a major difference being that the official statistics cover only the private business economy, while the BEA national accounts cover the total economy.[2] The BEA is charged with producing and publishing the official GDP estimates does not produce estimates of capital services. Recently the BEA and BLS developed an integrated industry level production account that embeds capital and labor estimates from the BLS into the GDP-by-Industry Accounts produced by the BEA (Rosenthal, Russell, D., H., & Usher, 2014). This integrated account allows us to compare the capital measures produced by BLS with others that have the same scope.

[2] The non-profit, owner occupied housing, and government sectors are included in the GDP accounts, but they are excluded from the official productivity statistics. The main reason for these exclusions is that output estimates for these sectors are generated using data on inputs—primarily wages and salaries—which tends to impart a downward bias on estimates of productivity growth.

In particular, we compare capital services measures based on the BLS methodology (produced for the integrated production account) to that based on the methods described in (Jorgenson, Ho, & Stiroh, 2005) (JHS) (produced to be consistent with the BEA accounts for research purposes). The major conceptual difference between the two is the specification of the age-efficiency function. The advantage of examining these via the lens of the integrated production account is that the capital estimates are equivalent in scope, i.e. both are constructed to be consistent with the GDP by Industry statistics published by the BEA. This allows for a straight forward assessment of the impact of the differences in capital measurement on the measured sources of growth. The paper is mostly empirical, although we discuss some of the conceptual issues that underlie the estimates.

In abstract, simple graphics can be informative about many of the differences in approaches. For example, the geometric depreciation rate produces a smooth convex efficiency decay of each asset, while a hyperbolic decay function allows for a concave pattern that many think is more realistic. The contribution of this paper is to go beyond the simple illustrative example to fully implement the measures at the industry level using the same source data to assess the empirical impact of these implementation choices.

Given that the two approaches use different methodologies and have different procedures for implementing their respective methodologies, it is not surprising that their estimates differ. In this paper, we compare the two methodologies. Our goal is to determine whether the differences in estimated capital input are due to differences in methodology (choices related to economic concepts) or implementation (how the concepts linked are to the data or put into practice), if the distinction can be drawn. We begin by briefly describing the BLS and JHS methodologies and identifying the main differences. We then compare estimates, and draw some preliminary conclusions. It appears that it is mostly differences in implementation that are driving the differences in estimates. In the next draft of this paper, we will examine implementation issues more closely.

The paper proceeds along the following outline: section 2 provides a general outline of capital services measurement, while sections 3 and section 4 cover the conceptual methodology currently used by the BLS, and that of (Jorgenson, Ho, & Stiroh, 2005), respectively. Section 5 compares the two approaches. Section 6 covers the implementation issues. Section 7 provides an overview of the framework that we use to compare the methods, and Section 8 presents the sources of U.S. economic growth over 1998-2012 using the two approaches. Section 9 uses a simple model to relate measured differences to industry characteristics and section 10 concludes.

2 A General Outline of Methodologies

Both BLS and JHS calculate aggregate measures of capital services using the same general methodology.

a. Calculate the productive capital stock from investment data using the perpetual inventory method (PIM). Under the PIM, past investments are adjusted to account for deterioration using the age-efficiency function, and then are aggregated into a capital stock. BLS and JHS estimate the productive capital stock at the asset-by-industry level. The use of the PIM, requires the assumption that investment is measured in constant-quality units.

b. Calculate the rental price for each industry × asset category cell. The rental price of capital represents the implicit rental cost of using the asset in production. As part of these calculations, BLS and JHS calculate a wealth stock and an internal rate of return for each industry that exhausts income across assets.
c. Aggregation – BLS and JHS aggregate the capital stock data to construct capital measures by industry for broad asset groups and aggregate measures of capital input.

3 BLS Methodology

Capital services measurement requires an estimate of the productive capital stock. Conceptually, this captures the quantity of capital assets that are available to yield a capital service flow into production at time t. As noted above, the PIM requires an assumption about how assets deteriorate over time. BLS assumes that assets deteriorate according to a hyperbolic age-efficiency function, which is given by:

$$(1) \qquad \lambda(a,\Omega)=\frac{(\Omega-a)}{(\Omega-\beta a)} \quad \text{if } a<\Omega \text{ (and 0 otherwise)},$$

where λ is the efficiency of that asset at age a relative to the efficiency at age = 0, Ω is the maximum service life of the asset, and β is a shape parameter. The hyperbolic function is very flexible, allowing for different deterioration patterns—convex, straight line, or concave—depending on whether β is less than, equal to, or greater than zero. BLS assumes β = 0.75 for structures and β = 0.5 for equipment.[3] These values result in a concave age-efficiency function,

[3] These values were chosen because they are close to values estimated by Hulten and Wykoff (1982) using actual data.

and reflect the casual empiricism that assets deteriorate more slowly when they are new and that they deteriorate more rapidly as they age.[4]

The age-efficiency function in equation (1) describes the deterioration of a single asset or a group of identical assets—that is, assets with the same maximum service life. But the investment data for asset categories includes assets with different service lives. For example, personal computers include top-of-the-line models, which may have a maximum service life of 5 or 6 years, but also low-end models that my last for only 2 or 3 years. To account for heterogeneity of service lives within asset classes, BLS assumes that asset service lives are distributed according to a modified truncated normal distribution, $\tilde{\phi}(\cdot)$.[5] BLS then computes a cohort age-efficiency function, which calculates the average efficiency of assets in an asset category that were purchased in the same year (cohort). It is calculated as a weighted average of the age-efficiency functions within an asset category, where the weight is the fraction of assets with a given maximum service life. The cohort age-efficiency function is defined over the interval, $[\Omega^{min}, \Omega^{max}]$, is given by:

$$(2) \qquad \bar{\lambda}(a, \overline{\Omega}) = \int_{\Omega^{min}}^{\Omega^{max}} \tilde{\phi}(a, k) \cdot \lambda(a, k) dk$$

[4] The age-efficiency function also accounts for obsolescence, time out of service for repairs, and failure. For example, a one year-old computer runs at about the same speed as a new computer. It is the introduction of new software that places greater demands on the computer that makes the computer obsolete. A one-year-old car and a five-year-old car provide the same service as a new car, but the five year-old car is more likely to be out of service for maintenance or repairs. A two year-old light bulb shines just as brightly as a new bulb, but is more likely to fail.

[5] The BLS assumes that $\tilde{\phi}(\cdot)$ is a modified truncated normal distribution with mean $\overline{\Omega}$ and $\sigma = 0.49\overline{\Omega}$. It is derived by truncating the normal distribution at ±2 standard deviations ($\overline{\Omega} \pm 0.98\overline{\Omega}$), shifting the density function downward so that it equals zero at the upper and lower bounds of the distribution, and then inflating the density function proportionately so that the final modified density, $\tilde{\phi}(\cdot)$, integrates to 1.

where the limits of the integral are the upper and lower bounds of the distribution of service lives (BLS assumes that $\Omega^{min} = 0.02\overline{\Omega}$ and $\Omega^{max} = 1.98\overline{\Omega}$).[6] The productive capital stock of asset *j* in industry *i* is given by:

$$K_{ij,t} = \sum_{a=0}^{\Omega^{max}} \bar{\lambda}(a, \overline{\Omega}) I_{ij,t-a} \tag{3}$$

As noted above, the rental price of capital is the opportunity cost of holding and using it for a period of time. BLS calculates the rental price along the lines of the specification in Hall and Jorgenson (1967), except that the BLS equation accounts for inflation in the price of new assets that was assumed to be zero in the original Hall and Jorgenson implementation.[7] The BLS calculates rental prices by industry and asset class using the following rental price formula:

$$c_{ij,t} = \frac{(1 - u_t z_t - e_t)\left(P_{ij,t-1} r_{i,t} + P_{ij,t-1} d_{ij,t} - \Delta P_{ij,t-1}\right)}{1 - u_t} + P_{ij,t-1} x_t \tag{4}$$

where:

u_t is the corporate income tax rate
z_t is the present value of \$1 of tax depreciation allowances (usually between .8 and 1.0)
e_t is the effective rate of the investment tax credit (zero since 1979)
r_{it} is the nominal (internal) rate of return on capital
d_{ijt} is the average rate of economic depreciation
P_{ijt} is the industry deflator for new capital goods
ΔP_{ijt} is the revaluation of assets due to inflation in new goods prices
x_t is the rate of indirect (property) taxes

[6] This assumption results in a wide range of service lives within each asset category.
[7] The BLS equation for the rental rate differs slightly from the original Hall and Jorgenson (1967) formulation. BLS includes an inflation term that was assumed to be zero in the Hall and Jorgenson (1967) implementation; this assumption has been dropped in subsequent work, such as (Jorgenson, Ho, & Stiroh, 2005). The equations also differ in the treatment of the investment tax credit. Both equations treat the tax benefits as a reduction in the purchase price of the asset, but it is additive in the BLS equation and multiplicative in the Hall and Jorgenson equation.

The price indexes are available at the industry by asset level. The depreciation rate is calculated from the wealth stock, which is given by:

$$K_t^W = \sum_{\tau=t}^{2t} p(\tau - t, \overline{\Omega}) \cdot I_{2t-\tau} \tag{5}$$

where I_t is investment in year t, and $p(a, \overline{\Omega})$ is the age-price function (the price of an *a* year-old asset [group] that has an average maximum service life of $\overline{\Omega}$). The age-price function is derived from the cohort age-efficiency function:

$$p(a, \overline{\Omega}) = \frac{\sum_{\alpha=a}^{\infty} \bar{s}(\alpha, \overline{\Omega}) \cdot (1-r)^{\alpha-a}}{\sum_{\alpha=0}^{\infty} \bar{s}(\alpha, \overline{\Omega})(1-r)^{\alpha}} \tag{6}$$

where *r* is the real discount rate, which is assumed to be 4% per year. Both the age-efficiency and the age-price function decline over time from 1.0, when the asset is new, to 0 at the end of its service life. The age-price function declines more quickly than the age-efficiency function, because it accounts for the decline in the remaining productivity capacity of the asset as it ages as well as the decline in current productive capacity.

The internal rate of return, r_{it}, is calculated using the accounting identity that capital income is equal to the price of capital services times the quantity of capital services:

$$Y_{i,t}^K \equiv K_{i,t}^S c_{i,t}, \tag{7}$$

where $Y_{i,t}$ is capital income in industry *i*, and $K_{i,t}$ is the productive capital stock in industry *i*.[8] It would be straightforward to calculate the internal rate of return by substituting equation (4)

[8] BLS estimates capital income for the non-corporate sector as follows: BLS calculates separate estimates of labor compensation and capital income for proprietors, sum them, and then proportionately inflates or deflates them so that they sum to proprietors' income. In the initial calculations. Note that in the initial calculations, BLS assumes that proprietors earn the same hourly wage as wage and salary workers and that non-corporate capital earns the same rate of return as corporate capital.

into equation (7) and solving for r_{it}. But capital income is available only at the industry level, which makes it necessary to modify equation (4) as follows:

$$(4') \qquad c_{it} = \frac{(1 - u_t z_t - e_t)\left(P_{i,t-1} r_{it} + P_{i,t-1} d_{i,t} - \Delta P_{i,t-1}\right)}{1 - u_t} + P_{i,t-1} x_t$$

The differences between equations (4) and (4') are that the prices and depreciation are now industry averages. The industry-level deflator for new capital goods is calculated as:

$$(8) \qquad P_{i,t-1} = \sum_{j \in J_i} \frac{K_{ij,t-1}}{\sum_{j \in J_i} K_{ij,t-1}} \cdot \frac{I^N_{ij,t-1}}{I^R_{ij,t-1}} = \sum_{j \in J_i} \frac{K_{ij,t-1}}{K_{i,t-1}} p_{j,t-1}$$

where $I^N_{ij,t-1}$ and $I^R_{ij,t-1}$ are nominal and real investment in asset *j* by industry *i*., and the capital stocks (K) are as defined in equation (3). The industry level depreciation rate is derived by aggregating industry × asset category wealth stocks into an industry wealth stock, and computing the depreciation rate as the percentage change in the wealth stock (excluding current-year investment). BLS calculates the internal rate of return by substituting equation (4') into equation 5 and solving for r_{it}.

BLS assumes that the flow of capital services is proportional to the productive capital stock. Industry × asset category capital stocks are Tornqvist aggregated, using capital cost shares for each cell as weights, into capital input. The cost shares are calculated using the capital stocks from equation (3) and the rental prices from equation (4). Capital composition is calculated by dividing capital input by the productive capital stock.

4 Jorgenson, Ho, Stiroh Methodology (JHS)

The second methodology that we consider is that of (Jorgenson, Ho, & Stiroh, 2005), which we will refer to as JHS. The main conceptual difference between the JHS and BLS methodologies is the age-efficiency function. Below, we present the relevant equations from JHS so that we can highlight the differences between the methodologies. Assuming geometric deterioration, the capital stock of asset *j* in industry *i* at time *t* is:

$$(9) \qquad K_{ijt}^{S} = \sum_{\tau=0}^{\infty} \left(1-\delta_j\right)^{\tau} I_{ij,t-1} = K_{ij,t-1}^{S}\left(1-\delta_j\right) + I_{ijt}$$

The service flow from the capital stock, capital input K_{ijt}^{I}, is assumed to be a constant proportion of the average of the current and lagged capital stock

$$(10) \qquad K_{ijt}^{I} = \kappa_{ij} \frac{1}{2}\left(K_{ijt}^{S} + K_{ij,t-1}^{S}\right)$$

where κ_{ij} is a time invariant constant of proportionality that transforms the capital stock to capital services. [9] κ_{ij} represents the "quality of capital" of type *j* and makes it clear that capital is measured in constant quality units.[10]

Because the age efficiency profile of each asset is geometric, the age-price price profile follows the same geometric pattern.[11] Thus, the tax-adjusted cost of capital is a function of this same depreciation rate and is specified as:

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[9] κ_{ij} drops out of growth accounting equations because its assumed that investment is in constant quality units, that is constant quality price deflators are used to estimate real investment.

[10] See (Jorgenson, Ho, & Stiroh, 2005)

[11] See (Jorgenson D. W., 1996)

(11) $$c_{ijt} = \frac{(1 - u_t z_{jt} - e_{jt})(P_{ijt-1} r_{ijt} + P_{ijt}\delta_j)}{1 - u_t} + P_{ijt-1} x_{it}$$

where the terms are defined as above.

The real rate of return on capital is r_{ijt}. This return is estimated using nominal rate of return that exhausts capital income across assets. In particular, the nominal rate of return is constructed to satisfy the following two equations:

(12) $$\sum_j c_{ijt} K^I_{ijt} = Y^K_{i,t}$$

and

$$r_{ijt} = \varphi[i_t - \pi_{ijt} + (1 - \varphi)[\rho_t - \pi_{ijt}]$$

where φ is the fraction of the industry's capital stock that is financed by debt, i_t is the nominal interest rate on debt[12], π_{ijt} is the asset-specific inflation rate, and ρ_t is the nominal rate of return on all assets in the industry. φ accounts for the economy's financing structure[13]; a portion of the capital is financed by issuing debt for which there is an observed interest rate, and the remainder if financed with equity which has an unobserved rate of return ρ_t that is assumed to be the same across assets.

[12] Set equal to the BAA bond rate.

[13] Calibrated with flow of funds data.

5 Comparing the Methodologies

The previous two sections described the BLS and JHS methodologies. These two methodologies follow the same basic steps, but they differ in the specifics. BLS assumes that assets deteriorate according to a hyperbolic age-efficiency function for the productive capital stock, whereas JHS applies geometric deterioration. As noted above, the hyperbolic age-efficiency function can take on different shapes depending on the value of β. BLS assumes values of β = 0.5 for equipment and β = 0.75 for structures, which result in a concave age-efficiency profile and contrasts with the convex profile implied by geometric deterioration. However, the BLS methodology accounts for heterogeneity of service lives within an asset category by constructing a "cohort age-efficiency function," which is a weighted average of age-efficiency functions of assets with different maximum service lives. (Sliker, 2015) contains a pertinent discussion of underlying weights. The JHS capital service approach simplifies the distinction between the age-price and age-efficiency of assets by assuming a geometric depreciation rate. Both BLS and JHS define the age-price function the same way, but the difference in the age-efficiency function implies that the age-price functions will take on different values. The rental prices of capital are defined similarly.

To illustrate some of the differences, we construct age-efficiency profiles for Photocopy and Related Equipment. BEA, in their estimates of wealth stock, estimates a depreciation rate of 18 percent based on an average service life of these assets of 9 years. BLS assumes that the average service life is 11 years.[14] Figure A shows the age-efficiency profiles for equipment (β = 0.5) with different service lives. All profiles are concave, but the 3-year service life profile is very steep and nearly linear. However the cohort age-efficiency function, which accounts for heterogeneity of service lives within asset categories, has a shape that is more convex. Figure B shows the corresponding cohort age-efficiency function and the geometric age-efficiency function. Note that both functions are convex, although the cohort age-efficiency function is slightly concave when the assets are new. The initial point in the geometric age-efficiency function is different from one because of the BEA mid-year convention, whereby the asset is assumed to depreciate by half the annual rate in the year of purchase. In contrast, BLS assumes no depreciation in the first year. The cohort age-efficiency function declines much more slowly than the geometric function, which implies that the BLS methodology should produce a higher estimated capital stock. The dotted line in Figure B shows the age-price function associated with the cohort age-efficiency function. Note that it lies below the cohort age-efficiency function, because it accounts for the reduction in the remaining life of the asset in addition to the decline in productive capacity. As noted above, BEA uses the same formula for the age-price function, except that the geometric age-efficiency function is used. It is well known that this age-price function reduces to the age-efficiency function.

To compare capital stocks, we continue our example using simulated data.[15] Figure C shows the capital stock for the BLS and BEA methodologies and for the BLS methodology using

[14] BLS compares alternative cohort age-efficiency profiles, and selects the one that most closely approximates the BEA depreciation rate on average. Typically BLS's average service lives are slightly longer that BEA's.

[15] We assume an initial investment of $1,000 in 1900, and that investment increases by 2.5% per year on average (actual growth rates are perturbed).

the BEA average service life. As expected the BLS methodology generates the largest estimated capital stock, while the BEA methodology generates the smallest. Figures D and E compare growth rates and log changes. Here we see much smaller differences. The largest differences are in the early years when investment is large relative to the capital stock. After about 1920, growth rates are remarkably similar, exhibiting only minor differences. Figure F shows wealth stocks. Here, we see much smaller differences between the BLS and BEA methodologies and that the BEA estimate of the wealth stock is slightly larger than the BLS estimate. Figures G-H show that, as with productive capital stocks, growth rates are quite similar.

Perhaps a clearer way to see the differences is to compare differences in the average growth rates to the average of the absolute value of differences in growth rates. Once we hit the steady state (from about 1920 onward), the two methodologies produced very similar long run growth rates of about 2.5 percent per year.[16] But comparing average absolute differences, we found a much larger difference of 0.15 of a percentage point. Thus the two methods produce very similar results in the long run, but can produce different results in the short run.

A second methodological difference is the calculation of the rental price. BLS uses an internal rate of return in its rental price formula, whereas the rate used in the JHS methodology is a weighted average of external and internal rates of return (weighted by the share of capital financed by debt and equity financing). To reiterate the point made above, the differences in the methodologies are relatively simple to compare in abstract, with graphics like those in

[16] This makes sense, because investment grows by about 2.5 percent per year, and the depreciation rate is about the same in all years under each methodology. If we start at 1911, the difference in average growth rate is about 0.2 of a percentage point. Therefore it is the depreciation in the early years that accounts for the difference in stocks. When we compute capital stocks as an index with a base year of 1920, we found that the stocks differed by only 0.2 of percentage point over the 95 year period.

figures A-E. We turn next to how these effect empirical estimates of capital services and the impact on measures of growth and productivity.

6 Data and Implementation

The purpose of this section is to describe the underlying source data and how the two methodologies differ in their implementation of the capital estimates described above.

6.1 Investment

BEA's fixed investment statistics are the primary data used by both BLS and BEA to estimate capital stock measures. Benchmark levels for fixed investment statistics are established every five years by BEA within an input-output (I-O) framework that balances and reconciles industry production and commodity usage.[17] Annual fixed investment statistics are interpolated between benchmark levels and extrapolated forward from the most recent benchmark statistics. In this section, we describe how these measures are developed for each of the broad types of fixed investment (i.e. equipment, structures and intellectual property products), including a discussion of selected adjustments necessary to conform these measures from a final* demand-based measure (reflected in the industry economic accounts (IEA) and the

[17] For detail on the most recent benchmark I-O account, see Donald D. Kim, Erich H. Strassner, and David B. Wasshausen, "Results of the Comprehensive Revision, Revised Statistics for 1997–2012," SURVEY 94 (February 2014): 1-18.

national income and product account (NIPA) statistics) to investment flows necessary for building capital stock (henceforth referred to as "capital stock-basis").[18]

6.1.1 Equipment

Estimates of fixed investment in equipment are largely constructed using the commodity-flow method, which is a "supply-side" approach that traces commodities from their domestic production or importation to their final purchase. Imports are added and exports are subtracted from estimates of domestic output (or domestic sales). The resultant measure is domestic supply that is then allocated among domestic purchasers (i.e. persons, business, and government). Further allocations are made within the business and government sectors to distinguish between investment and consumption. These allocations are based on a number of variables, including class of customer data collected in the Economic Census, as well as the detailed descriptions that underlie these commodities. (For example, a commodity description may explicitly include the phrase, "for commercial use" which we then allocate to the business sector.) Prices used to deflate these investment flows are largely based on Bureau of Labor Statistics (BLS) Producer Price Indexes (PPIs).

There are no adjustments necessary to transform investment flows for new equipment to a capital-stock basis; however, there are a couple of adjustments required for used equipment. First, dealers' margin on used equipment is removed from the IEA and NIPA-based measures of investment. Second, used automobiles are valued differently in the capital stock

[18] For example, a timing adjustment is made for fixed investment in electric power plants to account for when a plant is constructed (IEA and NIPA treatment) versus when it is actually placed into service (capital stock treatment).

measures versus the IEAs and the NIPAs. In BEA's capital stock statistics, used autos are valued at acquisition prices less depreciation; in the IEAs and in the NIPAs, net purchases of used autos by business from consumers are valued at wholesale prices of used automobiles.[19]

6.1.2 Structures

For structures, the benchmark I-O estimates are primarily based on detailed value-put-in-place data from the Census Bureau's monthly Survey of Construction Spending. The "value of construction put in place" is defined as the value of construction installed or erected at the construction site during a given period, regardless of when the work on the project was started or completed, when the structure was sold or delivered, or when payment for the structure was made. Prices used to deflate these investment flows reflect a variety of sources, including BLS PPIs, cost indexes from trade sources and government agencies, and the Census Bureau price index for single-family houses under construction.

There are two adjustments required for transforming fixed investment in structures to a capital stock-basis. First, a timing adjustment for electric power plants that reflects differences between when plants are built versus when they are put in use. The former is appropriate for final demand-based measures, while the latter is appropriate for developing capital stock. Second, similar to equipment, brokers' commissions on sale of nonresidential structures are excluded from investment flows used to construct capital stock.

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[19] Automobiles are the lone asset-type in which the physical inventory method is used to construct the capital stock. More information on this is presented in the capital stock section.

6.1.3 Intellectual property products

Fixed investment in intellectual property products (IPP) includes research and development (R&D), software and entertainment, literary and artistic originals. The unique, one-off nature of these types of assets presents some measurement challenges for both expenditures and prices. These assets also typically include a sizable portion of "own-account" production -- that is in-house production of an asset specifically for that entity's own use-- which is estimated using the costs of production because there are no observable market transactions. Expenditures for purchased R&D are based on information from the National Science Foundation's R&D surveys, while expenditures for purchased software are estimated using Census statistics and the commodity-flow procedure. Price indexes for IPP primarily reflect PPIs and input costs that implicitly miss changes in productivity. For research and development, an explicit adjustment is made to the input cost-based price index by subtracting the growth rate of BLS' private nonfarm business sector multi-factor productivity from the growth rate of the R&D input cost index.[20] There are no adjustments required for transforming final demand-based fixed investment in IPP to a capital stock-basis.

[20] For additional details on the methodology used to estimate expenditures and prices for research and development, see Marissa J. Crawford, Jennifer Lee, John E. Jankowski, and Francisco A. Moris, "Measuring R&D in the national Economic Accounting System," SURVEY 94 (November 2014): 1-15. For additional details on the methodologies used to estimate software and entertainment, literary and artistic originals, see chapter 6 of the NIPA Handbook: Concepts and Methods of the U.S. National Income and Product Accounts.

6.2 Wealth Stock

BEA's fixed assets accounts include estimates of wealth stocks and depreciation by industry, by type of asset and by legal form of organization. Investment flows based on the final demand-based fixed investment, as described in section 6.1, are the building blocks of these net stocks and depreciation. One of the primary purposes of constructing a *wealth* stock, as opposed to a *productive* stock, is to develop measures of economic depreciation (i.e. consumption of fixed capital or CFC), which is a charge against gross income earned from current period production. Subtracting consumption of fixed capital from gross domestic product yields a rough measure of the level of consumption that can be maintained while leaving capital assets intact. Similarly, deducting CFC from gross fixed investment provide an indicator of whether or not capital stocks are in a steady state, period of increase or period of decline.

The vast majority of BEA's net stocks are calculated using the perpetual inventory method (PIM) assuming geometric depreciation. The PIM is relatively simple way to indirectly measure stock by cumulating past investment flows. The following equation specifies the PIM:

$$K_t = K_{t-1}(1-\delta) + I_t\left(1-\frac{\delta}{2}\right)$$

where K_t represents net stock in period 't', δ represents the rate of depreciation and I_t represents investment in period '*t*'. Note, new assets (I_t) are assumed to be placed in service midyear; thus the depreciation rate is halved for period 't'.

In the perpetual inventory method, the pattern of depreciation charges for an asset of a given type is determined by its "depreciation profile"—that is, how the price of that type of asset declines as it ages in the absence of inflation. The profile for a given type of asset of a given vintage is assumed to be constant over time. However, the profile for one vintage of a given asset may differ from the profile of a different vintage of the same asset. For most asset types, BEA assumes geometric depreciation because the available data suggest it more-closely approximates actual price declines than straight-line depreciation. The geometric rates are determined by dividing the appropriate declining-balance rate by the assumed service life.[21] A numeric example illustrating the PIM using a geometric depreciation rate is presented in the appendix.

The physical inventory method applies independently estimated prices to a direct count of the number of physical units of each type of asset. The physical inventory method is more direct, but requires significantly more data than the perpetual inventory method. BEA uses the physical inventory method only for autos because they are the only type of asset for which detailed data on the prices and number of units of used assets in the stock of each vintage is available (from registration data). For all other assets, the perpetual inventory method is used.

[21] For additional detail on BEAs chosen depreciation profiles, see Barbara M. Fraumeni, "The Measurement of Depreciation in the U.S. National Income and Product Accounts," SURVEY 77 (July 1997): 7-23 and U.S. Department of Commerce, Bureau of Economic Analysis, "Fixed Assets and Consumer Durable Goods in the United States, 1925–99," September, 2003.

6.3 Capital Income

BEA's Industry Accounts provide information on the components of value added that are used in estimating capital income.[22] Conceptually, capital income includes all payments to capital assets used in production. In practice, capital income by industry is often computed as residual based on the accounting identity after subtracting off all payments to labor, thus we start with a description of the nominal accounts.[23] Data on the current dollar gross output are largely derived from the Census Bureau's Economic Census, the Annual Survey of Manufactures (ASM), the Services Annual Survey (SAS), and other annual and higher-frequency surveys, such as the Annual Retail Trade Survey and the Manufacturers' Shipments, Inventories, and Orders. Similarly, current-dollar intermediate inputs (energy, materials, and services) are based on the collection of business expense data from the Economic Census and the ASM. These include materials consumed data for the mining and manufacturing sector as well as other operating expenses that are collected on the ASM. For the service-sector industries, the Business Expenses Survey (BES) and the SAS provide the basis for the input structure. Compensation of employees is largely derived from wage data tabulated on the Quarterly Census of Employment and Wages (QCEW). Taxes on production and imports less subsidies are derived from the

[22] In particular, capital income is the difference between current-dollar gross output less the sum of current-dollar intermediate inputs, compensation of workers, and taxes on product and imports, less subsidies plus the value of property taxes paid. See (Strassner, 2013) for details on the methodology used to construct the BEA Industry Accounts.

[23] This exposition applies in the case of industries with no non corporate capital income. In cases where there is a significant non-corporate sector, corporate capital income is estimated from the GDP by Industry data, and non-corporate capital income is the residual from the accounting identity. This allows for modeling of the different tax components of the user cost in the non-corporate sector. See (Jorgenson, Ho, & Stiroh, Information Technology and the American Growth Resurgance, 2005) for additional detail. This implies of a portion of partnerships' and proprietors' income is reallocated from the gross operating surplus component of value added to account for labor income payments for the self-employed.

Internal Revenue Service and tabulations from state collection agencies. It is noteworthy that the published BEA GDP by Industry accounts do not include an estimate of self-employed labor compensation, but the BEA-BLS integrated account includes an estimate of self-employed labor in total labor compensation.

The principal source data for the capital income component of current-dollar value added are tabulations of tax returns for corporations, partnerships and proprietorships by the U.S. Department of Treasury Internal Revenue Service Statistics of Income (SOI) program. These data are used to estimate business income in the national income and product accounts (NIPA), which form the basis for constructing measures of gross operating surplus on an establishment basis in BEA's Industry Accounts.

6.4 Rates of Return

The calculation of capital services prices by industry and asset requires an estimate of the rate of return on capital. Both BLS and BEA use an internal rate of return, which exhausts capital income across assets, under the assumption that all income not paid to labor or intermediate input is a payment for capital services. Conceptually, the BLS and JHS rate of return is similar, but the JHS approach includes a weighted average rate of return over debt and equity financing. Although the calculation of capital rental prices is relatively straightforward, it is possible that calculated rental rates are negative.

JHS and BLS handle negative capital rental rates differently.[24] BLS uses external rates of return that are calculated at a higher level of aggregation. The JHS approach deals with this by

[24] Negative rental rates can happen when the capital gain term is large, or when capital income is low.

first reallocating labor income to capital income, and second by imputing capital income. This is mostly done in an ad hoc manner.

6.5 Capital Service Prices

As discussed above, the BLS and JHS approach to measuring capital service prices is similar. Like rates of return, actual capital service price estimates reflect implementation choices. The incorporation of asset specific rates of return into the estimate of capital services is important because it captures expected price changes investor face when making investment decisions. For example, (Jorgenson, Ho, & Stiroh, 2005) argue that expected price declines in IT equipment are related to the relatively high marginal product of IT equipment. That is, investors anticipate the price decline, and this price decline must be compensated with a higher rental rate, and the marginal product reflects this. While there is no theoretical reason that rental prices cannot be negative, negative prices present a problem when aggregating assets because the capital cost shares are used as weights. To avoid negative capital service prices, conditional on a reasonably positive rate of return, JHS smooths the capital gains terms. This is done in a mostly arbitrary fashion; for example, the land capital gains are taken to be the average for the period as a whole, but for other assets centered averages around the problematic years replace the observed capital gain. Because the rate of return is endogenous, this involves re estimating the rate of return and recalculating the prices of all assets.

7 Growth Accounting Framework

We compare the sources of aggregate output growth using industry-level production account framework described in (Rosenthal, Russell, D., H., & Usher, 2014). These accounts cover 63 industries that span the entire domestic U.S. economy for the 1998-2012 period.[25]

We use a growth accounting framework to examine how estimates of the contribution of capital to growth differ when using the two capital measures. The derivation of the equations is described in detail in (Jorgenson, Ho, & Stiroh, 2005), but we present the relevant equations here. The starting point is an industry growth accounting equation:

$$\Delta \ln MFP_i = \Delta \ln Q_i^G - s_{K,i} \Delta \ln K_i^I - s_{L,i} \Delta \ln L_i - s_{X,i} \Delta \ln X_i$$

where Q_i^G is the quantity index for industry growth output, K_i^I is capital input quanitty, L_i is labor input quantity, and X_i is industry intermediate input growth. Each input growth is weighted by its value share in gross output, $s_{K,i}$, $s_{L,i}$, $s_{X,i}$, and MFP growth is the residual. This equation is implemented for each time period and the value shares are the average of period t and t-1. By construction, the value shares sum to one.

Capital input can be decomposed into the contribution of increases in the capital stock and changes in the composition of asset types in the capital stock:

$$K_{i,t}^I = K_{i,t}^I \cdot \frac{K_{i,t}^S}{K_{i,t}^S} = \frac{K_{i,t}^I}{K_{i,t}^S} \cdot K_{i,t}^S$$

And the growth of capital input can be decomposed as follows:

[25] The account of (Rosenthal, Russell, D., H., & Usher, 2014) covers 63 industries. For consistency in coverage, this account aggregates transportation equipment and government. (Jorgenson, Ho, & Samuels, 2011) covers the 65 in which the transportation equipment components are disaggregated, and government covered federal and state and local, subdivided into general and government enterprise.

$$\Delta \ln K_{i,t}^{I} = \left(\Delta \ln K_{i,t}^{I} - \Delta \ln K_{i,t}^{S}\right) + \Delta \ln K_{i,t}^{S},$$

so that the industry growth accounting equation is rewritten as

$$\Delta \ln MFP_i = \Delta \ln Q_i^G - s_{K,i}\left(\left[\Delta \ln K_{i,t}^{I} - \Delta \ln K_{i,t}^{S}\right] + \Delta \ln K_{i,t}^{S}\right) - s_{L,i}\Delta \ln L_i \ - s_{X,i}\Delta \ln X_i$$

In our empirical exercise, differences attributed productive stock growth reflect differences in how the productive stock is constructed, while differences attributed to capital quality growth also reflect differences in capital services prices.

To analyze how the capital measures impact the aggregate sources of growth, we use aggregate across industries using the "direct aggregation across industries" method.[26] In particular, aggregate real value added growth is a weighted contribution of industry input growth and industry MFP growth.

$$\Delta \ln V = \sum_i \overline{w_i}\frac{\bar{s}_{K,i}}{\bar{w}_{V,i}}\Delta \ln K_i^I + \overline{w_i}\frac{\bar{s}_{L,i}}{\bar{w}_{V,i}}\Delta \ln L_i^I + \overline{w_j}\frac{1}{\bar{w}_{V,i}}\Delta \ln MFP_i$$

Where $\overline{w_j}$ is the industry value added share in aggregate value added, $\bar{s}$ is each input's share industry in industry gross output, and $\bar{w}_{V,j}$ is the industry value added share in the value of industry gross output.[27] We modify this to explicitly account for the contributions of productive capital stock and capital quality, where the weighted change of $\Delta \ln K_i^I - \Delta \ln K_i^S$ represents that contribution of industry capital quality growth to aggregate value added growth.

$$\Delta \ln V = \sum_i \overline{w_i}\frac{\bar{s}_{K,i}}{\bar{w}_{V,i}}(\Delta \ln K_i^I - \Delta \ln K_i^S + \Delta \ln K_i^S) + \overline{w_i}\frac{\bar{s}_{L,i}}{\bar{w}_{V,i}}\Delta \ln L_i^I + \overline{w_j}\frac{1}{\bar{w}_{V,i}}\Delta \ln MFP_i$$

[26] Described in (Jorgenson, Ho, & Stiroh, 2005)

[27] Bar indicates average of period t and t-1 shares.

8 Capital Alternatives and the Sources of Growth

8.1 Aggregate Sources of Growth

Table 1 shows the sources of growth based on the two methodologies. By construction, aggregate value added growth and the contribution of labor inputs are the same under the BLS and JHS treatments of capital. Here, we see that the sources of aggregate output growth is not sensitive to the measure of capital input used. Based on our earlier simulation results, this should not be too surprising. Note that we held the cost shares constant to isolate the effect methodological differences. So the small differences in the contribution of capital input reflect only differences in the growth rates of capital input. For the 1998-2012 period, capital input contributed 1.13 percentage points per year over the period based on the BLS method, and 1.11 percentage points per year based on the DJA method. The difference between the two capital measures results in very minor differences in aggregate measured MFP growth per year over that period. By the subperiods that we consider, the differences are minor as well, with relatively faster MFP growth of the BLS measure for the 2007-2009 period being mostly offset by relatively slower growth in the 2009-2012 period, resulting in very similar estimates for the 2007-2012 period as a whole.

8.2 Industry Sources of Growth

Figure 1 compares the capital input growth rates by industry. Overall, the growth rates of the two measures are similar, but they differ for some industries. The correlation between the growth rates of the measures is about 0.93 overall, with notable outlier being the capital input growth rates of Securities, Commodities, and Investments; Educational Services; Computer Systems Design; Management of Companies; Air Transportation; and Other Services.

The growth rate comparisons in Figure 1 show the sum the growth rates of the productive capital stock and capital composition (quality) as described above. Figure 2 shows that the productive capital stock estimates are even more highly correlated than the capital input growth rates. The overall correlation coefficient is 0.97, and even the outliers—Data Processing, Internet Publishing, and Information Services; Amusements; Support Activities for Mining; and Forestry and Fishing—are fairly close to the 45-degree line. Figure 3 shows that many of the differences in capital input growth are due to differences in estimates of capital quality. The overall correlation between the two estimates of capital quality is 0.74, with relatively large differences in Management of Companies; Educational Services; Securities, Commodity Contracts, and Investments; and Other Services.

The impact of the capital on the industry sources of growth also depends on the capital income share. That is, even large differences in estimates of capital input growth may not lead to significant differences in the estimates of the industry sources of growth if the capital income share is small. Figure 4 shows the differences in the capital contribution to output growth by industry; note that by construction these represent the differences in MFP growth by industry as well. The relatively large differences in Management of Companies, for example, is dampened by the relatively low capital share in that industry. So that estimates of the capital contribution to growth no longer appear as a significant outlier. On the other hand, in industries with a relatively large capital share, such as Pipelines, differences in the capital input growth rate are magnified by a relatively large capital share.

Finally, differences in industry contributions to aggregate growth are shown in Figure 5. These differences reflect differences in the capital contribution by industry and each industries Domar weight. Figure 5 shows that overall differences in industry contributions to aggregate MFP growth were mostly in line, except for the Real Estate sector. Because the Real Estate industry (which includes owner occupied housing) has a large weight, even relatively small differences have the potential to impact aggregate MFP estimates.

9 Capital Quality Differences and Industry Characteristics

The results in Figures 1-5 indicate that much of the difference between the two capital estimates are due to differences in the estimates of capital composition. The framework for estimating the capital composition in the two approaches is similar, thus we speculate that a large portion of the difference relates to implementation choices in the user cost (discussed above). In this section, we relate these differences to observable industry characteristics. We do this for two reasons. The first is to highlight industry characteristics that result in different estimates of capital composition; the second is to identify industry characteristics that may be problematic when generating estimates of capital services. Table 6 shows the results of a regression of the absolute value of the difference in capital quality growth by industry between the two methods and selected industries characteristics, in particular the growth of the capital stock, the land share in the capital stock, the gross operating surplus as a share of industry value added, the industry's share in aggregate value added, and the share of self-employed workers in the industry.[28] At the 5% level, industries with relatively rapid capital stock growth are associated with larger differences in capital quality growth, while industries with a higher share of gross operating surplus have relatively less of a difference in the estimates for capital quality growth. The latter results is intuitive and relates to the implementation of the user cost in that industries with a low gross operating surplus often have a low rate of return. With low enough nominal rates of return, the real rate of equation may become negative, leading to some of the implementation issues discussed above.

10 Conclusion

Industry production accounts are a useful tool for analyzing the drivers of growth, especially when they are integrated with the national accounts and are internationally comparable. There is broad consensus on the importance of measuring capital services for construction of these production accounts, e.g. (OECD, 2009). The World KLEMS and EUKLEMS initiatives are consortiums of academics, researchers, and practitioners that follow the same basic approach

[28] Thanks for Barbara Fraumeni for suggesting that the share of self-employed workers may complicate estimates of capital income and thus the rate of return to capital.

to capital services measurement so that datasets are internationally comparable. The estimation of capital services requires a number of assumptions, such as how assets deteriorate over time, how internal rates of return are calculated, and how individual assets are aggregated.

In this paper, we have compared two approaches to capital services measurement, the approach used in the official statistics of the BLS, and that used by (Jorgenson, Ho, & Stiroh, 2005) in research work. Between these approaches, for the period that we consider (1998-2012) the two approaches generated growth rates of aggregate productive capital stocks that were similar. Capital services growth rates by industry are mostly in line as well, but some there were differences in some industries. Because the productive stocks are similar, the differences between the two approaches to measuring capital input manifests mostly in differences in capital quality. Capital quality reflects underlying differences in the weights used in aggregating assets to industry-level capital input. Because the formulation of these weights in the two approaches is conceptually similar, we attribute the large majority of the differences in the estimates on the contribution of capital to economic growth across industries to implementation choices related to negative rates of return and negative capital services prices. We conclude that future research should focus on developing standards for dealing with these implementation issues. Nevertheless, for analysts and researchers that desire a long time series, the results in this paper suggest that combining historical capital inputs from (Jorgenson, Ho, & Samuels, 2016), for example, with more recent data from the BEA-BLS integrated production account produces a reasonably consistent historical time series of capital input by industry.

The similarity between the sources of growth based on the two methodologies that we consider does not mean that the methodological differences are trivial. Estimation of service lives and depreciation rates is an ongoing area of research, as is whether the scope of capital assets should include investments such as spending on organizational capital. Measuring rates and return, and understanding why measured rates differ across industries depends on capital measurement, and understanding shorter term fluctuations in economic activity requires consideration of utilization rates and investment flows over the business cycle. The methods that we have considered for capital service measurement are predicated on holding quality fixed; whether current methods sufficiently adjust for quality change in a subset of capital goods, such and residential structures and land, deserves further research attention.

Figure 1: Capital Input Growth Rates 1998-2012: JHS versus BLS

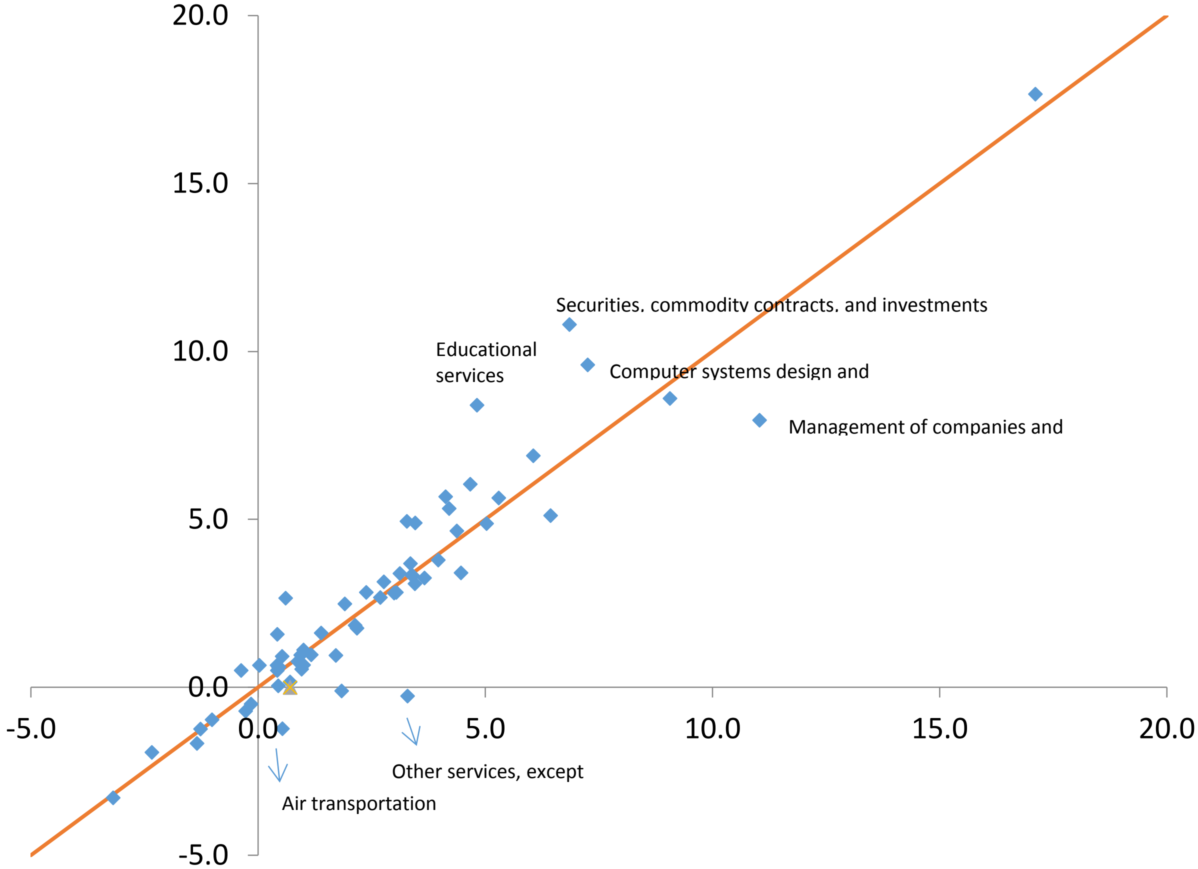

Figure 2: Productive Capital Stock Growth Rates 1998-2012: JHS versus BLS

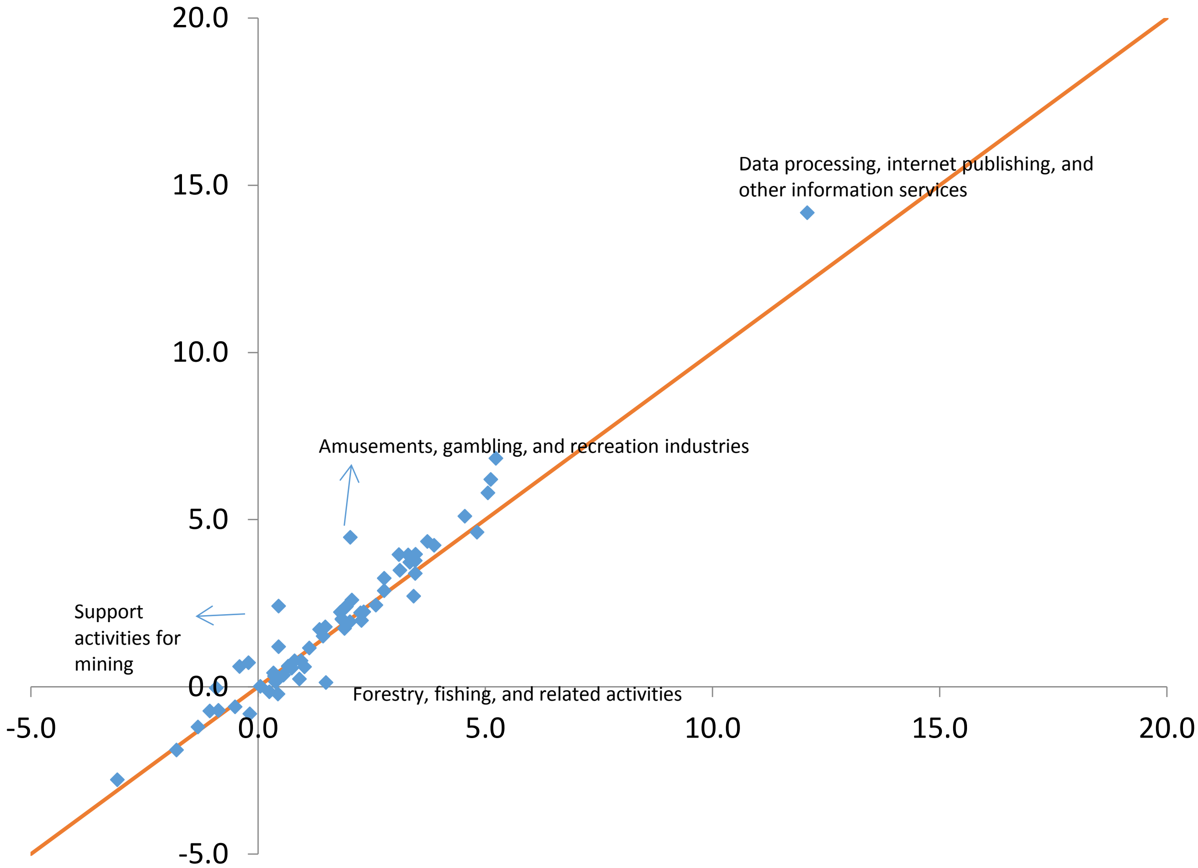

Figure 3: Capital Quality Growth Rates 1998-2012: JHS versus BLS

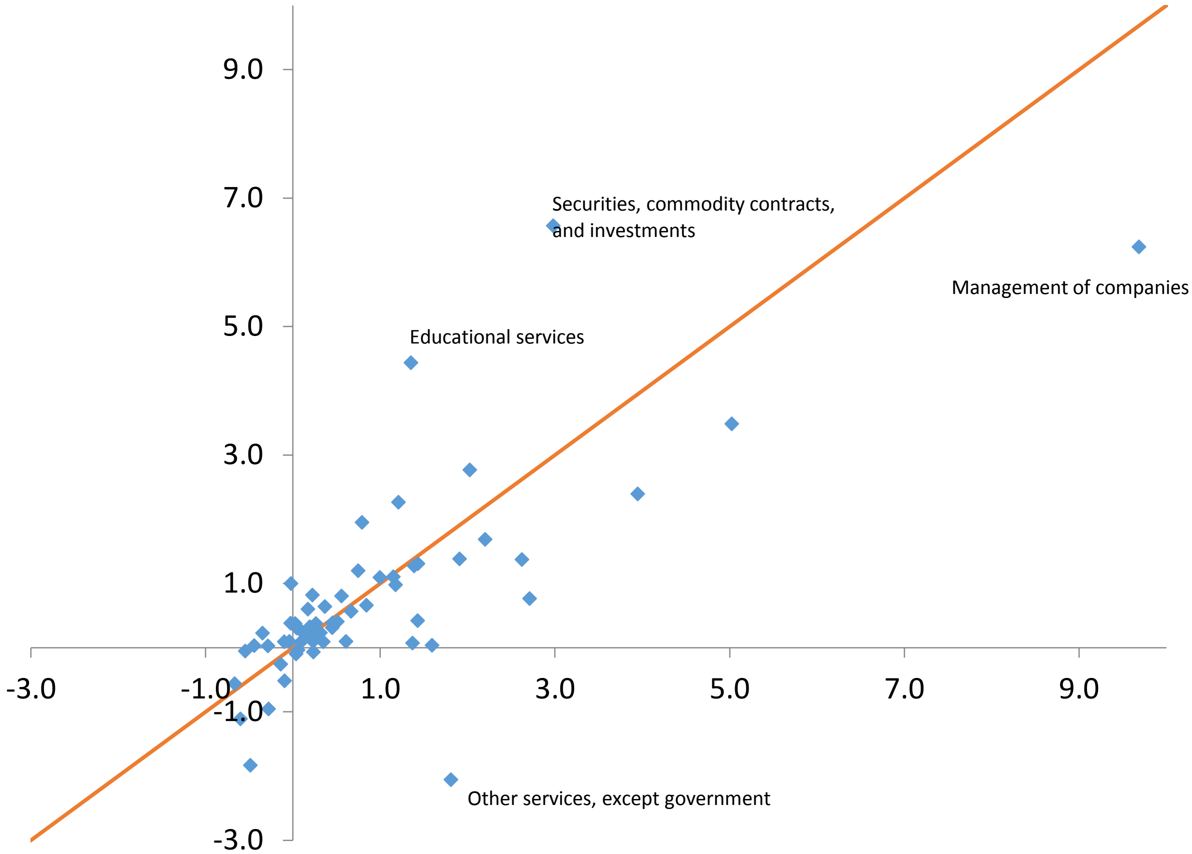

Figure 4: Capital Contributions to Industry Growth 1998-2012:JHS versus BLS

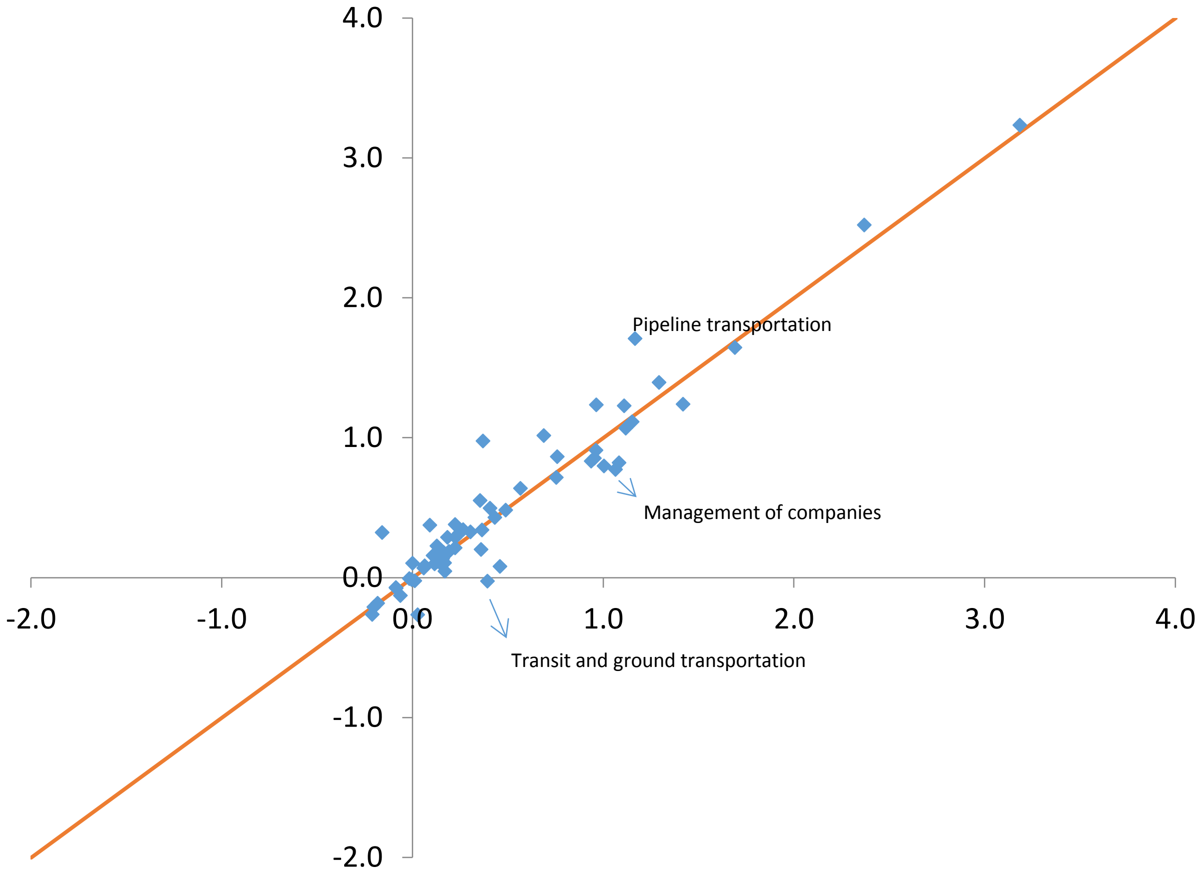

Figure 5: Industry Contributions to Aggregate MFP Growth

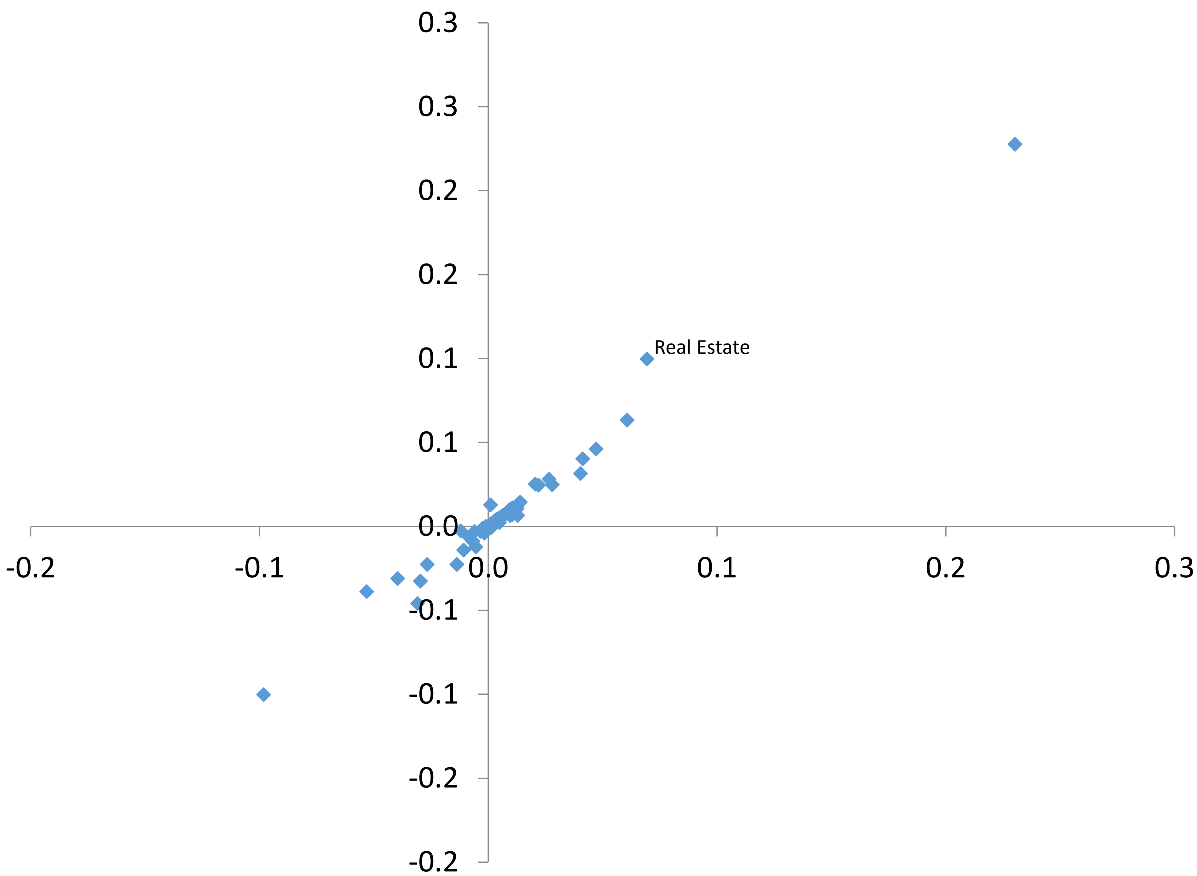

Table 6 Regression Results

VARIABLES	(1) Abs value of Difference in Capital Quality Growth
Stock Growth	0.0878***
	(0.0314)
Land Share of Stock	-0.333
	(0.742)
Operating Surplus/VA	-1.572**
	(0.590)
Industry Share in Agg. GDP	-3.524
	(5.809)
Industry Self-employed Worker Share	-0.0678
	(1.043)
Constant	1.142***
	(0.289)
Observations	60
R-squared	0.240

Dependent variable is the absolute value of the annual average difference in capital quality growth over the period, by industry. Independent variables are averages over the period.
Robust standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

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Table 1: Growth in Aggregate Value-Added and the Sources of Growth
Direct Aggregation across Industries

	1998-2012	1998-2007	2007-2012	2007-2009	2009-2012
		Contributions BLS			
Value-Added	1.94	2.71	0.56	-1.69	2.06
Capital Input	1.13	1.51	0.44	0.60	0.34
Labor Input	0.38	0.63	-0.09	-1.38	0.77
College Labor	0.55	0.66	0.35	-0.11	0.66
Non-college Labor	-0.17	-0.02	-0.44	-1.27	0.11
MFP	0.44	0.56	0.21	-0.91	0.96
		Contributions JHS			
Value-Added	1.94	2.71	0.56	-1.69	2.06
Capital Input	1.11	1.48	0.45	0.70	0.29
Labor Input	0.38	0.63	-0.09	-1.38	0.77
College Labor	0.55	0.66	0.35	-0.11	0.66
Non-college Labor	-0.17	-0.02	-0.44	-1.27	0.11
MFP	0.45	0.59	0.20	-1.01	1.01

Notes: Average annual percentages. Aggregate value added growth is the aggregate of share weighed industry value added growth. The contributior is the domar-weighted industry contributions.

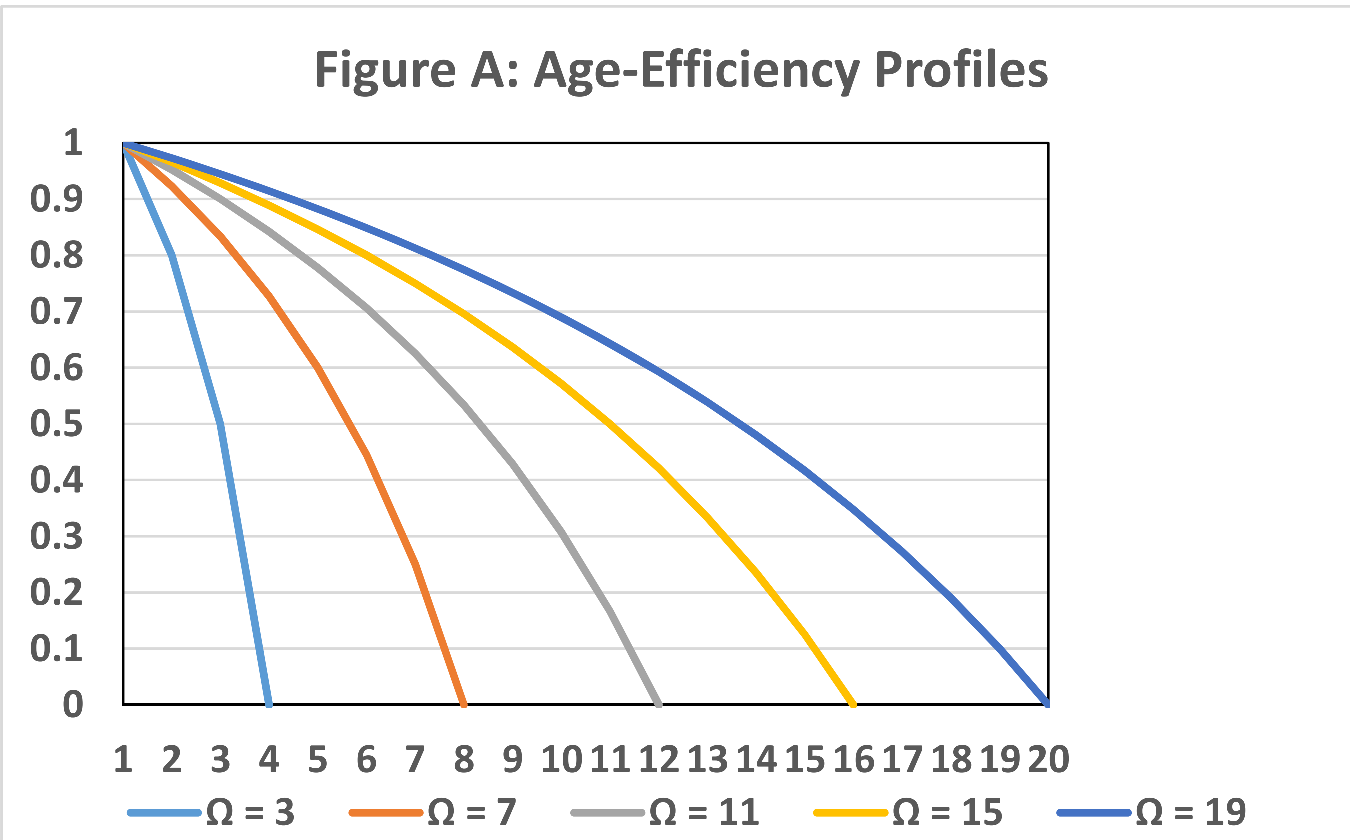
Figure A: Age-Efficiency Profiles
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20
Ω = 3
Ω = 7
Ω = 11
Ω = 15
Ω = 19

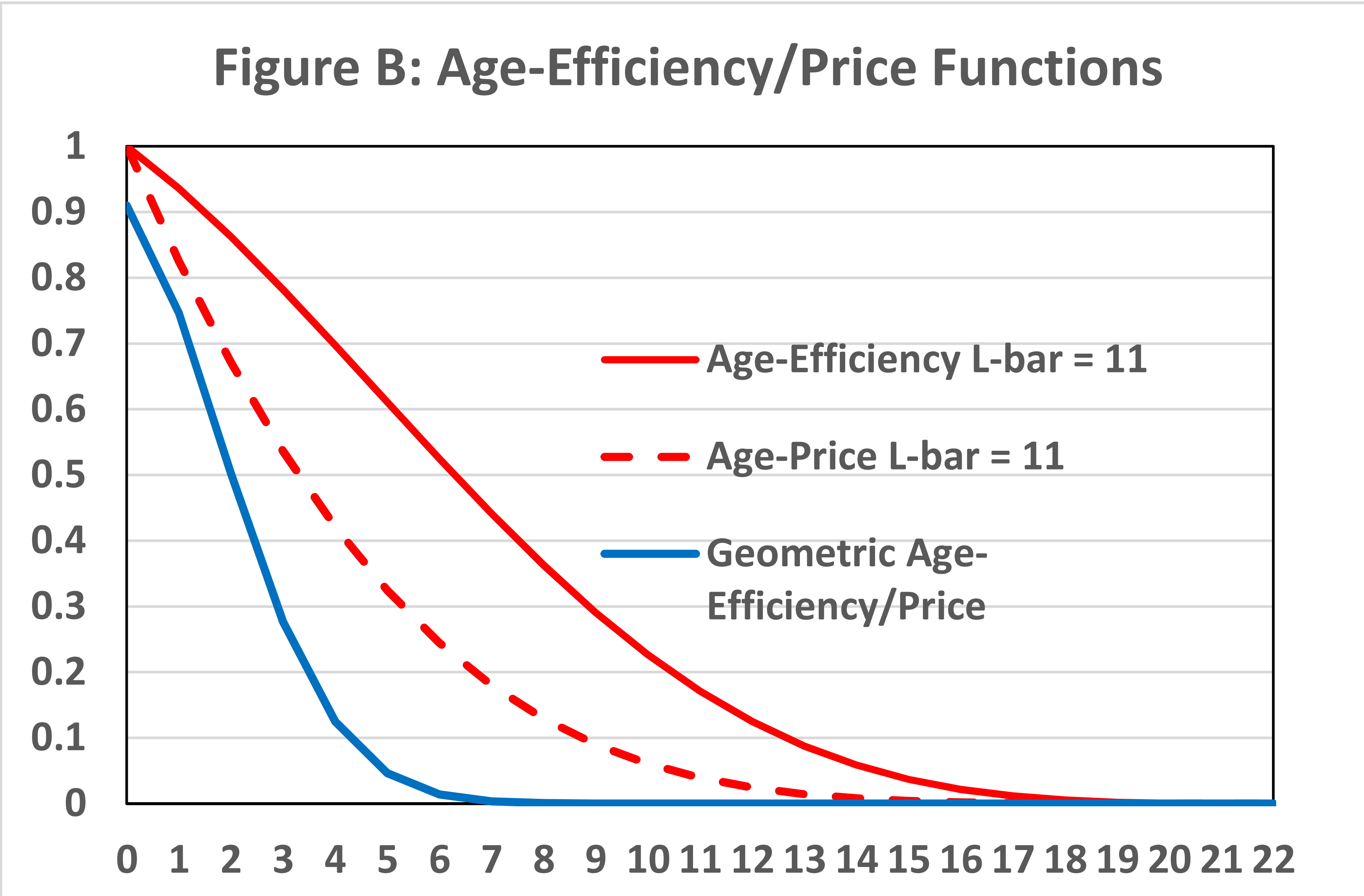
Figure B: Age-Efficiency/Price Functions
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22
Age-Efficiency L-bar = 11
Age-Price L-bar = 11
Geometric Age-Efficiency/Price

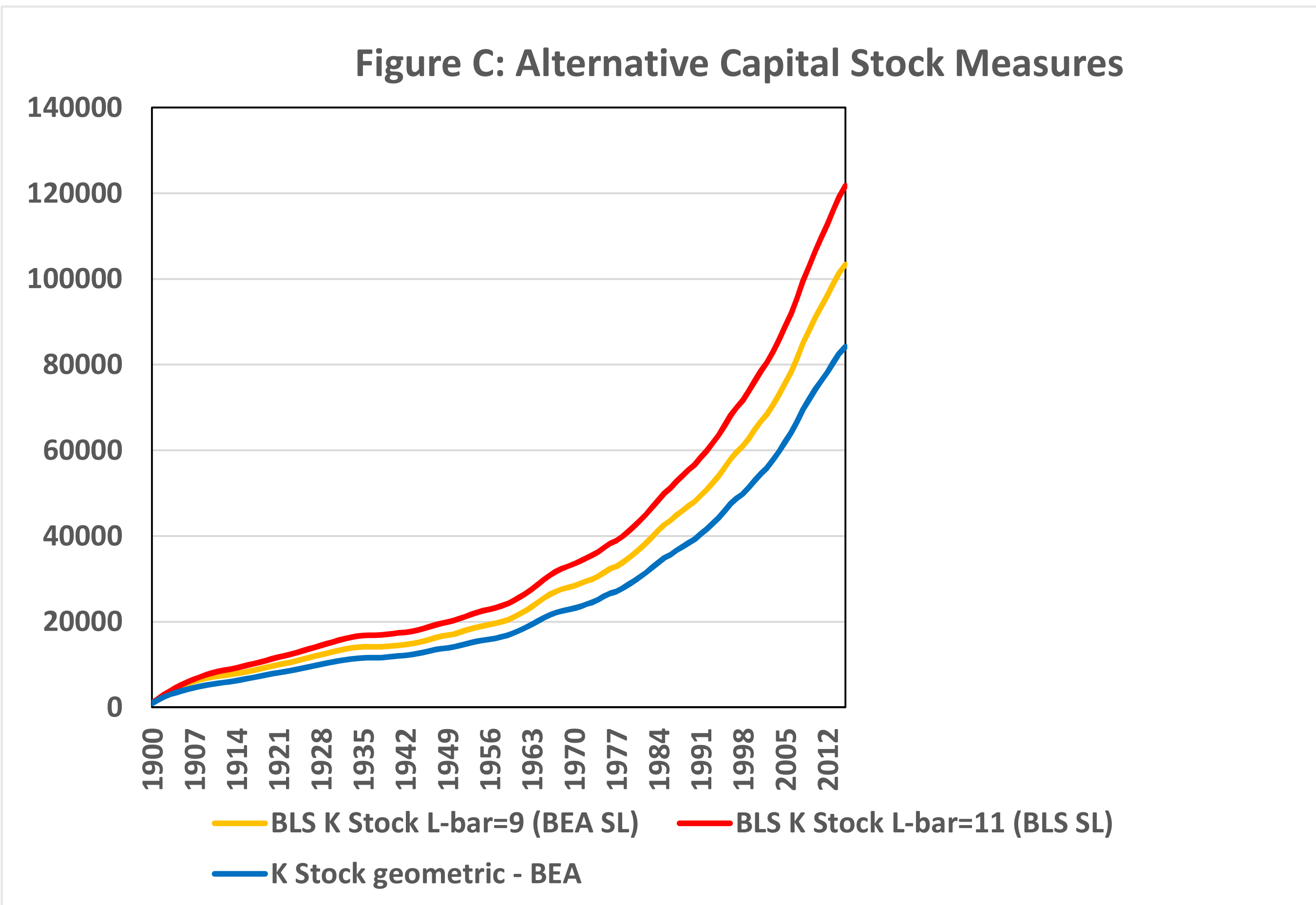
Figure C: Alternative Capital Stock Measures
140000
120000
100000
80000
60000
40000
20000
0
1900
1907
1914
1921
1928
1935
1942
1949
1956
1963
1970
1977
1984
1991
1998
2005
2012
BLS K Stock L-bar=9 (BEA SL)
BLS K Stock L-bar=11 (BLS SL)
K Stock geometric - BEA

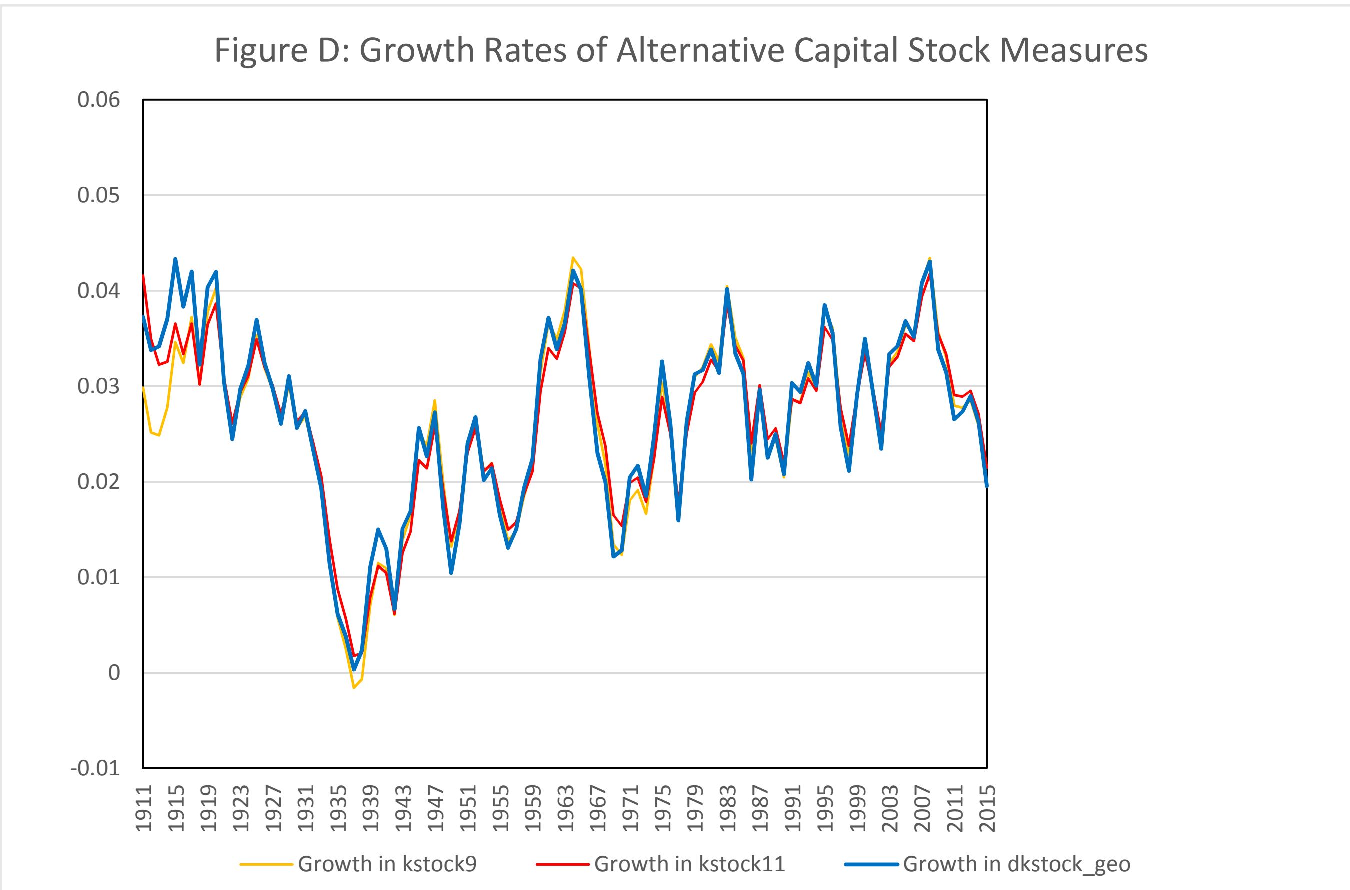
Figure D: Growth Rates of Alternative Capital Stock Measures
0.06
0.05
0.04
0.03
0.02
0.01
0
-0.01
1911
1915
1919
1923
1927
1931
1935
1939
1943
1947
1951
1955
1959
1963
1967
1971
1975
1979
1983
1987
1991
1995
1999
2003
2007
2011
2015
Growth in kstock9
Growth in kstock11
Growth in dkstock_geo

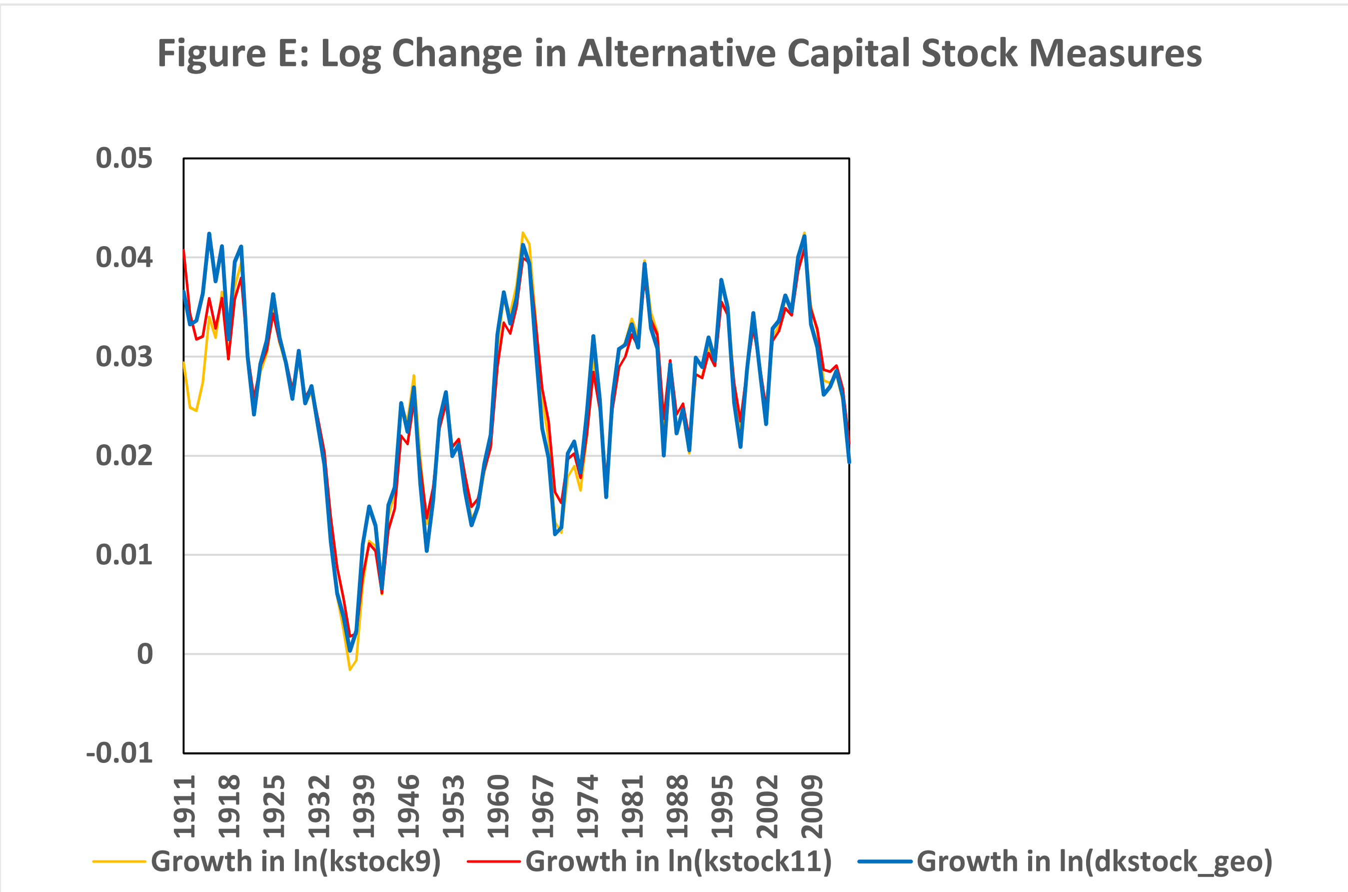
Figure E: Log Change in Alternative Capital Stock Measures
0.05
0.04
0.03
0.02
0.01
0
-0.01
1911
1918
1925
1932
1939
1946
1953
1960
1967
1974
1981
1988
1995
2002
2009
Growth in ln(kstock9)
Growth in ln(kstock11)
Growth in ln(dkstock_geo)

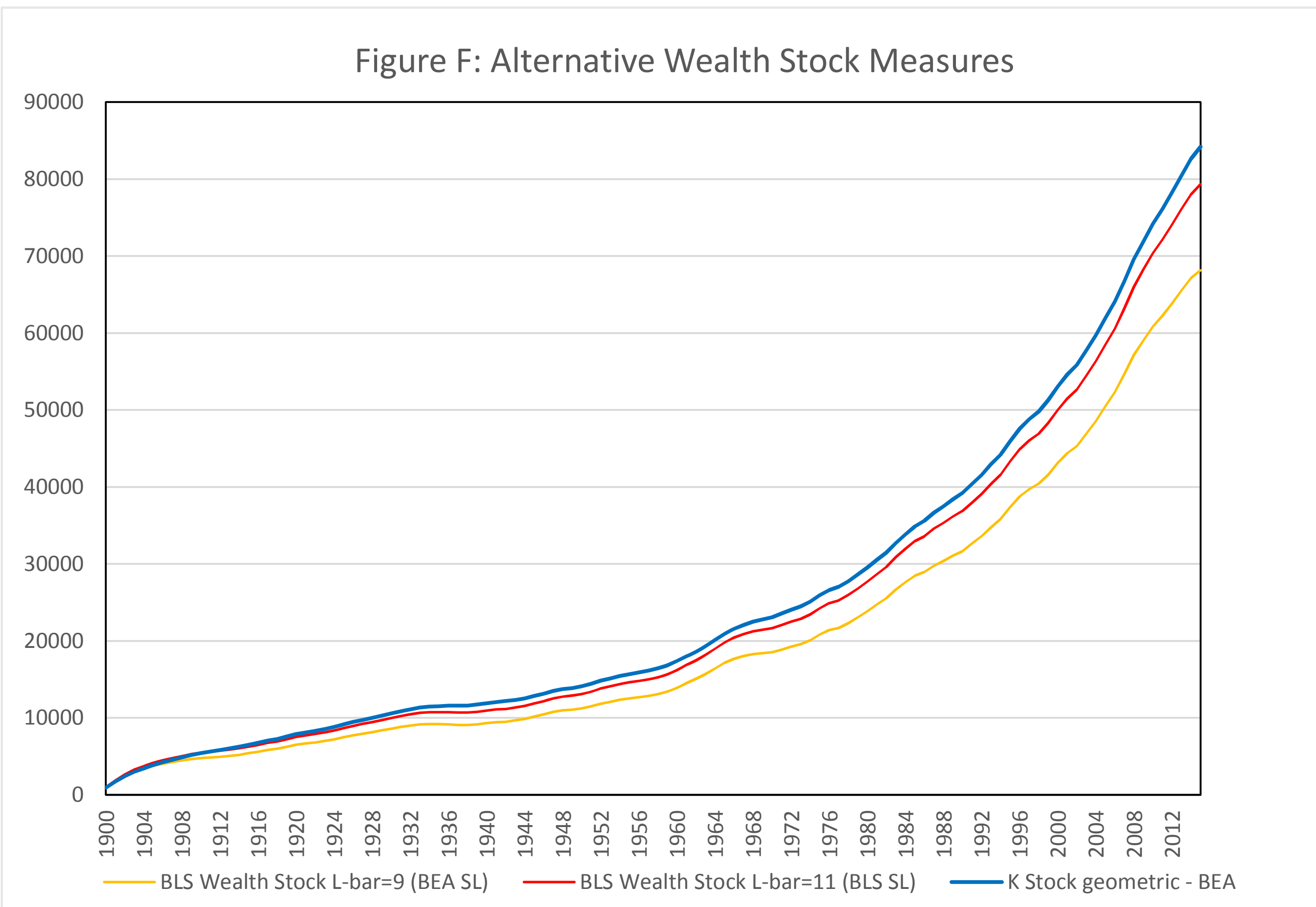

Figure F: Alternative Wealth Stock Measures
90000
80000
70000
60000
50000
40000
30000
20000
10000
0
1900
1904
1908
1912
1916
1920
1924
1928
1932
1936
1940
1944
1948
1952
1956
1960
1964
1968
1972
1976
1980
1984
1988
1992
1996
2000
2004
2008
2012
BLS Wealth Stock L-bar=9 (BEA SL)
BLS Wealth Stock L-bar=11 (BLS SL)
K Stock geometric - BEA

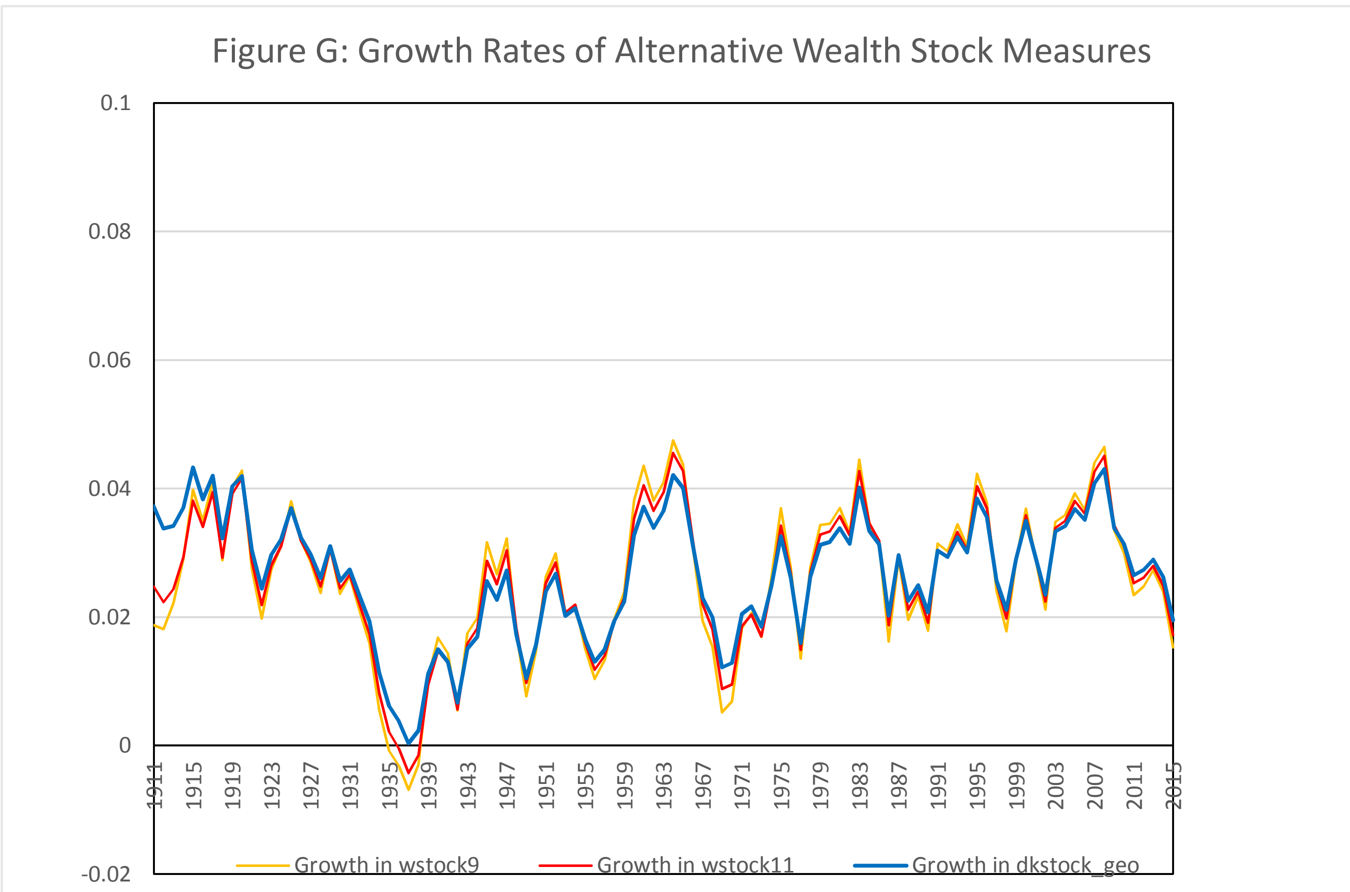
Figure G: Growth Rates of Alternative Wealth Stock Measures
0.1
0.08
0.06
0.04
0.02
0
-0.02
1911
1915
1919
1923
1927
1931
1935
1939
1943
1947
1951
1955
1959
1963
1967
1971
1975
1979
1983
1987
1991
1995
1999
2003
2007
2011
2015
Growth in wstock9
Growth in wstock11
Growth in dkstock_geo

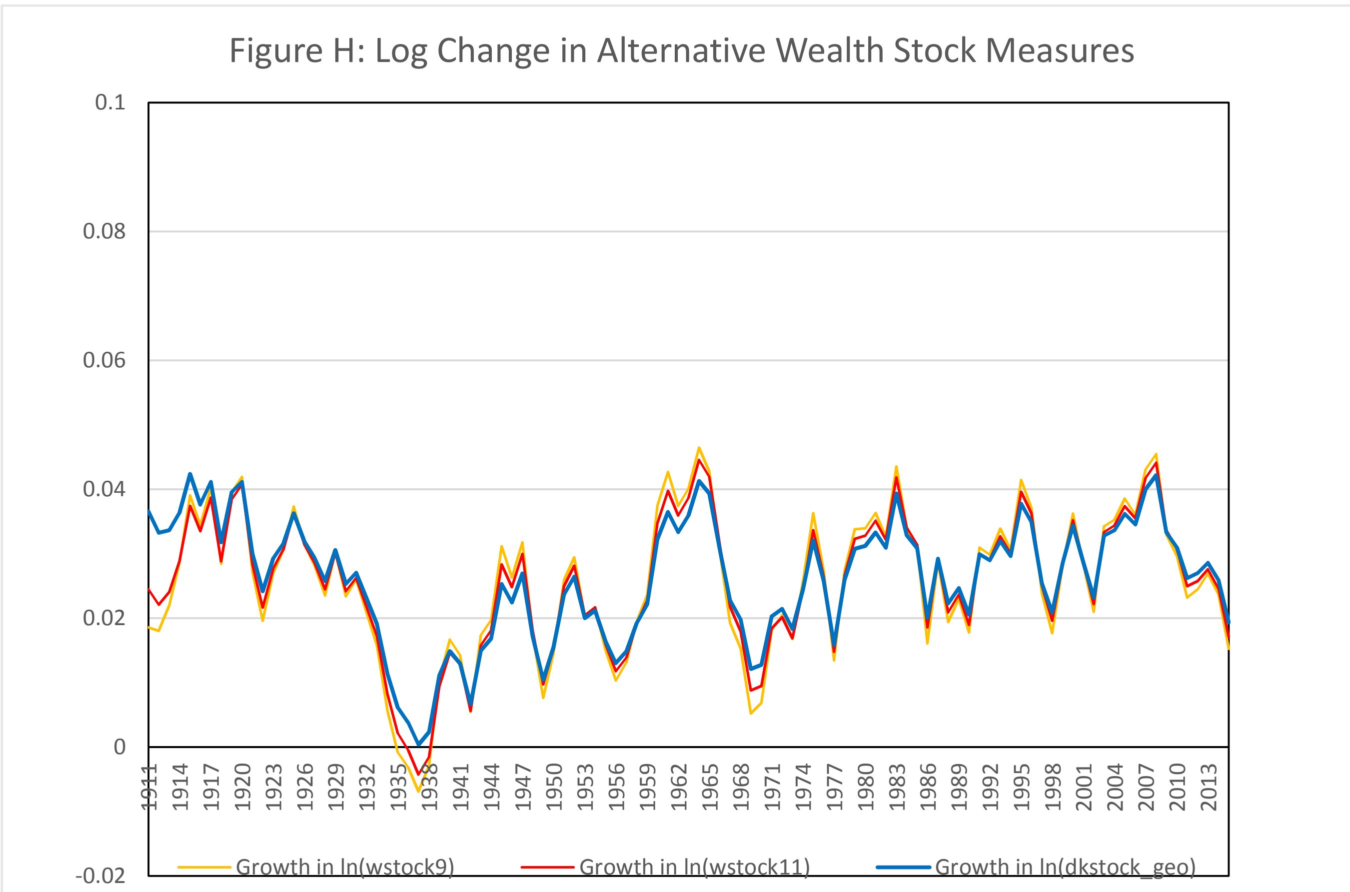
Figure H: Log Change in Alternative Wealth Stock Measures
0.1
0.08
0.06
0.04
0.02
0
-0.02
1911
1914
1917
1920
1923
1926
1929
1932
1935
1938
1941
1944
1947
1950
1953
1956
1959
1962
1965
1968
1971
1974
1977
1980
1983
1986
1989
1992
1995
1998
2001
2004
2007
2010
2013
Growth in ln(wstock9)
Growth in ln(wstock11)
Growth in ln(dkstock_geo)