Measuring Labour Services: Quality-Adjusting the Entry and Exit of Workers

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Abstract

Many statistical agencies use the sum of hours worked when measuring labour services. This implies that all workers provide work of equal quality. Various indices for adjusting for labour quality have been employed in a large body of literature. However, this literature has not yet addressed the issue of how to quality-adjust the impact of workers entering and exiting the labour market. We outline a theoretical framework for dealing with quality adjustment of labour services caused by workers entering and exiting employment. To illustrate the theoretical framework, we use the case of Norway in the period 1997 – 2013. The impact on labour services due to our quality adjustment of net entry is found to be cyclical. While the adjustment for the quality of net entry amounts to about -0.3 percentage point annually during expansions, it is offset by about the same magnitude during contractions.

Keywords: Labour services, Index numbers.

JEL classification: C43, E24, J24.

1 Introduction

Many statistical agencies use the sum of hours worked when measuring labour services. A weakness of using hours worked as a measure of labour services is the implicit assumption that workers are a homogeneous group – every hour worked is equally productive. For example, an hour worked by a lawyer and an hour worked by a paperboy are treated as equal amounts of labour.

To overcome this weakness, it is common in the literature to apply a two-tier approach to adjust for the quality of labour. In the first tier, workers are aggregated into groups categorised according to certain

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characteristics, such as gender, age and educational attainment level, using the sum of hours worked. In the second tier, workers are aggregated by means of an index formula that takes into account that the groups contain workers with different characteristics. The Törnqvist index is often applied at this stage. This two-tier approach generalises the standard measure that sums hours worked, as it measures compositional changes between groups. For example, demographic trends such as increasing educational achievement and/or ageing of the population can be identified using this approach.

The literature on quality adjustment of labour services dates back to at least Jorgenson and Griliches (1967). Jorgenson et al. (2005), Cao et al. (2009), Nilsen et al. (2011) and work based on the EU KLEMS database such as that of O'Mahony and Timmer (2009) and Timmer et al. (2010) apply the two-tier approach. Zoghi (2010) discusses the use of predicted wages in calculating the weighting scheme in the second tier. Diewert and Lippe (2010) add to this literature by showing explicitly the reduction in the bias achieved by switching from the sum of hours worked to the two-tier approach.

Some statistical agencies have begun to use the two-tier approach to measure quality-adjusted labour services. The Bureau of Labor Statistics in the U.S. publishes a labour composition index that measures the effect of shifts in the age, education and gender composition of the work force. The Office for National Statistics in the UK publishes quality-adjusted labour input based on the characteristics age, gender, education and industry, see Acheson (2011). The national statistical offices of Australia, Canada and New Zealand also publish measures of quality-adjusted labour services; see OECD (2016).

A major weakness of the two-tier approach, however, is that quality adjustment within groups is not accounted for. Since the groups defined in the second tier are broad, the potential for shifts in labour quality within groups is large. As pointed out by Zoghi (2010), "the ideal extreme case allows each worker to function as a unique input, by virtue of his unique set of relevant characteristics" (p. 459). In particular, the two-tier approach does not deal with how to quality-adjust the entry and exit of workers. Hours worked by workers entering employment are treated as equal to the amounts of labour by the workers already employed. Similarly, it is assumed that the hours worked by workers exiting employment are equal in terms of labour services to those of workers who remain employed. Compositional changes from workers entering and exiting employment may have large impacts on the quality of the workforce and should be taken into account when calculating labour services.

In this paper, we contribute to the literature by formulating a theoretical framework that quality-adjusts the entry and exit of workers. In particular, we propose a three-tier aggregation framework that encompasses the two methods used in the literature so far. In the first tier, workers are aggregated into elementary aggregates consisting of workers who are assumed to be perfect substitutes. This represents a generalisation of the methods used in the literature as they implicitly assume workers in the elementary aggregates to be a homogenous group, i.e., both perfect substitutes and of equal quality. At this stage, we build upon the results of Feenstra (1994), who showed how to incorporate new product varieties into a constant elasticity of substitution (CES) aggregate of import prices. Several papers have applied the Feenstra (1994) framework. For example, Broda and Weinstein (2006) use it to analyse the value to U.S. consumers of expanded import product varieties. Harrigan and Barrows (2009) analyse how the end of the multifibre arrangement impacted prices and quality. Feenstra et al. (2013) consider how an increase in product varieties affected the measurement of U.S. productivity growth. But, to our knowledge, this framework has not previously been used to calculate labour services. An important insight by Feenstra (1994) is that when varieties are perfect substitutes, any change in expenditure stemming from new varieties should be interpreted as a change in volume, not a change in price. Correspondingly, any nominal value change in labour costs stemming from the entry or exit of workers in an elementary aggregate should be interpreted as a change in labour services (volume) and not a change in wages. In the second tier, workers are aggregated into broad groups by age, gender, education etc. In the third-tier, these broad groups are aggregated into an overall index of labour services.

A practical problem with the theoretical framework in this paper is that it may be impossible to identify the elementary aggregates either because some worker characteristics are unobservable or because data on other types of characteristics are unavailable. To overcome this practical problem, we propose an index that approximates the theoretically derived index and which can be applied without it being necessary to create broad or elementary aggregates.

To illustrate the framework, we use the case of Norway in the period between 1997 and 2013. By using register data, we allow each worker to function as a unique input. We find a mean quality adjustment of about minus one percentage point annually, most of which comes from continuous workers. This implies that there is a lot to be gained in terms of quality adjustment when aggregating from the individual worker instead of from broad aggregates such as gender, age, industry and immigrant background. Although the quality adjustment from net entry is small over the entire sample period, there are relatively large contributions from net entry over the business cycle. In particular, lower quality workers enter the labour market during expansions and lower quality workers exit the labour market during downturns and this quality adjustment ranges from -0.5 to 0.4 percentage points, annually.

The paper is set out as follows: In Section 2, the theoretical framework is outlined. It is explicitly shown how to quality-adjust the impact from workers entering and exiting employment. In Section 3, the data are described and the three possible approaches to measuring labour services are empirically compared. Section 4 provides a conclusion.

2 Aggregation of labour services

In this section, we first outline the theoretical basis of the three-tier framework for aggregating labour services and then show how the current practices for measuring labour services fit into this framework. Our objective is to construct an index that generalises current practices and that can also be used to identify the impact from workers entering and exiting the labour force. To this end, we will outline the general function that aggregates labour services at all three stages and illustrate the explicit assumptions applying to the possible approaches at each stage. Finally, we construct an index that serves as an approximation of the theoretically derived index and which can be applied without it being necessary to explicitly split the workforce into broad and elementary aggregates.

2.1 An overview of the three-tier aggregation framework

The approaches in the current literature on aggregation of labour services can be encompassed by a three-tier aggregation framework, as illustrated in Figure 1. Aggregation starts from the level of each individual worker. The first tier of the framework is the aggregation into elementary aggregates. An elementary aggregate consists of workers who perform the same type of work and share the same qualities. These qualities are narrowly defined in terms of characteristics such as education, age, gender, cultural background, industry, unionisation, geographic location, tasks performed and experience. For instance, immigrant workers may in the first step be placed in a different elementary aggregate from native workers. An immigrant to Norway from say Sweden will be placed in a different elementary aggregate from non-Scandinavian immigrants, since these immigrants are of a different type. The three-tier aggregation framework thus corresponds to the aggregation of consumer and producer prices, which also starts from a narrowly defined elementary aggregate: "An elementary aggregate consists of the expenditures on a small and relatively homogenous set of products defined within the consumption classification used in the CPI.", see p. 355 in ILO et al. (2004). We proceed with the analysis of labour services by assuming that such elementary aggregates exist. Under this assumption, we provide an index that can be applied even without it being necessary to explicitly identify these elementary aggregates. The aggregation framework outlined in Figure 1 is conceptual and aggregation could equally well begin at a worker-job match level, where workers with more than one job would be counted more than once, and could possibly be included in more than one elementary aggregate.

The second tier of the framework involves aggregating the elementary aggregates into broad aggregates. These broad aggregates represent workers classified by a broader set of characteristics. Broad aggregates thus differ from elementary aggregates in terms of both the number of characteristics and the different types of elements within each characteristic. For example, while a broad aggregate may consist of only 5 to 10

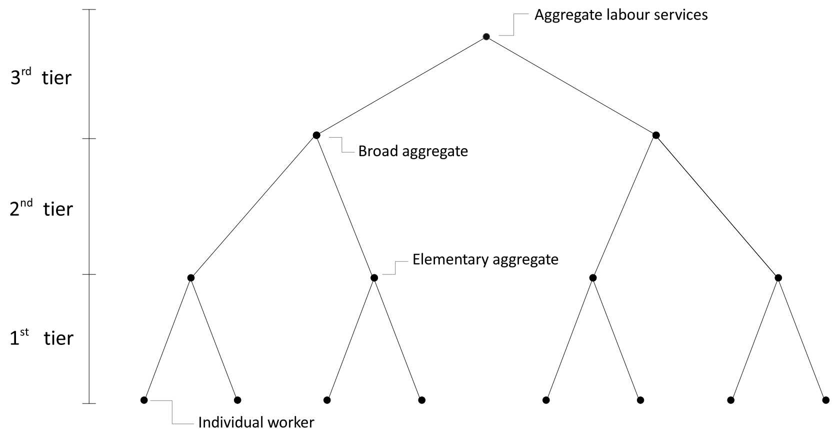

Figure 1 – The three-tier aggregation framework.

different industry characteristics, an elementary aggregate may be defined by more than 1 000 different industry characteristics. An elementary aggregate may also be defined by geographic location and tasks performed, while these characteristics may not be included in the definition of a broad aggregate. The third and final tier involves aggregating the broad aggregates into the total aggregate of labour services.

The three-tier framework represents our starting point because it encompasses aggregation methods commonly used in the literature. We can thus think of the methods in the literature as operationalisations of the three-tier framework. Although we only consider a three-tier framework, the results in this paper can be generalised to a multi-tier framework by introducing more aggregation steps between the elementary aggregate and the broad aggregate. For notational convenience, and without loss of generality, we use the three-tier approach to illustrate the concept.

Aggregation will be based on a constant elasticity of substitution (CES) production function in all three tiers.[1] A CES function contains two types of parameters that express the interrelationships between workers: a quality parameter and an elasticity of substitution. The former shows the relative quality of workers, while the latter illustrates how easy it is to substitute one worker with another. Ideally, one should choose an aggregate index for labour services that does not place any restrictions on the elasticities of substitution or quality parameters. Neither the methods currently described in the literature nor the approach we propose in this paper live up to that ideal, as we illustrate below. However, the approach that we propose is a generalisation of the other methods applied in the literature, as it puts fewer restrictions on the quality parameters and the substitution elasticities.

Table 1 illustrates how the method we propose generalises the other approaches described in the literature.

[1] Our approach to measuring labour heterogeneity based on nested CES functions is closely related to the literature on modelling the effects of immigration on wages; cf. Ottaviano and Peri (2012) for a recent contribution. They specify a model of worker heterogeneity using nested CES functions with nationality at the lowest level. However, this literature does not distinguish between workforce entries and exits as we do.

Table 1 – Three operationalisations of the three-tier aggregation framework

	Hours worked (HW)	Two-tier restricted (2TR)	Three-tier restricted (3TR)
3rd tier	Perfect substitutes & equal quality	No restrictions	No restrictions
2nd tier	Perfect substitutes & equal quality	Perfect substitutes & equal quality	No restrictions
1st tier	Perfect substitutes & equal quality	Perfect substitutes & equal quality	Perfect substitutes

In particular, it compares the method we propose with two others that we label "hours-worked" (HW) and the "two-tier restricted" (2TR) approach. The hours-worked index is simply the ratio of a summation of hours worked across workers in two different time periods. In the three-tier aggregation framework outlined above the HW approach can be viewed as imposing the conditions that the quality parameters are the same for all workers and that all workers are perfect substitutes. In other words, it is implicitly assumed that quality parameters and substitution elasticities are equal at all three tiers of aggregation. The 2TR approach generalises the method of summing hours worked and represents the quality adjustment approach often described in the literature. It is based on the HW framework in the first two tiers of aggregation. The 2TR approach, however, employs an index that does not impose any restrictions on either the elasticity of substitution or the quality parameters in the third tier of aggregation.[2] It is thus in the third tier of aggregation that the 2TR approach generalises the HW framework.

Our proposed approach denoted "three-tier restricted" (3TR) does not restrict either the quality parameters or the substitution elasticities in the 2nd or 3rd tier of aggregation, nor does it restrict the quality parameters in the elementary aggregates in the 1st tier of aggregation. The only restriction imposed in this framework is that workers within each elementary aggregate are assumed to be perfect substitutes. Such a restriction is plausible, since workers are categorised into narrowly defined elementary aggregates that should be characterised by a high degree of substitutability. Although the 3TR framework does impose a restriction on the elasticity of substitution in the 1st tier, the framework is nevertheless a generalisation of both the HW and the 2TR approach.

A practical problem with the 3TR framework is that it may be impossible to identify the narrowly defined elementary aggregates in the 1st tier. These narrowly defined elementary aggregates can be thought of as theoretical constructions, assumed to exist, but which may be unidentifiable in practice because some worker characteristics may be unobservable or because data for other types of characteristics may be unavailable.

[2] In practice, many studies use the Törnqvist index, which corresponds to the Translog aggregation formula, and not the Sato-Vartia index (Sato 1976, Vartia 1976), which corresponds to the CES aggregation formula, in the 3rd-tier of aggregation, see e.g., Jorgenson et al. (1987).

To overcome this practical problem, we propose an index that approximates the 3TR framework and which can be applied without the need to create neither broad or elementary aggregates.

2.2 Functional forms underlying the three-tier framework

We proceed by outlining the explicit functional form of the three-tier aggregation framework when no restrictions are imposed. The upper level aggregation function for period t is a CES aggregate of broad aggregation categories

$$H_t = \left(\sum_{b \in B} \gamma_b H_{bt}^{(\sigma-1)/\sigma} \right)^{\sigma/(\sigma-1)}, \tag{2.1}$$

where H_{bt} is the quantity of labour services in a broad aggregate that is defined below, $\gamma_b > 0$ represents a quality parameter, σ is the elasticity of substitution among the broad aggregates and B is the set of broad aggregates. A broad aggregate represents an aggregate of labour defined by characteristics such as gender, education level, experience and industry.

Moving to the next tier, we define each broad aggregate as a CES aggregate of elementary aggregates

$$H_{bt} = \left(\sum_{e \in E_b} \gamma_{be} H_{bet}^{(\sigma_b-1)/\sigma_b} \right)^{\sigma_b/(\sigma_b-1)}, \tag{2.2}$$

where H_{bet} is the quantity of labour services in an elementary aggregate e belonging to the broad aggregate b, $\gamma_{be} > 0$ represents a quality parameter for each elementary aggregate, σ_b is the elasticity of substitution among the elementary aggregates and E_b is the set of elementary aggregates in the broad aggregate b.

Moving to the last tier, we define each elementary category as a CES aggregate of the hours worked by each worker

$$H_{bet} = \left(\sum_{m \in M_{bet}} \gamma_{bem} H_{bemt}^{(\sigma_{be}-1)/\sigma_{be}} \right)^{\sigma_{be}/(\sigma_{be}-1)}, \tag{2.3}$$

where H_{bemt} is the number of hours worked by a worker m in the elementary aggregate e belonging to the broad category b, $\gamma_{bem} > 0$ represents a quality parameter for each worker, and σ_{be} is the elasticity of substitution among the individual workers in the particular elementary aggregate. M_{bet} is the set of workers in the elementary aggregate e belonging to the broad category b. Ideally, each elementary aggregate should consist of a perfectly homogeneous set of workers, i.e., the elasticity of substitution would approach infinity ($\sigma_{be} \to \infty$) and the quality parameters would equal unity ($\gamma_{bem} = 1$). In practice, however, no pair of workers has exactly the same qualities and even the most detailed classification would imply a deviation from the

Table 2 – Classification of workers within elementary aggregates

	Continuing	Entering	Exiting
Time period t	■	■	
Time period $t-1$	■		■

■ denotes positive working hours.

ideal elementary aggregate characterised by $\sigma_{be} \to \infty$ and $\gamma_{bem} = 1$.

Note that the set of workers within each elementary aggregate, M_{bet}, may vary for different time periods due to workers entering and exiting employment. To illustrate, and to introduce notation that will be useful below, let C_{bet} denote the set of workers who were employed in two consecutive time periods, say $t-1$ and t. We refer to these as *continuing* workers; see Table 2. Workers *entering* employment were employed in period t but not in period $t-1$. Workers *exiting* employment worked in period $t-1$ but not in period t. Let N_{bet} and X_{bet} denote the sets of entering and exiting workers, respectively. It then follows that the number of workers within each elementary aggregate in period t is the union of the set of continuing workers and the set of entering workers, i.e., $M_{bct} = C_{bet} \cup N_{bet}$. Correspondingly, the number of workers within each elementary aggregate in period $t-1$ can be written as the union of the set of continuing workers and the set of exiting workers, i.e., $M_{bct-1} = C_{bet} \cup X_{bet-1}$.

The three-tier approach outlined above is extremely flexible since no restrictions have been placed on the number of elementary aggregates. It is assumed, however, that every entering worker will be in an elementary aggregate with at least one continuing worker, and that every exiting worker will be in an elementary aggregate with at least one continuing worker. Given these assumptions, there can at a maximum be as many elementary aggregates as there are continuing workers.

To create the aggregate index of labour services, we apply the results of Sato (1976), Vartia (1976) and Feenstra (1994). Sato (1976) and Vartia (1976) showed how to calculate a price and a quantum index when the number of categories is constant for different periods. This is useful for the last two tiers of aggregation since the numbers of broad and elementary aggregates in Equation 2.1 and Equation 2.2 are independent of time. Feenstra (1994) generalised the results of Sato (1976) and Vartia (1976) to handle situations where the number of categories changed through time, which is the case for the number of workers in each elementary aggregate in Equation 2.3.

We begin by showing the Sato-Vartia index corresponding to Equation 2.1. Let W_{bt} be the wage index of the broad category b and let the quantity of labour services in the broad aggregation categories H_{bt} be cost-minimising. The Sato-Vartia index for the labour services in Equation 2.1 is then given by a geometric mean of changes in the individual labour services in each broad category

$$\Delta \ln H_t = \sum_{b \in B} w_{bt} \Delta \ln H_{bt}, \tag{2.4}$$

where the weight w_{bt} is the logarithmic mean of the wage shares between two time periods

$$w_{bt} = \left(\frac{s_{bt} - s_{bt-1}}{\ln s_{bt} - \ln s_{bt-1}} \right) \Bigg/ \sum_{b \in B} \left(\frac{s_{bt} - s_{bt-1}}{\ln s_{bt} - \ln s_{bt-1}} \right), \tag{2.5}$$

and where the wage shares are defined as $s_{bt} = \frac{W_{bt} H_{bt}}{\sum_{b \in B} W_{bt} H_{bt}}$. A remarkable feature of the Sato-Vartia index is that it is independent of the quality parameters and elasticity of substitution. Moving to the second tier, let W_{bet} be the wage index of an elementary aggregate e and let the quantity of labour services in the elementary aggregate H_{bet} be cost minimising. The Sato-Vartia index for the labour services in Equation 2.2 is then given by a geometric mean of the individual labour service changes in each elementary aggregate

$$\Delta \ln H_{bt} = \sum_{e \in E_b} w_{bet} \Delta \ln H_{bet}, \tag{2.6}$$

where the weights w_{bet} are the logarithmic mean of the wage shares between two time periods

$$w_{bet} = \left(\frac{s_{bet} - s_{bet-1}}{\ln s_{bet} - \ln s_{bet-1}} \right) \Bigg/ \sum_{e \in E_b} \left(\frac{s_{bet} - s_{bet-1}}{\ln s_{bet} - \ln s_{bet-1}} \right), \tag{2.7}$$

and where the wage shares are defined by $s_{bet} = \frac{W_{bet} H_{bet}}{\sum_{e \in E_b} W_{bet} H_{bet}}$.

To calculate the labour services index for the elementary aggregates we apply the results of Feenstra (1994). He showed that the total index could be decomposed into contributions from a standard Sato-Vartia index across continuous workers and separate contributions from entering and exiting workers. Let s_{bet}^N denote the total wage share of entering workers in an elementary aggregate: $s_{bet}^N = \sum_{m \in N_{bet}} W_{bemt} H_{bemt} / \sum_{m \in M_{bet}} W_{bemt} H_{bemt}$. Also, let s_{bet}^X denote the total wage share of exiting workers in an elementary aggregate, i.e., $s_{bet}^X = \sum_{m \in X_{bet}} W_{bemt} H_{bemt} / \sum_{m \in M_{bet}} W_{bemt} H_{bemt}$. Moreover, let the weight w_{bemt} represent the logarithmic mean of the wage shares for continuous workers in two different time periods

$$w_{bemt} = \left(\frac{s_{bemt} - s_{bemt-1}}{\ln s_{bemt} - \ln s_{bemt-1}} \right) \Bigg/ \sum_{m \in C_{bet}} \left(\frac{s_{bemt} - s_{bemt-1}}{\ln s_{bemt} - \ln s_{bemt-1}} \right), \tag{2.8}$$

where the expenditure shares are defined by $s_{bemt} = \frac{W_{bemt} H_{bemt}}{\sum_{m \in C_{bet}} W_{bemt} H_{bemt}}$. Applying the results of Feenstra

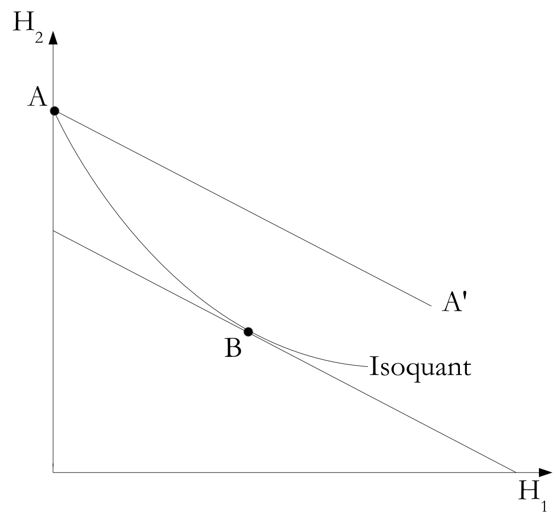

Figure 2 – Impacts on wage costs due to workers entering and exiting employment

(1994) and the product rule, the index for labour services in Equation 2.3 can then be written

$$\Delta \ln H_{bet} = \sum_{m \in C_{bet}} w_{bemt} \Delta \ln H_{bemt} + \left(\frac{\sigma_{be}}{1-\sigma_{be}} \right) \ln \left(1 - s_{bet}^{N}\right) - \left(\frac{\sigma_{be}}{1-\sigma_{be}} \right) \ln \left(1 - s_{bet-1}^{X}\right). \tag{2.9}$$

The first term is the standard Sato-Vartia index across continuous workers. The second and third terms are the contributions to overall labour services due to workers entering and exiting the workforce.

It follows from the Feenstra (1994) index that, ceteris paribus, the higher the elasticity of substitution, the higher the contribution to labour services due to entering workers. The intuition behind this result is that a higher elasticity of substitution means that there is less to be gained from a new worker in terms of reduced costs. This is implicitly illustrated in Figure 2 which shows how workers entering and exiting employment impact wage costs. The isocost line AA' shows the combination of hours worked by the two workers that yields the same cost. To minimise costs for a given level of production, the problem is to find a point on the isoquant where the associated isocost curve has the minimum vertical intercept. In time period $t-1$, only worker H_2 is available and employment is at point A. In time period t, however, both workers are available. When both workers are available, the isocost curve with the minimum vertical intercept goes through point B. The entry of a new worker thus reduces wage costs for a given level of production.

The size of the wage cost reduction depends on the curvature of the isoquant, or how easy it is to substitute one worker for another, as expressed by the elasticity of substitution. When there is some sort of complementarity between workers, i.e., a worker's efficiency increases when working with others, the isoquant line will show a curvature as illustrated in Figure 2. However, if workers are perfect substitutes, the elasticity of substitution tends to infinity and the isoquant becomes a straight line. There is thus no longer a wage cost reduction from having a new worker available for production. Any increase in labour costs attributable to new workers is thus a result of increased labour services.

The aggregate index of labour services can be calculated in three steps, using Equation 2.9, Equation 2.6, and Equation 2.4. These equations thus constitute the three-tier aggregation framework. So far, no restrictions have been imposed on the quality parameters or the elasticities of substitution in any of the three steps. In the following we will show how the three-tier aggregation framework encompasses current practices, as outlined in Table 1, and we will propose a generalised index that is suitable when data on hours worked and wages are available for all workers.

We start by comparing the three-tier aggregation framework with the current approach that is the sum of hours worked across workers, labelled hours worked (HW) in Table 1. This method, which is widely used in national accounts, is tantamount to imposing the restriction that the quality parameters equal unity and the elasticities of substitution approach infinity in all three stages, i.e., $\gamma_b = \gamma_{be} = \gamma_{bem} = 1$ and $\sigma, \sigma_b, \sigma_{be} \to \infty \; \forall b, e, m$ in the three-tier aggregation framework. Given these restrictions, the three steps in Equation 2.1, Equation 2.2 and Equation 2.3 can be written as $H_t = \sum_{b \in B} H_{bt}$, $H_{bt} = \sum_{e \in E_b} H_{bet}$ and $H_{bet} = \sum_{m \in M_{bet}} H_{bemt}$, respectively. Hence, total labour services is the simple sum of hours worked across all workers.

Consider next the quality-adjustment approach labelled 2TR in Table 1. It is calculated by summing hours worked in the broad aggregates and then applying the Sato-Vartia index in Equation 2.4 to the broad aggregates. This approach is tantamount to imposing the restrictions that the quality parameters equal unity and that the elasticities of substitution approach infinity in all broad aggregates and in all elementary aggregates, i.e., $\gamma_{be} = \gamma_{bem} = 1$ and $\sigma_b, \sigma_{be} \to \infty \; \forall b, e, m$. This approach is thus more general than using a simple sum of hours worked across all workers since no restrictions are placed on the quality parameters, γ_b, or on the elasticity of substitution, σ, at the upper level.

We now consider the generalisation of the HW and 2TR indices, which we have called the three-tier restricted (3TR) approach. Assume that the only restrictions imposed on the three-tier aggregation framework are that the elasticity of substitution tends to infinity in all elementary aggregates; i.e., $\sigma_{be} \to \infty \; \forall b, e$. These restrictions are consistent with the idea that elementary aggregates should consist of workers with similar characteristics and thus that it should be relatively easy to substitute one worker for another within an elementary aggregate. Since the quality parameters in the elementary aggregates (γ_{bem}) have not been restricted, there is still room for quality differences and wage differences among workers in each elementary aggregate. Imposing the assumption $\sigma_{be} \to \infty \; \forall b, e$ on each elementary aggregate and inserting Equation 2.9

into Equation 2.6 and Equation 2.4 yields the aggregate index for labour services

$$\Delta \ln H_t = \sum_{b\in B} w_{bt} \sum_{e\in E_b} w_{bet} \sum_{m\in C_{bet}} w_{bemt} \Delta \ln H_{bemt} - \sum_{b\in B} w_{bt} \sum_{e\in E_b} w_{bet} \ln\left(1 - s_{bet}^{N}\right) + \sum_{b\in B} w_{bt} \sum_{e\in E_b} w_{bet} \ln\left(1 - s_{bet-1}^{X}\right), \quad (2.10)$$

where the first term is the index calculated across continuous workers, the second term is the contribution from workers entering employment and the third term is the contribution from workers exiting employment. Equation 2.10 thus provides a theoretically consistent way of identifying the impact from workers entering and exiting the labour market.

2.3 Approximating the theoretical index

Some practical problems arise when the above index is implemented. It may be impossible to identify the elementary aggregates in practice, as some worker characteristics may be unobservable or data for other types of characteristics may be unavailable.

To overcome these practical problems we propose an index that serves as an approximation of the theoretically derived index in Equation 2.10 and which is applicable with data on hours worked and wages for all workers. To this end, we introduce notation that is independent of the particular stages of aggregation. Let the sets of continuing, entering and exiting workers be denoted by $C_t = \cup_{b\in B} \left(\cup_{e\in E_b} C_{bet}\right)$, $N_t = \cup_{b\in B} \left(\cup_{e\in E_b} N_{bet}\right)$ and $X_{t-1} = \cup_{b\in B} \left(\cup_{e\in E_b} X_{bet-1}\right)$, respectively. The total set of workers in period t is the union of the sets of continuing and entering workers, $I_t = C_t \cup N_t$, and the total set of workers in period $t-1$ is the union of the sets of continuing and exiting workers, $I_{t-1} = C_t \cup X_{t-1}$. Moreover, let s_t^N denote the total wage share of entering workers, $s_t^N = \sum_{i\in N_t} W_{it}H_{it} / \sum_{i\in I_t} W_{it}H_{it}$, and let s_t^X denote the total wage share of exiting workers, $s_t^X = \sum_{i\in X_t} W_{it}H_{it} / \sum_{i\in I_t} W_{it}H_{it}$. Given these definitions, we now introduce the alternative index (3TR) for aggregate labour services

$$3\text{TR} = \sum_{i\in C_t} w_{it} \Delta \ln H_{it} - \ln\left(1 - s_t^N\right) + \ln\left(1 - s_{t-1}^X\right), \quad (2.11)$$

where H_{it} is the number of hours worked by worker i in period t and w_{it} represents the logarithmic mean of the wage share for a continuous worker $i \in C_t$ between two time periods

$$w_{it} = \left(\frac{s_{it} - s_{it-1}}{\ln s_{it} - \ln s_{it-1}}\right) \Big/ \sum_{i\in C_t} \left(\frac{s_{it} - s_{it-1}}{\ln s_{it} - \ln s_{it-1}}\right). \quad (2.12)$$

The proposed index in Equation 2.11 is independent of the different stages of aggregation. Like the theoretically derived index in Equation 2.10, the proposed index in Equation 2.11 also includes separate contributions due to continuing, entering and exiting workers. Note that the index in Equation 2.11 resembles the Sato-Vartia-Feenstra index in Equation 2.9 calculated over the entire set of workers. In particular, the impact from entering and exiting workers in Equation 2.11 is consistent with the Sato-Vartia-Feenstra index applied for the entire set of workers when the elasticity of substitution is infinite. In addition, the impact from continuous workers only is consistent with the Sato-Vartia index for any value of the elasticity of substitution.

To compare the proposed index in Equation 2.11 with the theoretically derived index in Equation 2.10, we first state the definition of differential approximation and then state the conditions for when these indices differentially approximate each other:

Definition 1 (Differential approximation) A functional form f differentially approximates F at x^* if, and only if, (i) $f(x^*) = F(x^*)$ and (ii) $f_x(x^*) = F_x(x^*)$.

Proposition 1 *The proposed index in Equation 2.11 differentially approximates the theoretically derived index in Equation 2.10 at any point where wages and quantities across continuous workers for the two periods are equal,* $W_{it} = W_{it-1}$ *and* $H_{it} = H_{it-1}$ *for all* $i \in C_t$ *and where entering and exiting workers are evenly distributed across elementary aggregates,* $s^N_{be_1t} = s^N_{be_2t}$ *for all* $e_1, e_2 \in N_t$ *and* $s^X_{be_3t-1} = s^X_{be_4t-1}$ *for all* $e_1, e_2 \in N_t$ *and* $e_3, e_4 \in X_{t-1}$.

Proof. See the appendix, Section 5.1. □

Proposition 1 states that for data with small variations and where the variation in entering and exiting workers is small across the elementary aggregates, the proposed index in Equation 2.11 will be approximately equal to the theoretically derived index in Equation 2.10. Proposition 1 therefore provides a rationale for applying the proposed index in Equation 2.11. It resembles the results of Diewert (1978), who showed that the known superlative indices at the time, such as the quadratic mean, were approximately consistent in aggregation.[3] Proposition 1 can thus be viewed as a generalisation of the results of Diewert (1978), since the case when the set of workers differs for different time periods is also explicitly considered. That the impact of entering and exiting workers in Equation 2.11 differentially approximates the impact from entering and exiting workers in Equation 2.10 follows from the weights summing to unity.

The impact from entering and exiting workers in the 3TR index in Equation 2.11 can also be linked to the three-tier aggregation framework in a more fundamental respect. Since the theoretically derived index in Equation 2.10 does not satisfy the conditions of the factor reversal test, the final index number will depend on the order of aggregation. Aggregating wages and then calculating the quantity index by means of the

[3] Later, Barnett and Choi (2008) showed that also the Sato-Vartia index can be defined as superlative.

product rule will not yield the same result as calculating quantities directly. Interestingly, by aggregating wages, instead of quantities, in the three-tier approach and then backing out the quantity index using the product rule yields the same entering and exiting terms as in Equation 2.11. This can be seen by the fact that if wages are aggregated, and if the elasticity of substitution equals infinity in all elementary aggregates, then the entry and exit of a worker will not impact the overall wage index. Any contribution to a change in the nominal value of labour costs is then the result of a quantity change. To be explicit, let *labour costs* refer to the nominal value of compensation paid to employees for their work, $V_{it} = W_{it}H_{it}$, where W_{it} and H_{it} denote the hourly wage cost and the number of hours worked for employee i in period t. The ratio of labour costs in two consecutive time periods can then be written

$$\begin{aligned}
\underbrace{\left(\frac{\sum_{i\in I_t} V_{it}}{\sum_{i\in I_{t-1}} V_{it-1}}\right)}_{\text{TOTAL}} &= \underbrace{\left(\frac{\sum_{i\in C_t} V_{it}}{\sum_{i\in C_t} V_{it-1}}\right)}_{\text{CONTINUING}} \times \underbrace{\left(1+\sum_{i\in N_t} V_{it}\Big/\sum_{i\in C_t} V_{it}\right)}_{\text{ENTERING}} \times \underbrace{\left(1+\sum_{i\in X_t} V_{it-1}\Big/\sum_{i\in C_t} V_{it-1}\right)^{-1}}_{\text{EXITING}} \\
&= \underbrace{\left(\frac{\sum_{i\in C_t} V_{it}}{\sum_{i\in C_t} V_{it-1}}\right)}_{\text{CONTINUING}} \times \underbrace{\left(1-s_t^N\right)^{-1}}_{\text{ENTERING}} \times \underbrace{\left(1-s_{t-1}^X\right)^{-1}}_{\text{EXITING}}, \qquad (2.13)
\end{aligned}$$

where s_t^N and s_t^X denote the nominal wage share of entering and exiting workers, respectively, $s_t^N = \sum_{i\in N_t} V_{it}/\sum_{i\in I_t} V_{it}$ and $s_t^X = \sum_{i\in X_t} V_{it}/\sum_{i\in I_t} V_{it}$. By taking logs of Equation 2.13 it follows that the overall growth contribution of entering and exiting workers is equivalent to the growth contribution of entering and exiting workers to labour services in Equation 2.11. The proposed index in Equation 2.11 is thus consistent with the view that entering and exiting workers should only impact labour services (quantity) if workers are perfect substitutes at the elementary aggregate level.

The index in Equation 2.11 will be compared with the index based on the sum of hours worked and the 2TR framework. Both of these approaches can be decomposed into contributions from workers who are continuing, entering and exiting. To compare the three approaches, we now show the explicit decomposition of the sum of hours worked and the 2TR framework. The ratio of the sum of hours worked between two time periods (HW) can be decomposed by taking logs in the same way as in Equation 2.13

$$\text{HW} = \ln\left(\frac{\sum_{i\in C_t} H_{it}}{\sum_{i\in C_t} H_{it-1}}\right) - \ln\left(1-h_t^N\right) + \ln\left(1-h_{t-1}^X\right), \qquad (2.14)$$

where h_t^N and h_t^X denote the shares of hours worked by entering and exiting workers, respectively, $h_t^N = \sum_{i\in N_t} H_{it}/\sum_{i\in I_t} H_{it}$ and $h_t^X = \sum_{i\in X_t} H_{it}/\sum_{i\in I_t} H_{it}$. The 2TR framework is calculated by summing hours worked within broad aggregates and then aggregating the broad aggregates using Equation 2.4. Decomposing hours worked within each broad aggregate using Equation 2.14 and inserting the result into Equation 2.4 yields the 2TR framework

$$2\text{TR} = \sum_{b \in B} w_{bt} \ln \left(\frac{\sum_{i \in C_{bt}} H_{it}}{\sum_{i \in C_{bt}} H_{it-1}} \right) - \sum_{b \in B} w_{bt} \ln \left(1 - h_{bt}^{N}\right) + \sum_{b \in B} w_{bt} \ln \left(1 - h_{bt-1}^{X}\right), \qquad (2.15)$$

where we have defined the sets of continuing, entering and exiting workers in a broad aggregate b as $C_{bt} = \cup_{e \in E_b} C_{bet}$, $N_{bt} = \cup_{e \in E_b} N_{bet}$ and $X_{bt-1} = \cup_{e \in E_b} X_{bet-1}$, respectively. The total set of workers in period t in a broad aggregate is the union of the sets of continuing workers and entering workers, $I_{bt} = C_{bt} \cup N_{bt}$ and the total set of workers in period $t-1$ is the union of the sets of continuing and exiting workers, $I_{t-1} = C_t \cup X_{t-1}$. h_{bt}^{N} and h_{bt}^{X} denote the hours worked shares of continuing and exiting workers within each broad aggregate, $h_{bt}^{N} = \sum_{i \in N_{bt}} H_{it} / \sum_{i \in I_{bt}} H_{it}$ and $h_{bt}^{X} = \sum_{i \in X_{bt}} H_{it} / \sum_{i \in I_{bt}} H_{it}$.

There are clear differences in the impact from the entry and exit of workers in the three methods. Table 1 showed that both the HW and the 2TR index assume that workers are both perfect substitutes and of equal quality at the elementary aggregate level, which implies that an hour worked by a worker entering employment is treated as equal in terms of amount of labour to those workers that continue to be employed. Correspondingly, both indices are based on the assumption that the hours worked by workers exiting employment are treated as equal in terms of amount of labour to those workers that continue to be employed. Nevertheless, the expression for the contribution of workers entering employment differs somewhat between the two methods. In the HW index, the contribution from entering workers is defined by the hours-worked share of entering workers, i.e., $-\ln\left(1 - h_t^{N}\right)$. The higher the hours-worked share of entering workers, the higher the overall contribution from entering workers to overall growth in hours worked. In contrast, in the 2TR index, the contribution from entering workers is given by a weighted sum of the hours-worked share of entering workers across the broad aggregates $-\sum_{b \in B} w_{bt} \ln\left(1 - h_{bt}^{N}\right)$. The difference between the two expressions is a result of the quality adjustment in the 3$^{\text{rd}}$ tier in the 2TR index. In contrast to these two approaches, the 3TR index does not impose the assumption that workers entering or exiting employment are of equal quality to those that are continuously employed. Hence, the contribution from entering workers is defined by the nominal labour cost share of entering workers: $-\ln\left(1 - s_t^{N}\right)$. Since the ln function increases monotonically, it follows that the entering contribution in the 3TR index will be higher than in the HW index if $s_t^{N} > h_t^{N}$. This is tantamount to the wage per hour of entering workers being higher than the wage per hour of continuing workers. The intuition behind this result is that wages reflect quality differences in the 3TR index. If the quality of entering workers is higher than that of continuing workers, and if wages reflect quality, then the contribution from entering workers will be higher in the 3TR index than in the HW index, which does not quality-adjust entering workers. The opposite result holds for exiting workers, where the 3TR index will be higher than in the HW index if $s^X_{t-1} < h^X_{t-1}$, i.e., if the wage per hour worked of exiting workers is lower than the wage per hour worked of continuing workers. *Quality adjustment* can thus be operationalised as the difference between the 3TR and the HW index. In the following, we show how the three indices compare in practice and calculate the size of the quality adjustment in the case of Norway.

3 Empirical example

In the following empirical example, the HW index in Equation 2.14 will be compared with the 2TR framework in Equation 2.15 and the 3TR index in Equation 2.11. Our annual dataset holds information about hours worked and labour costs for all employed persons in Norway between 1997 and 2013. Employment increased from 2.1 million to 2.7 million workers in this period. The data are based on information from the Register of Employers and Employees and the Pay Statements Register. Labour costs are measured by wage income and include wages and other remuneration.[4] Hours worked are measured by contractual annual working hours. In line with Aukrust et al. (2010), we have adjusted hours worked in the human health activities industry by setting an upper limit of 37.5 hours per week. Further details of the register-based microdata and how they compare to data from e.g., the Labour Force Survey can be found in Villund (2009) and Aukrust et al. (2010). To calculate the two-tier restricted index we have categorised workers by industry, gender, age and immigrant background, resulting in a total of 97 broad aggregates.[5]

Prior to implementing the theoretical framework in practice, it is crucial to define what exactly constitutes a new worker and an exiting worker.[6] In many cases this may emerge from the structure of the data. For example, the incidence of workers entering and exiting employment depends on data frequency (weekly, monthly, annual). Although weekly data will register a two-week absence from work as an exit in one week and an entry in another week, such a short absence may not suffice to categorise the worker as entering and exiting employment. In our empirical example, which is based on annual data, we follow Table 2 when classifying workers as continuing, entering or exiting.

Figure 3 compares labour services in terms of hours worked, the two- and three-tier restricted framework. Note that the Norwegian economy went through almost two full business cycles during the sample period. There was a peak in 1998, another large boom in 2007 and a recovery after the financial crisis that resulted in a third boom, albeit more modest, in 2013. Interestingly, there is little difference between the HW index and the 2TR index in the course of the sample period. On average, the HW index was 1.10 per cent, while the 2TR index was 1.17 per cent, indicating a quality adjustment of 0.07 percentage point. By contrast, there is a large difference between these indices and the 3TR index. The 3TR index shows consistently lower

[4] It also includes income earned at sea and company benefits such as a car or a phone.
[5] See the appendix, Section 5.2 for a detailed description of the categories.
[6] Thanks to an anonymous referee for pointing this out.

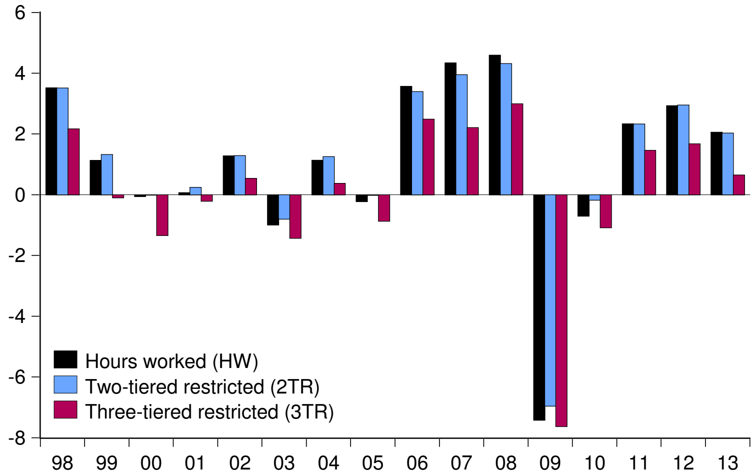

Figure 3 – Labour services. Per cent. HW, 2TR and 3TR are calculated using Equation 2.14, Equation 2.15 and Equation 2.11, respectively. Source: Statistics Norway, authors' calculations

growth through the entire sample period. On average, labour services grew by only 0.12 per cent annually when measured by the 3TR index. This indicates an average annual quality adjustment of -0.98 percentage points relative to the HW index. The large difference also seen between the 2TR index and the 3TR index implies that there is a lot to be gained in terms of quality adjustment by aggregating from the individual worker. In this respect, it should be noted that the educational dimension is not included in the data. Some of the gap between the 2TR and 3TR index could possibly be reduced if the level of education was included as a separate category.

Figure 4 shows how quality adjustment is distributed across workers that are continuing and workers that are either entering or exiting (net entry). In the course of the sample period, it is the quality adjustment attributable to continuous workers that contributes most to the overall quality adjustment. It explains -0.95 percentage point of the total bias of -0.98 percentage point. Although net entry accounts for only -0.03 out of -0.98 percentage point for the entire sample period, the quality adjustment due to net entry is more pronounced at different stages of the business cycle. For example, in the boom years 2007 and 2013, the quality adjustments due to net entry of workers were -0.46 and -0.38 percentage point, respectively. In contrast, in the two contractionary years 2001 and 2009, the quality adjustments due to net entry were 0.41 and 0.25 percentage point, respectively.

Table 3 shows the underlying data for the quality adjustment between the 3TR index and the HW index. In contrast to Figure 4, Table 3 shows quality adjustment for workers entering and exiting separately. The high level of quality adjustment in 2007 and 2008 of about -1.7 percentage points due to workers entering should be viewed in conjunction with the surge in immigration to Norway after the enlargement of the EU

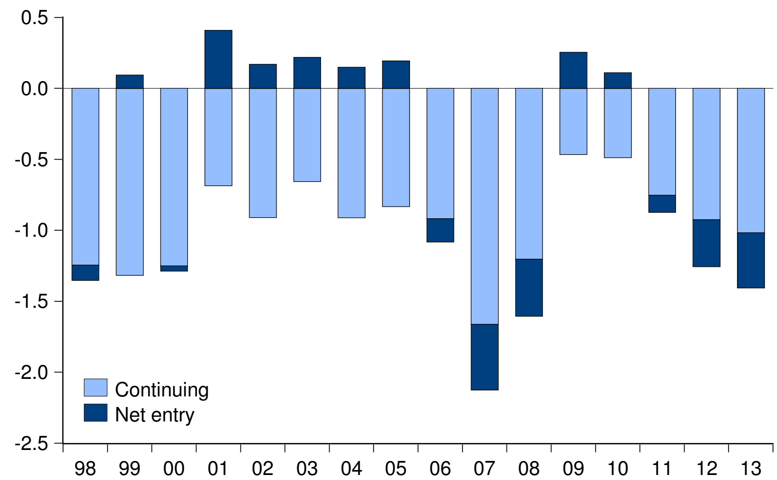

Figure 4 – Quality adjustment. The total quality adjustment is measured as the difference between the 3TR index in Equation 2.11 and the HW index in Equation 2.14. "Continuing" is measured as $\sum_{i\in C_t} w_{it}\Delta \ln H_{it} - \Delta \ln \sum_{i\in C_t} H_{it}$, while 'Net entry" is measured by $-\ln\left(1-s_t^N\right)+\ln\left(1-s_{t-1}^X\right)-\left(-\ln\left(1-h_t^N\right)+\ln\left(1-h_{t-1}^X\right)\right)$. Source: Statistics Norway, authors' calculations

in 2004. While the average quality adjustment due to entering workers was -1.25 percentage points, the average quality adjustment due to exiting workers was 1.22 percentage points. It follows from the theoretical discussion that the unit wages of both entering and exiting workers are lower than those of continuing workers. It is thus workers with lower quality that enter the labour market during expansions and lower quality workers that exit the labour market during contractions. All in all, the results point to quality adjustment due to net entry being highly cyclical.

4 Conclusions

In this paper, we have developed a theory-based index for labour services that also quality-adjusts the impact of workers entering and exiting the labour market. To this end, we have formulated a three-tier framework that encompasses the methods used in the literature. In contrast to other indices that implicitly impose the condition that workers entering and exiting the labour market are equal in quality to those who are continuously employed, our proposed index allows for workers being of different quality. To illustrate the theoretical framework, we have used the case of Norway in the period between 1997 and 2013. We found a mean quality adjustment of about minus one percentage point annually. Most of the adjustment is attributable to continuous workers. This implies that there is a lot to be gained in terms of quality adjustment by aggregating from the individual worker instead of using broad aggregates such as gender, age, industry and immigrant background. Although the quality adjustment emerging from net entry is small for the sample as a whole, there are relatively large contributions from net entry in the course of the business cycle. In particular, it is mainly lower quality workers who enter the labour market during expansions and lower quality workers who exit the labour market during downturns, and this quality adjustment was found to range from -0.5 to 0.4 percentage point annually.

Table 3 – Comparing the hours-worked method with the three-tier restricted index

	Hours worked (HW)[a]				Three-tier restricted (3TR)[b]				Quality adjustment[c]			
	Total	Continuing	Entering	Exiting	Total	Continuing	Entering	Exiting	Total	Continuing	Entering	Exiting
1998	3.52	2.37	5.61	−4.46	2.17	1.13	4.44	−3.40	−1.35	−1.25	−1.17	1.07
1999	1.13	1.23	4.87	−4.97	−0.10	−0.09	3.79	−3.80	−1.22	−1.32	−1.08	1.18
2000	−0.06	0.79	4.97	−5.82	−1.34	−0.46	3.95	−4.84	−1.29	−1.25	−1.02	0.98
2001	0.07	0.02	5.86	−5.81	−0.21	−0.66	4.73	−4.28	−0.28	−0.69	−1.13	1.54
2002	1.28	1.55	5.34	−5.61	0.54	0.64	4.12	−4.22	−0.74	−0.91	−1.22	1.39
2003	−0.99	−0.31	5.12	−5.80	−1.43	−0.96	4.06	−4.52	−0.44	−0.66	−1.06	1.28
2004	1.14	1.28	4.90	−5.04	0.37	0.37	3.85	−3.84	−0.76	−0.91	−1.05	1.20
2005	−0.23	0.15	4.55	−4.93	−0.87	−0.68	3.44	−3.63	−0.64	−0.83	−1.11	1.31
2006	3.57	1.91	6.17	−4.51	2.49	0.99	4.67	−3.18	−1.08	−0.92	−1.49	1.33
2007	4.34	2.51	6.42	−4.59	2.21	0.85	4.69	−3.33	−2.12	−1.66	−1.72	1.26
2008	4.60	2.56	6.50	−4.47	2.99	1.36	4.81	−3.18	−1.61	−1.20	−1.69	1.29
2009	−7.42	−7.97	5.09	−4.53	−7.63	−8.44	3.98	−3.17	−0.21	−0.47	−1.11	1.36
2010	−0.70	−0.17	4.26	−4.79	−1.08	−0.66	3.14	−3.56	−0.38	−0.49	−1.12	1.23
2011	2.34	1.54	4.97	−4.17	1.46	0.79	3.71	−3.04	−0.87	−0.75	−1.25	1.13
2012	2.93	1.77	4.95	−3.78	1.68	0.84	3.55	−2.71	−1.26	−0.93	−1.40	1.07
2013	2.05	0.99	4.67	−3.60	0.65	−0.03	3.31	−2.63	−1.41	−1.02	−1.36	0.97
Mean	1.10	0.64	5.27	−4.81	0.12	−0.31	4.02	−3.58	−0.98	−0.95	−1.25	1.22

[a] Hours worked (HW) is decomposed into continuing, entering and exiting workers using Equation 2.14.
[b] The three-tier restricted (3TR) index is calculated using Equation 2.11.
[c] Quality adjustment is calculated as the difference between the 3TR index and the HW index.
Measured as the logarithmic difference in per cent. Source: Statistics Norway, authors' calculations.

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5 Appendix

5.1 Proof: Proposition 1

The proof of Proposition 1 can be decomposed into the three parts, related to continuing, entering and exiting workers. We utilise the fact that if the function of n variables f can be written $f(y) = f^1(y) + f^2(y) + f^3(y)$, the function F can be written $F(y) = F^1(y) + F^2(y) + F^3(y)$ and that if f^i differentially approximates F^i at y^* for $i = 1, 2, 3$, then f differentially approximates F. Let f represent the function in Equation 2.10 then $f^1 = \sum_{b\in B} \sum_{e\in E_b} \sum_{m\in C_{bet}} w_{bt}\, w_{bet}\, w_{bemt}\, \Delta \ln H_{bemt}$, $f^2 = -\sum_{b\in B} w_{bt} \sum_{e\in E_b} w_{bet} \ln\left(1 - s^N_{bet}\right)$ and $f^3 = \sum_{b\in B} w_{bt} \sum_{e\in E_b} w_{bet} \ln\left(1 - s^X_{bet-1}\right)$. Correspondingly, if F represents the function in Equation 2.11, then $F^1 = \sum_{i\in C_t} w_{it} \Delta \ln H_{it}$, $F^2 = -\ln\left(1 - s^N_t\right)$ and $F^3 = \ln\left(1 - s^X_{t-1}\right)$.

We start by showing that f^1 differentially approximates F^1 to the second order. Diewert (1978) proved that the Törnqvist index is approximately consistent in aggregation since it differentially approximates the Vartia index to the second order and since the Vartia index is consistent in aggregation. By showing that the Sato-Vartia index differentially approximates the Törnqvist index to the second order we have thus shown that f^1 differentially approximates F^1 to the second order. Consider the following index, which encompasses both the Törnqvist and the Sato-Vartia index:

$$\Delta \ln H_t = \sum_{i\in I} (g_{it}\, s_{it-1}) \Delta \ln H_{it} \tag{5.1}$$

where the weights include the generalised logarithmic means of the ratio of the value share for periods t and $t-1$, i.e., $g_{it} = \left(\frac{(s_{it}/s_{it-1})^\alpha - 1}{\alpha(s_{it}/s_{it-1} - 1)}\right)^{1/(\alpha-1)}$. Whereas $\alpha = 2$ yields the Törnqvist index, $\alpha = 0$ yields the Sato-Vartia index (L'Hôpital's rule). It follows from the two lemmas below that the Sato-Vartia index differentially approximates the Törnqvist index to the second order at $s_{it} = s_{it-1}$, and thus that f^1 differentially approximates F^1 to the second order:

Lemma 5.1 *Suppose that: (i) the function of n variables g differentially approximates the function g^* to the second order at the point y^*; i.e., $g(y^*) = g^*(y^*)$, $g_y(y^*) = g^*_y(y^*)$ and $g_{yy}(y^*) = g^*_{yy}(y^*)$; (ii) the function of n variables f differentially approximates the function f^* to the first order at y^*; i.e,. $f(y^*) = f^*(y^*)$ and $f_y(y^*) = f^*_y(y^*)$; and (iii) $g(y^*) = g^*(y^*) = 0$. Then $h(y) = f(y)g(y)$ differentially approximates $h^*(y) = f^*(y)g^*(y)$ to the second order at point y^*.*

Proof. See Diewert (1978, p. 893). □

Lemma 5.2 *The generalised logarithmic mean function $g(a, \alpha) = \left(\frac{a^\alpha - 1}{\alpha(a-1)}\right)^{1/(\alpha-1)}$ differentially approxim-*

ates the generalised logarithmic mean function $G(a,\beta)=\left(\frac{a^{\beta}-1}{\beta(a-1)}\right)^{1/(\beta-1)}$ *to the first order at* $a=1$.

Proof. $g(1,\alpha)=G(1,\beta)$ follows from L'Hôpital's rule. Since $g_a(a,\alpha)=\frac{1}{\alpha-1}\left(\frac{a^{\alpha}-1}{\alpha(a-1)}\right)^{(2-\alpha)/(\alpha-1)}\left[\frac{\alpha a^{\alpha-1}(a-1)-(a^{\alpha}-1)}{\alpha(a-1)^2}\right]$, it follows from L'Hôpital's rule that $g_a(1,\alpha)=1/2=G_a(1,\beta)$. $\square$

That f^2 differentially approximates F^2 and that f^3 differentially approximates F^3 follows when the following lemmas are applied:

Lemma 5.3 *Suppose that: (i) x is a vector of M variables, (ii) y is a vector of N variables that are a subset of the variables in x , (iii) there are N weighting functions $h_n(x)$ such that $\sum_{n=1}^{N}h_n(x)=1$, (iv) there are N weighting functions $H_n(x)$ such that $\sum_{n=1}^{N}H_n(x)=1$, (v) the weights are equal at x^*, i.e., $h_n(x^*)=H_n(x^*)$and (vi) $G(y_n)$ differentially approximates $g(y_n)$. Then the function $F=\sum_{n=1}^{N}H_n(x)\ln\left[G(y_n)\right]$ differentially approximates the function $f=\ln\left(\sum_{n=1}^{N}h_n(x)\,g(y_n)\right)$ at the point x^* where $y_i^*=y_j^*$ for all values of i,j.*

Proof. $F(x^*)=f(x^*)$ follows from Jensen's inequality. For any variable i in the vector y we have $F_{y_i}=H_nG_{y_i}/G(y^*)+\sum_{n=1}^{N}\partial H_n/\partial y_i\ln\left[G(y^*)\right]=H_nG_{y_i}/G(y_i^*)$ and $f_{y_i}=\frac{1}{\sum_{n=1}^{N}h_n\,g(y^*)}\left(h_ig_{y_i}+\sum_{n=1}^{N}\partial h_n/\partial y_i\ln(g(y^*))\right)=h_ng_{y_i}/g(y^*)$. For any variable j in the vector x that is not a part of the vector y we have $F_{x_j}=f_{x_j}=0$, and thus $F_x(x^*)=f_x(x^*)=0$. $\square$

Lemma 5.4 *Suppose that: (i) the set N can be partitioned as a union of disjoint sub-aggregates, $N=\cup_{b\in B}N_b$, (ii) the sets N_b can for all b be further partitioned as a union of disjoint sub-aggregates, $N_b=\cup_{e\in E_b}N_{be}$, (iii) the weights w_b and w_{be} sum to unity within each aggregate, i.e., $\sum_{b\in B}w_b=1$ and $\sum_{e\in E_b}w_{be}=1$ for all values of b. Then the function $F=\sum_{b\in B}w_b\sum_{e\in E_b}w_{be}X_{be}$ can be written $F=\sum_{i\in N}h_iX_i$, where the weights h_i sum to unity, i.e., $\sum_{i\in N}h_i=1$*

Proof. Let $\bar{b}$ denote the number of elements in B and let $\bar{e}_b$ denote the number of elements in E_b. Then $F=\sum_{b=1}^{\bar{b}}\sum_{e=1}^{\bar{e}_b}w_bw_{be}X_{be}=(w_1w_{11}X_{11}+\cdots w_1w_{1\bar{e}_b}X_{1\bar{e}_b})+\cdots+(w_{\bar{b}}w_{\bar{b}1}X_{\bar{b}1}+\cdots w_{\bar{b}}w_{\bar{b}\bar{e}_b}X_{1\bar{e}_b})=\sum_{i\in N}h_iX_i$, where $h_1=w_1w_{11},h_2=w_1w_{12},\ldots,h_{\bar{b}}=w_{\bar{b}}w_{\bar{b}1},\ldots,h_{\bar{b}\times\bar{e}_b}=w_{\bar{b}}w_{\bar{b}\bar{e}_b}$. Since the weights w_b and w_{be} sum to unity, it follows that $\sum_{i\in N}h_i=\sum_{b=1}^{\bar{b}}w_b\sum_{e=1}^{\bar{e}_b}w_{be}=1$. $\square$

The conditions of Proposition 1, $s_{it}=s_{it-1}$ for all $i\in C_t$ and $s_{be_1t}^{N}=s_{be_2t}^{N}$ for all $e_1,e_2\in N_t$ and $s_{be_3t-1}^{X}=s_{be_4t-1}^{X}$ for all $e_3,e_4\in X_t$, imply that $w_{bet}=s_{bet}$ and $w_{bt}=s_{bt}$ for all $b\in B$, $e\in E_b$ since the logarithmic and arithmetic mean coincide when evaluated for equal numbers. The functions f^2 and F^2 can then be written $f^2=-\sum_{b\in B}s_{bt}\sum_{e\in E_b}s_{bet}\ln\left(1-s_{bet}^{N}\right)$ and $F^2=-\ln\left(\sum_{b\in B}s_{bt}\sum_{e\in E_b}s_{bet}\left(1-s_{bet}^{N}\right)\right)$. Lemma 5.4 implies that $f^2=-\sum_{i\in I}h_{it}\ln\left(1-s_{it}^{N}\right)$ and $F^2=-\ln\left(\sum_{i\in I}h_{it}\left(1-s_{it}^{N}\right)\right)$ and Lemma 5.3

implies that these functions differentially approximate each other. By the same reasoning, f^3 differentially approximates F^3.

5.2 Broad aggregates in the two-tier restricted index

The broad aggregates in the two-tier restricted index are:

- Gender
  - Male
  - Female
- Age
  - 16 – 24
  - 25 – 40
  - 41 – 59
  - 60 +
- Industry (SIC 2007[7])
  - Agriculture, forestry and fishing (A)
  - Mining and quarrying (B)
  - Manufacturing (C)
  - Other commodity production (D, E, F)
  - Market-oriented services (G, H, I, J, K, L, M, N, T)
  - Non-market-oriented services (O, P, Q, R, S, U)
- Immigrant background
  - Born in Norway to Norwegian-born parents
  - Other
- Missing data for at least one category

The categories above amount to a total of $2 \times 4 \times 6 \times 2 + 1 = 97$ broad aggregates. It may be the case that hours worked by the same person are split between groups if for example the person changes jobs from one industry to another during a year or holds more than one job in different industries concurrently.

[7] The Standard Industrial Classification (SIC) codes can be found at http://stabas.ssb.no/ItemsFrames.asp?ID=8118001&Language=en