



# **How Should We Measure Residential Property Prices to Inform Policy Makers?**

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# How should we measure residential property prices to inform policy makers?

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## 1 Motivation

Data on house prices provide valuable information as a key macroeconomic indicator for identifying price signals, as an indicator for monetary policy impact analyses via the monetary transmission mechanism and, furthermore, as a tool for measuring an economy's real property assets. The data are also used to assess asset price bubbles as well as weaknesses and sources of potential risks in the financial sector, thus forming a basis for financial stability.

In order to make a statement about the residential real estate market as a whole, aggregation of the available price information is required. This can be done by forming the average using weights covering two different populations. On the one hand, the building stock – that is, all residential buildings existing in an economy – can be used as a basis; this results in a wealth perspective. On the other hand, the calculation can be made using transactions. This reflects market activity. It is appropriate to use different measurement approaches and weights depending on the specific analysis objective. Therefore, a single indicator cannot satisfy all user requirements equally.

This paper examines the various motivations for the analysis of house prices and the alternative measures to be applied in each case. Since for short-term business cycle analysis, the most recent developments are at the centre of attention, aggregation should be performed using transactions. In the case of national accounts, housing price data are needed to convert nominal values into real values. If the price-induced change in the property stock is to be measured, as a component of an economy's assets, and not just traded properties, it is appropriate to apply stock weighting. From a financial stability point of view, the potential build-up of asset price bubbles and the risks of banks' credit exposures associated to the financial soundness of private households are most relevant. Much like in short-term business cycle analyses, transactions can be used as a proxy for financings in order to provide valuable clues on the build-up of risks in banks' new business. It should be

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<sup>1</sup> This paper represents the authors' personal opinions and does not necessarily reflect the views of the Deutsche Bundesbank or its staff.

noted however, that important information on the regional heterogeneity is lost through aggregation.

Owing to newly available data sources for weighting from the 2011 Census of buildings and housing and data on the number of transactions and transaction values for Germany, this paper expands the previous method of calculating house price indices at the Deutsche Bundesbank with regard to weighting schemes. Starting from a methodological framework, this paper examines the distinctive purposes of house price indices and the appropriate information on prices and weights to be applied. The analysis then turns to assessing the differences in the weights as well as a comparison of the resulting price indices. The final section intends an empirical quantification of differences in the results.

## 2 Conceptual framework

The market value of a specific building depends on a variety of factors, such as the location, fittings, age and size of the property. The breakdown of this value<sup>2</sup> into its three main components – price, quality and quantity – can be shown as follows.

$$\text{Value} = \underbrace{\text{Price} \times \text{Quality}}_{\text{Unit value}} \times \text{Quantity} \quad (1)$$

In the calculation, quantity is measured in square metres, for example. The unit value is calculated as the value divided by quantity, ie as the value in euro per square metre. It is thus dependent on the quality of the building concerned and contains not just pure price movements but also changes in quality over time. The quotient of value and price is termed volume and describes the real change in value, adjusted for pure price movements. It can also express, for example, an increase in effective expenditure if this comes about due to energy refitting or modernisation (ie improved quality).

The price in euro per square metre shown in the equation is given with all quality factors eliminated, so that quality appears as a dimensionless mark-up (or mark-down). The intertemporal comparison of prices therefore shows how much more or less would have to be spent today compared with previously under the assumption that the same property would have identical price-relevant fittings and characteristics.

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<sup>2</sup> The derivation of this breakdown is based to a lesser extent on theoretical model considerations, eg portfolio theory, on the value of a reproducible, durable consumer good such as a residential building, but more on the breakdown of the value into a price and quantity component while taking into account changes in quality, as is customary in index theory (and hence for consumer prices as well).

In order to make a statement about the residential real estate market as a whole, aggregation is required. This can be done by forming the average using weights covering two different populations. On the one hand, the building stock – that is, all residential buildings existing in an economy – can be used as a basis; this results in a wealth perspective. On the other hand, the calculation can be made using transactions. This reflects market activity.

### **3 Macroeconomic perspective**

#### **3.1 Identification of price signals to allocate resources**

In a market economy, prices provide signals about relevant shortages through the balance of supply and demand. In this way, enterprises and consumers receive important indications for their production and purchase decisions. Prices and the changes in them thus play a role in the saving and investment decisions of households and commercial investors.

Housing prices are also a key macroeconomic indicator. Rising housing prices tend to stimulate construction activity. What is more, there are indications that inflation in housing prices is linked to transaction frequency. In particular, price rises for housing, which entail increases in value for the owners, can indirectly boost household consumption via wealth effects.

As an indicator for monetary policy, housing prices constitute a key component of headline inflation measurement.<sup>3</sup> The prices of owner-occupied housing may be included in the Harmonised Index of Consumer Prices in future. As with other durable consumer goods, the net acquisition approach is also to be applied here. This takes as its basis expenditure on the acquisition of new housing and on the maintenance and insurance of existing apartments and houses.

Measuring prices based on constant quality and quantity of a well-defined good is crucial to identifying undistorted signals. The measurement objective of a price index is not to portray the development of average expenditure on the acquisition of houses and apartments, which also incorporates higher or lower spending on changes in quality or quantity. Rather, the measurement objective is to record pure price developments under the assumption of identical price-relevant fittings and characteristics. To do this, prices have to be normalised to a uniform standard, which means eliminating quality-related differences.

Aggregation on the basis of transactions, which only incorporates price information on properties actually sold, should be undertaken for business cycle analysis. Ideally, the relevant purchase transactions would be used here as a weighting matrix, which reflect structural differences in the

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<sup>3</sup> The same applies to interregional and international comparisons.

transaction frequency for different property types or regions. Cyclical fluctuations in the weights should be avoided, however.

### 3.2 Uses in the national accounts

Housing price data are also needed to convert nominal values into real values (deflating) in the national accounts.<sup>4</sup> In simplified terms,<sup>5</sup> the volume can be derived as follows.

$$\text{Volume} = \frac{\text{Value}}{\text{Price}} \tag{2}$$

This requires a pure price index for this asset class, which is also termed a deflator.

If the price-induced change in the property stock<sup>6</sup> is to be measured, as a component of an economy's assets, and not just traded properties, it is appropriate to apply stock weighting.<sup>7</sup> In conceptual terms, a prerequisite for this is the availability of price information on both sold and unsold properties. Depending on the source of the price data, it is possible that information will only be available for sold properties; however, their price development can differ from that of unsold properties.

Deflators are additionally used in other sections of the national accounts. First, for overall sales of housing, to depict the real production value of real estate and housing services. Second, a price index for the production value of new buildings is needed, which forms part of gross (fixed) capital formation.<sup>8</sup> Deflating these variables requires a transaction-weighted price index which comprises only the prices of new properties for the new buildings component.

## 4 Application to financial stability

From a financial stability perspective, besides the possible emergence of asset price bubbles, the market risk associated with households' debt sustainability posed to lending banks is of particular relevance. In this connection, the change in value of the financed properties must be noted, taking into account two dimensions: risks involved in newly granted loans and changes in the value of properties in the loan portfolio.

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<sup>4</sup> Naturally, the nominal figures are also justified in their own right as a key indicator.  
<sup>5</sup> This document does not address the problem of breakdown into a country and structural component in further detail.  
<sup>6</sup> The value of the property stock is a significant component of an economy's assets. For example, gross domestic capital stock as reported at replacement cost at year-end 2013 in the area of dwellings constitutes around 265% of nominal gross domestic product for the same year.  
<sup>7</sup> This approach would also be appropriate in terms of estimating wealth effects, as the values of households' individual asset portfolios are influenced by changes in housing prices, which diverge between regions.  
<sup>8</sup> There is a link between the definitions of stock and transaction values. The stock value at the beginning of the period plus the net change in that period gives the stock at the beginning of the subsequent period. Depreciation (devaluations and disposals) and write-ups (owing to construction and renovation, for example) also have to be taken into account.

However, aggregation can result in important information about regional heterogeneity being lost.<sup>9</sup> After all, in line with the experiences of other countries with exaggerations in the housing market, regional trends can definitely develop systemic relevance. Ultimately, undesirable regional developments in lending that initially arise in isolation can multiply, allowing the rise in housing prices to continue gaining ground.<sup>10</sup> Disaggregated price levels are therefore required to examine geographical transmission channels in more detail.

#### **4.1 Assessment of the emergence of asset price bubbles at the current juncture**

The emergence of asset price bubbles is often associated with misallocations, for example on account of a significant increase in construction investment and the corresponding capacities which, in the case of a trend reversal, involves higher default risk in the non-financial corporate sector, amongst other things. However, the acquisition of housing by households, which is credit-financed to a significant degree, merits particular attention. In this context, the value of a property at the time of purchase plays a particular role in lending to households. Thus, for example, the initial ratio of the loan amount to the market value is a key figure in macroprudential analysis.<sup>11</sup> The price dynamics have to be assessed in connection with additional financing indicators. The concurrent increase in lending and easing of lending standards, which can be observed in typical house price booms, is especially risky.

Like the user requirements for business cycle and price analysis, transaction weighting of the properties sold on the market, as an approximation value for financing and construction investment,<sup>12</sup> can provide important information for assessing the build-up of risk positions in new business.

#### **4.2 Observation of the development of financed properties over time**

Another relevant variable that is an important indicator is the change in value over time – changes in price including quality. This is because, with respect to the banks' default risk, the residual value of a property is only of interest when there is a default on loan payments

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<sup>9</sup> For further details, see Deutsche Bundesbank, The determinants and regional dependencies of house price increases since 2010, Monthly Report, October 2013, pp 13-29.

<sup>10</sup> For further details, see Financial Stability Committee, Erster Bericht an den Deutschen Bundestag zur Finanzstabilität in Deutschland, June 2014.

<sup>11</sup> This kind of monocausal analysis falls short of prudential practice in that there are more factors between property appraisal and customer rating than just the loan-to-value ratio, such as the posting of collateral. Furthermore, the normal loan-to-value ratio in Germany is not necessarily a good measure of a property's actual value, as this "may not exceed the value resulting from a prudent appraisal of the future marketability of a property" pursuant to section 16 (2) of the Pfandbrief Act (Pfandbriefgesetz).

<sup>12</sup> For the construction sector, however, there are vastly more suitable indicators available to directly measure activity.

(exposure), as the property would then revert to them and might have to be sold on the market.<sup>13</sup>

As shown in equation (1), the value of an individual property is made up of the three variables of price, quality and quantity. The quantity (eg living space) of a property is generally approximately constant over time. However, the price and quality change over time. Thus, the change in value from the time of house purchase until possible default of the loan amounts to:

$$\text{Change in value} = \text{Price change} + \text{Change in quality.} \quad (3)$$

The condition of the house, ie its quality, is not a fixed variable in the equation, however; rather, a discount is subsequently assumed as a constant annual depreciation factor. A property's value is thus correlated with the price change on the market.<sup>14</sup>

Consequently, only the price developments of bank-financed properties would be relevant from a macroprudential perspective. Equally, the portfolio to be analysed should incorporate only these properties into the weighting scheme. This is crucial in that its composition changes over time. Newly financed buildings and apartments are added, and others are removed, as the loans granted for them were paid off.

For the purposes of financial stability, supplementary institution-specific data for the identification of sources of potential risk are therefore imperative.<sup>15</sup> The question of the breakdown's borders naturally cannot be answered using the available data. The weighting scheme which comes closest to the measurement objective discussed in this section is probably weighting based on the building stock.<sup>16</sup>

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<sup>13</sup> Of course, this is only weighed against the average probability of default in the loan portfolio. In principle, the market value of a property can also fall below the loan amount. As long as households can still make the interest and redemption payments, however, these non-defaulted loans do not play a role in the effectiveness of banks' risk management.

<sup>14</sup> In this context, the absolute residual value of the property is not the decisive factor, but rather the ratio to the outstanding loan amount at the time of the possible loan default. Particularly in the first few years of the mortgage term, however, the principal component of the annuity is very low, while the rate of depreciation here was assumed to be constant, which means that the outstanding loan amount/residual value ratio normally initially deteriorates compared with the time the loan was granted.

<sup>15</sup> In addition, developments broken down by year of loan granting are interesting as these can express the prevailing regime at the time in the form of lending standards.

<sup>16</sup> Nonetheless, this approximation is rough at best. For example, the situation regarding households' ownership of real estate property is as follows, based on the income and consumption sample for 2008: just over half of all households live in rented housing and another fifth own the property without a mortgage loan on it; only around one-quarter of households own housing for which they still have to settle an outstanding loan.

## 5 Data sources

Previously in Germany, there were no granular data nor any statistical register about the property stock available. The results of the building and apartment count in the 2011 Census, which was a complete survey,<sup>17</sup> can now be used as a source of data on the housing stock in Germany and the supply of housing to the population at the granular level. The survey must be repeated every ten years in future.<sup>18</sup> The results thus form a new data set for the building and apartment stock, which also contains information on the number of freehold apartments and single and two-family houses.

The Bundesbank obtains data on the number of transactions and transaction values for freehold apartments as well as for single and two-family houses at the administrative district level from a subsidiary of the Association of German Pfandbrief Banks (Verband deutscher Pfandbriefbanken) called vdpResearch GmbH. In methodological terms, these data are analyses of the market reports issued by the relevant appraisal committees, which are supplemented with estimates.

To determine the weights for new buildings and existing properties, the information on weighting shares in the house price index (HPI) calculated by the Federal Statistical Office (Destatis) can be used. A large number of data sources are used to derive the weights.<sup>19</sup> This involves a breakdown of transaction values which are attributable to new buildings and existing properties; a regional breakdown of the purchase transactions is not possible. Since the HPI is calculated as a chain-linked index on the basis of European requirements, these weights are updated annually.<sup>20</sup> The breakdown of new buildings and existing properties that underlies the Bundesbank's index is the arithmetic average of the data from 2010 to 2012, based on the selected period for the number of transactions.<sup>21</sup>

The underlying price information is provided by bulwiengesa AG.<sup>22</sup> This refers to price data which are quality-adjusted using the method of typical cases<sup>23</sup> which means that only the prices of similar properties are included in the index calculation.<sup>24</sup>

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<sup>17</sup> All of the approximately 17.5 million owners and administrators of real estate were surveyed, by post, on the kind of building, the number of apartments in the building, the building type, the proprietorship, the year of construction and various fittings and fixtures of the apartments.

<sup>18</sup> Regulation (EC) No 763/2008 of the European Parliament and of the Council of 9 July 2008 on population and housing censuses.

<sup>19</sup> For a precise description of how the weights are derived, see J Dechent, *Preisindizes für Wohnimmobilien*, *Wirtschaft und Statistik*, November 2011, pp 1126-1134.

<sup>20</sup> Commission Regulation (EU) No 93/2013 of 1 February 2013 laying down detailed rules for the implementation of Council Regulation (EC) No 2494/95 concerning harmonised indices of consumer prices, as regards establishing owner-occupied housing price indices.

<sup>21</sup> See section 3.2 for the calculation of the mean value for transaction-based figures.

<sup>22</sup> bulwiengesa AG uses information taken, among other sources, from its own appraising activity, from building and loan associations, research institutes, appraisal committees, estate agent associations, chambers of industry and commerce as well as from independent experts. The calculations thus also incorporate valuations, meaning that the data are not purely transaction prices.

## 6 Deriving the weights

In principle, the available information permits two options for weighing together the properties – freehold apartments, terraced houses and detached houses – within an administrative district or city as well as for condensing these data into an aggregate for Germany as a whole. Specifically, one averaging based on stocks and another based on purchase transactions.

It should be noted that weighting is based on space data (stocks or turnover) since the price data provided by bulwiengesa AG are absolute figures in euro per square metre or are converted into such using the classification of building types.<sup>25</sup>

### 6.1 Stock-weighting

Data from the Census as at 9 May 2011, broken down by municipality, are available on apartments in residential buildings containing one apartment, broken down by construction type (detached house, semi-detached house and terraced house), and on apartments in residential buildings divided up according to the Act on Cooperative Apartments and Proprietary Leases (*Wohnungseigentumsgesetz*). This allows apartments in single-family houses to be clearly distinguished from freehold apartments.

Semi-detached houses and terraced houses are accounted for together in the "terraced houses" category. The space available for the property types in question can be determined directly on this basis. Additionally, the above-mentioned transaction-based HPI weights are used consistently for the breakdown into first-time occupancy and re-sale for all administrative districts and cities as well as for property types. The space is formally calculated as follows.

$$\text{Total space}_{i,j,k} = \text{Stock}_{i,k} \times \text{Share}_j \times \text{Living space}_i, \quad (4)$$

where  $i$  refers to the three above-mentioned property types,  $j$  reflects re-sales or first-time occupancies and  $k$  represents the relevant municipality.

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<sup>23</sup> See Deutsche Bundesbank, Price indicators for the housing market, Monthly Report, September 2003, pp 45-58.

<sup>24</sup> The standardised properties are freehold apartments with living space of around 80 square metres and terraced houses with living space of around 110 square metres, each providing comfortable living conditions and in average to good locations, as well as existing single-family houses with living space of around 175 square metres, standard fittings and a standard property size.

<sup>25</sup> To do this, the price data are divided by 110 square metres in the case of terraced houses and by 175 square metres in the case of detached houses. These values are also used for multiplying the stock figures and the transactions; for freehold apartments, 80 square metres are used. For more information, see footnote 3.

Although this weighting scheme results in a simple and complete solution, the next count of buildings and apartments will only be conducted in 2021, meaning that a new basis can only be introduced with a time lag.

## 6.2 Transaction-weighting

Purchase transactions (Purch trans) for freehold apartments (FA) and single and two-family houses (STFH) are available at the level of the rural districts and urban municipalities. The latter category thus first needs to be broken down into terraced houses (TH) and detached single-family houses (DSFH). The Census data are used to do this; however, this only applies to the relationship between these two types of house. The ratio of freehold apartments to houses, on the other hand, is specified from the transaction data. For the cities, a discount must additionally be made on the data from the districts; the Census results are likewise used for this purpose. A factor specific to the property type is determined from the ratio of the stocks in a city to the districts. Using the above notation, the equation can be explicitly presented as follows.

$$\text{Purch trans}_{i,k} = \text{Purch trans}_{\text{STFH},k} \times \frac{\text{Stock}_{i,k}}{\sum_{l \in \{\text{TH}, \text{DSFH}\}} \text{Stock}_{l,k}}, \quad i \in \{\text{TH}, \text{DSFH}\}, \quad (5)$$

$$\text{Purch trans}_{i,m} = \text{Purch trans}_{i,k(m)} \times \frac{\text{Stock}_{i,m}}{\text{Stock}_{i,k(m)}}, \quad i \in \{\text{FA}, \text{TH}, \text{DSFH}\}, \quad (6)$$

where m refers to the cities and k(m) to the districts to which the cities belong.

The breakdown into newly constructed properties and existing properties again uses the HPI weights, and the previously mentioned property spaces are used to provide the relevant turnover of space.

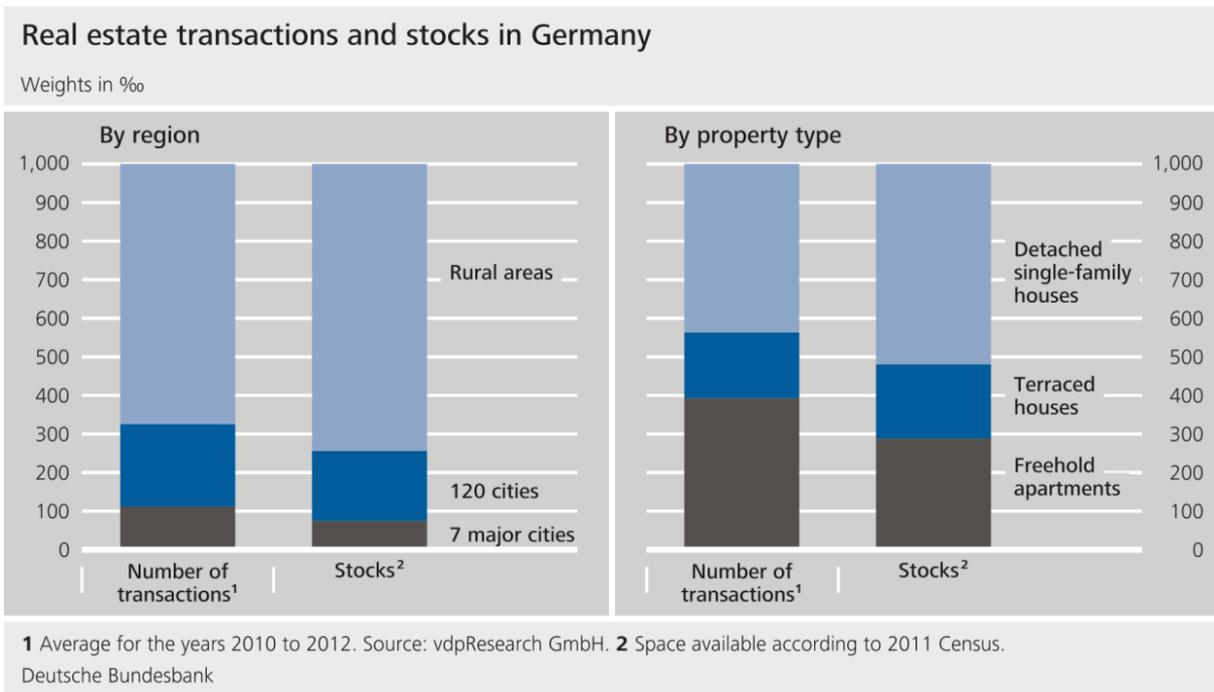
Figures from the years 2010 to 2012 are used for the transactions since purchase transactions are subject to cyclical fluctuations.<sup>26</sup> This is not least because in a chain-linked index where there is a (positive) correlation between prices and transactions, there is the risk of index drift.<sup>27</sup> No chain-linked index was calculated here.

<sup>26</sup> Even when the weighting scheme is based on the transactions from a single year, there were no notable differences in the aggregates considered here.

<sup>27</sup> For an exact description of this effect, see J Hoffmann and A Lorenz, Real estate price indices for Germany: past, present and future, paper presented at the OECD/IMF workshop on real estate price indices in Paris from 6 to 7 November 2006.

## 7 Differences in the weights

The differences in the weights can be explained by structural differences in the sales rates (the ratio of purchase transactions to housing stock) of both between municipalities and property types. For example, freehold apartments are given a higher weight in a transaction-weighting scheme. Their sales rate is higher than that of houses. A breakdown by municipality also reveals that urban regions have a higher share when weights are transaction-based. The reason for this is, again, the higher sales rate compared with rural

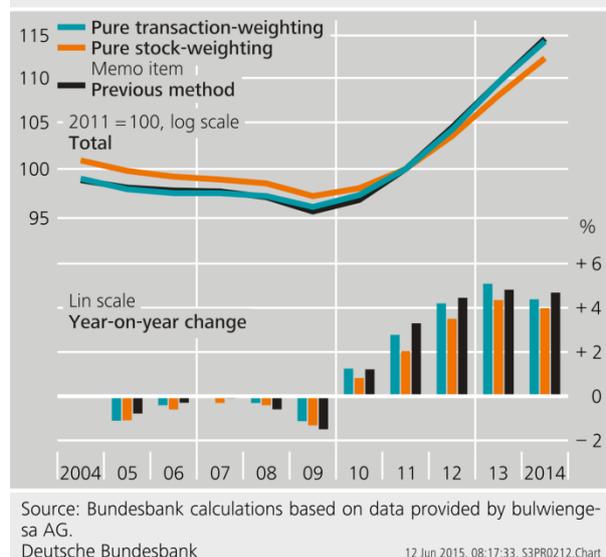


municipalities, not just for apartments but also for houses. The convention can therefore hold that the sales rate is higher in cities than in rural areas and is likewise higher for apartments than for houses.

## 8 Comparison of the price indices

In principle, no fundamentally different statements can be made on trend patterns and the timing of turning points than could be made on the basis of the previous method. Equally, the signs of the annual rates of change remain unaffected, as does the determination of an acceleration or deceleration of price dynamics.

### Comparison of purchase prices for residential real estate in Germany according to different weighting schemes



In principle, however, the stock-weighted data show a flatter pattern compared with the price indices calculated using transactions.

## 9 Empirical quantification of differences

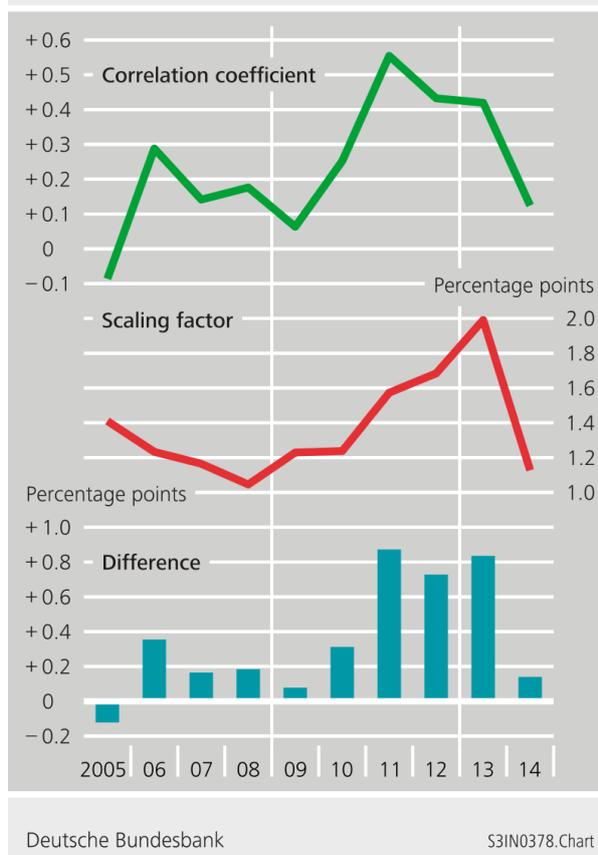
Based on a variation of the theorem of von Bortkiewicz (see annex), we derived a breakdown of the difference in the year-on-year rates of the weightings using purchase transactions and housing stocks.<sup>28</sup> It can be seen that the difference is the product of two factors.

1. The correlation between the price changes of properties in cities and the corresponding sales rates (the ratio of purchase transactions to housing stock).
2. A "scaling factor", the temporal development of which is driven by the standard deviation of the price changes.

Considering the results regarding the differences in weights, the stylised fact can hold that the sales rate is generally higher in cities than in rural areas and is likewise higher for freehold apartments than for terraced houses and detached single-family houses.

At the current end, it can be seen that the fact that the difference between the results on the basis of the two weighting schemes is now only minor can be explained by the correlation having fallen sharply and the standard deviation having decreased.<sup>29</sup> With regard to the results of the calculations for urban housing, the price dynamics of the market segments houses and apartments and also those of the seven largest cities and the 127 cities moved closer together last year.

Breakdown of the difference in the year-on-year rates of the weightings using purchase transactions and housing stocks in 127 cities



<sup>28</sup> For information on the theoretical foundations, see v d Lippe, P (2007), "Index Theory and Price Statistics", P Lang, pp 194-196.

<sup>29</sup> The latter is partly due of course to the overall weaker price developments, as the standard deviation depends on the level of the price changes. With regard to the generally positive correlation, the literature points out that it is transactions that follow developments in prices (and not the reverse).

The evident connection between price changes and sales rates on the one hand and the difference between transaction and stock weightings on the other can therefore be proven analytically and thus also quantified empirically (see the chart for 127 cities).

## Derivation of the adaptation of the theorem of von Bortkiewicz on the difference in year-on-year rates

The absolute data for the average prices per square metre in a year  $t$  are produced based on the new weighting of housing prices as follows

$$P^t = \sum_{l \in L} p_l^t \cdot \frac{\bar{q}_l}{\sum_{l \in L} \bar{q}_l},$$

where the summation can be carried out using the property types, re-sales or first-time occupancies, and administrative districts or cities. The price  $p$  stated in the equation is in euro per square metre, the quantities  $q$  are either space available (“ $B$ ”) or take-up of space (“ $U$ ”), each in square metres.

Consequently, the published Laspeyres price indices with the base year 2011 are as follows

$$P^{2011:t} = \frac{P^t}{P^{2011}} = \frac{\sum_{l \in L} p_l^t \cdot \bar{q}_l}{\sum_{l \in L} p_l^{2011} \cdot \bar{q}_l}.$$

A rebased Lowe price index is used for the year-on-year change

$$\frac{P^{2011:t}}{P^{2011:t-1}} = \frac{\sum_{l \in L} p_l^t \cdot \bar{q}_l}{\sum_{l \in L} p_l^{t-1} \cdot \bar{q}_l} = \frac{P^t}{P^{t-1}} = P^{t-1:t}.$$

The theorem of von Bortkiewicz provides the following equation for comparing year-on-year change based on transaction and stock weightings

$$\frac{P_U^{t-1:t}}{P_B^{t-1:t}} - 1 = \frac{\text{Cov} \left[ \frac{p^t}{p^{t-1}}, \frac{\bar{q}_U}{\bar{q}_B} \right]}{\text{E} \left[ \frac{p^t}{p^{t-1}} \right] \cdot \text{E} \left[ \frac{\bar{q}_U}{\bar{q}_B} \right]},$$

where the ratio of take-up of space to space available reflects the sales rate and all moments are weighted with  $p^{t-1} \cdot \bar{q}_B$ , which gives  $\text{E} \left[ \frac{p^t}{p^{t-1}} \right] = P_B^{t-1:t}$ .

This results in the following equation for the difference in the year-on-year rates of both weightings

$$P_U^{t-1:t} - P_B^{t-1:t} = \frac{\text{Cov} \left[ \frac{p^t}{p^{t-1}}, \frac{\bar{q}_U}{\bar{q}_B} \right]}{\text{E} \left[ \frac{\bar{q}_U}{\bar{q}_B} \right]}.$$

Finally, the breakdown is shown as the product of two factors

$$P_U^{t-1:t} - P_B^{t-1:t} = \text{Corr} \left[ \frac{p^t}{p^{t-1}}, \frac{\bar{q}_U}{\bar{q}_B} \right] \cdot \sqrt{V \left[ \frac{p^t}{p^{t-1}} \right]} \cdot \frac{\sqrt{V \left[ \frac{\bar{q}_U}{\bar{q}_B} \right]}}{\text{E} \left[ \frac{\bar{q}_U}{\bar{q}_B} \right]} \sim \text{Corr} \left[ \frac{p^t}{p^{t-1}}, \frac{\bar{q}_U}{\bar{q}_B} \right] \cdot \sqrt{V \left[ \frac{p^t}{p^{t-1}} \right]},$$

where the rough proportionality is the result of the approximate constancy of the variation coefficient of the sales rates over time.