Polarization and Convergence: Measurement in the Absence of Cardinality

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Abstract.

Polarization and Convergence, terms employed in many situations in the social sciences, can be interpreted as describing types of transition process between populations observed at departure and arrival points. Typically the processes are calibrated in the context of variates that convey a sense of distance between, and concentration within, groups in a resultant "snap shot" arrival state distribution, so the phenomena can only be established by comparing indices at two points. Yet in many environments the terminologies are applied to groupings that are categorical and ordinal in nature, devoid of a meaningful cardinal distance measure (for example, socio-economic, political, educational and health status variables) and which relate to transitions between non-comparable states (for example parental social class to child educational outcome). In such contexts measurement must rely upon the anatomy of the transition process without dependence on a cardinal distance measure or the comparison of cardinally non-comparable states. Accordingly here Polarization/Convergence indices and tests based upon the structure of an underlying transition process are proposed and implemented. The tools have many diverse applications, 3 examples from Canadian Generational relationships, the world size distribution of Gross National Product per capita and Chinese Social to Educational Class Structures illustrate their use and revealing many interesting features of polarizing and converging behaviors.

Introduction.

Polarization, "the production of polarity, a sharp division, as of a population or group, into opposing factions" and Convergence, "an act or instance of converging, the contraction of a vector field"[1] are terms that have been applied in many diverse economic and social contexts. Recently the disappearance or emergence of classes in a society has attracted interest[2]. Conceptually similar themes have found expression in contemporary conflict, segregation and social polarization literatures which have shifted emphasis toward emerging dissimilarities as matters for study[3]. In the equality of opportunity and labor mobility literatures typically relating Socio-economic Status to education or health outcomes or changes in earnings status over time, transitions from circumstances (departure states) to achievements (arrival states) are evaluated in terms of the extent to which the outcome distributions of different circumstance classes differ[4]. These all have a "converging" or "polarizing" interpretation in the sense that there is some increasing commonality (i.e. absence of dominance) or dissimilarity in the arrival state distributions of different departure classes. Measurement and calibration in the vast

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[1] Definitions drawn from Websters and Concise Oxford Dictionary.

[2] See inter alia Acemoglu 1999, Autor, Katz and Kearney 2006, Autor and Dorn 2013, Beach and Slotsve1996, Borgen 2015, Davies and Zarifa 2012, Heisz 2007, Foster and Wolfson 1992, Goos and Manning 2007, Goos, Manning, and Salomons 2009, Kharas 2010, Triventi 2013, US Department of Commerce 2010, Zhang and Kanbur 2001.

[3] Here the Herfindahl-Hirschman fractionalization index (Herfindal (1950)) has frequently been employed, see also Lichbach 1986, Easterly and Levine 1997, Rodriguez, and Salas 2002, Reynal-Querol 2002, Fearon and Laitin 2003, Collier and Hoffler 2004, Miguel, Satyanath and Sergenti 2004, Echenique and Fryer2007 Esteban and Schneider 2008, Frankel and Volij 2011, Gentzkow and Shapiro. 2011, Ray and Esteban 2011, D'Ambrosio C., Bossert W. and E. La Ferrara 2011, D'Ambrosio and Permanyer 2013 , Mele 2013 Ballester and Vorsatz. 2014.

[4] Kanbur and Stiglitz 1982, 1986, 2016, Dardanoni 1993, Fields and Ok 1996, Maasoumi 1999, Van de Gaer, Schokkaert and Martinez 2001, Fields 2008, Lefranc et al 2008, 2009, Dardanoni, Fiorini and Forcina 2010, Cutler, Lleras-Muney and Vogl 2011, Ramos and Van de Gaer 2014.

majority of this work relies upon measurement of some aspect of the anatomy of the final state distribution or distributions.

There has also been a developing theoretical and empirical literature on polarizing and converging groupings of countries in the development and growth literatures which has more of a dynamic flavor but these works rely upon cardinal measures national income concepts[5]. The seminal work on the phenomena (Barro and Sala-i-Martin 1992 Esteban and Ray 1994, Duclos, Esteban and Ray 2004) defined and calibrated Polarization and Convergence on a continuously measured variate, relying upon a sense of changing distance between, and concentration within, polarizing or converging groups[6]. While Polarization Indices are axiomatically developed on the basis of dynamic movements (slides and squeezes) of class size distributions of fixed size, the measures end up characterizing the anatomy of an arrival state distribution so that indices have to be compared between states in order to establish polarization. On the other hand early work on convergence (Barro and Sala-i-Martin 1992) focused on the nature of the dynamic process in a regression to the mean typology, almost without reference to the (possible) changing nature of the distribution of interest.

However by definition, the phenomena each describe the nature of a process of change in and between states which are not necessarily cardinally ordered or based upon the same cardinal ordering. They speak to the division and/or amalgamation of subgroups (i.e. vector field

[5] See Barro, Sala-i-Martin 1992, Quah 1993, 1996, Galor 1996, , Keefer and Knack 2002, Anderson 2004, 2004a, , Anderson, Linton and Leo 2012, Pittau, Zelli and Johnson 2010, Anderson, Pittau and Zelli 2016.

[6] Although Polarization describes the process of transition between states, its indices focus on the anatomy of the final state distribution, Convergence is usually interpreted in terms of the adjustment coefficient of a transition to a stationary process, both in terms of continuously measured variates.

expansions or contractions) implying changes in group sizes without particular reference to cardinal measures of distance. Indeed in many of the above cited references, transition is between categorical (though usually ordered) classes for which no distance metric is available. In such situations movements of agents between the classes is the sole issue. In the statistics literature the evolution of states is studied in the context of Markov Chain Processes (Billingsley 1961, Shorrocks 1976, Geweke et. al. 1986) and Copula theory (Nelsen 1999, Jaworski, Durante, Härdle and Rychlik 2010) characterizing transitions and mobility between classes, frequently without reference to a metric by which those classes can be differentiated. In this literature mobility indices are frequently based upon square matrices, precluding differences in the numbers of classes in departure and arrival states and thus not allowing for the disappearance or emergence of classes. They do not necessarily address movements in the cardinal values associated with classes that they are ranked by, with good reason, because such values do not exist, i.e. classes only have an ordinal ranking.

Here Polarization and Convergence indices will be developed in the context of patterns of transition based upon the balance of probabilities of movement between groups without reliance on a cardinal measure of distance between them. In the following, Section 1 relates the axiomatic basis for transition based indices to the axioms currently employed in the cardinal characterization and Section 2 outlines a family of transition based polarization / convergence indices and tests. These are exemplified in section 3 with 3 examples. The first, from Parent – Child educational transitions, illustrates differences between male and female transitions and the changing nature of the process in Canada over the last half century. The second, from a study of national GDP per capita groupings illustrates how transition analysis can add additional insight to cardinal measures when they are available. The third, from a study of Chinese Grand Parent to Grand Child social class to educational class transitions illustrates how polarizing behavior can be measured when the transition is across differently defined variables.

Section 1. Developing Polarization/Convergence Indices for Transition Matrices.

Esteban and Ray (1994) and Duclos, Esteban and Ray (2004) formulated polarization indices[7] for discrete and continuous destination distributions by positing a collection of axioms whose consequences should be reflected in a Polarization measure. The axioms are founded upon an Identification-Alienation nexus wherein notions of polarization are fostered jointly by an agent's sense of increasing within-group identity (measured by smaller aggregate distances between within group agents) and between-group distance or alienation (measured by greater aggregate distances between agents of different groups). They were couched in terms of movements of subgroup distributions defined over a metric (note in the discrete case for definitional purposes variate values were allowed to wander on the real line). Thus in the discrete case π_i, i=1,..,n corresponds to the probability that outcome at location x_i i=1,…,n will occur and in the continuous case f(x) is the pdf of x.

The four axioms may be summarized as follows:

[7] See also Wang and Tsui 2000, Wolfson 1994, Anderson 2004, 2004a, 2012, Anderson, Linton Leo 2012.

Axiom 1: A mean preserving reduction in the spread of a distribution cannot increase polarization.

Axiom 2: Mean preserving reductions in the spread of sub-distributions at the extremes of a density cannot reduce polarization.

Axiom 3: Separation of two sub-densities towards the extremes of the distributions range must increase polarization.

Axiom 4: Polarization measures should be population-size invariant.

The general polarization indices developed as a consequence of these axioms for discrete and continuous distributions at the arrival state may be written respectively as:

$$P_\alpha \propto \sum_{i=1}^{n}\sum_{j=1}^{n} | x_i - x_j | \pi_i^{1+\alpha} \pi_j \quad [1]$$

$$P_\alpha(F) \propto \int_y f(y)^\alpha \int_x | y - x | dF(x) dF(y) \quad [1a]$$

In both discrete and continuous versions $\alpha > 0$ is a polarization sensitivity factor chosen by the investigator. It may be seen that $\alpha = 0$ yields the Gini coefficient, essentially the mean standardized average distance between all points in the distribution, Polarization measures rescale the component distances by a monotonically non decreasing function of the pdf based upon α.

The axiomatic construction of these indices is couched in terms of potential changes in location of and squeezes (reductions in the spread) of the basic densities describing sub-populations defined on a particular cardinal metric. The populations of basic densities remain intact, they just change locations and domains, the possibility of the relative size of component populations changing (i.e. people switching classes) is not entertained, neither is the possibility of classes polarizing when they only possess an ordinal ranking since $|x-y|$ is not defined in this case. In this regard the development of a measure of transitional polarization requires that the axioms be articulated in the absence of, or at least without reliance upon, a cardinal ranking of sub populations in the context of a transition matrix.

As far as the evaluation of societal wellbeing in terms of inequalities engendered by transition structures is concerned, Kanbur and Stiglitz (1982, 1986[8]), Dardanoni (1993) and Dardanoni, Fiorini and Forcina (2010) consider an expected infinite lifetime welfare ranking of 2 monotone mobility matrices with the same long run stationary solution. In essence they demonstrate the equivalence of the welfare ranking of the two processes with a dominance ranking between respective rows of the two transition matrices, posing and answering the question as to which transition matrix is offering an agent emerging from each class in period t the best lottery on outcomes in period t+1, if dominance prevails for all inheritance classes then the dominating transition matrix will yield the highest welfare[9] (see also Lefranc et al. (2008, 2009)). A problem with these results in the present context is that the measures of the values of the social classes (as referred to in Dardanoni, Fiorini and Forcina (2010)) are not commensurable across transition matrices because of the lack of cardinality.

Here Axioms 1 and 2 are re - interpreted in terms of the effect of a particular transition matrix on a probability distribution without reference to a cardinal ordering with Axiom 1 addressing

[8] Revisited in Kanbur and Stiglitz (2016).

[9] An interesting feature of the Kanbur - Stiglitz results is that the standard equality of opportunity transition matrix outcome does not always yield the highest welfare in terms of minimizing dynastic inequality.

transitions that transfer mass to the center of the distribution and Axiom 2 addressing transitions that concentrate mass at the extremities of a distribution[10]. In essence the polarization measures summarize, in an index form, the effects on the anatomy of a distribution that has been the subject of such transformations. Axiom 3 addresses changes in the location of sub-distributions in the form of movements over the metric away from the center of the distribution, which could be reflected in movements of mass away from the center of a distribution. Axiom 4 is just a size invariance property. Fundamentally, in the absence of a metric, the axioms are about the balance of two opposing forces, converging forces that transfer mass from groups on the peripheries to groups at the center and polarizing forces that transfer mass from groups at the center to groups at the peripheries and a transition matrix based polarization/convergence measure should reflect the balance of such forces. Consider the following 2 axioms.

A1 Transitions that promote net relocation of mass toward the peripheries of a distribution increase (reduce) polarization (convergence) indices I(T).

A2 Transitions that promote net relocation of mass toward the center of a distribution reduce (increase) polarization (convergence) indices I(T).

The Normalization, and Period Consistency Axioms of Shorrocks (1978a), are pertinent and since the initial and outcome states may only possess an ordinal ranking and be associated with some arbitrary scale, any index would need to satisfy the usual scale invariance and scale independence axioms found in the inequality index literature (see for example Kobus and Mił

[10] However they could also be contemplated in terms of locational movements of sub distributions when a metric is available see Appendix 1.

o's 2012). So A1 and A2 will be augmented to cover these ideas. Let C be the set of scaling functions associated with the classes such that in a typical k x 1 vector c, elements c_i and c_j are such that $c_i < c_j$ for all i < j i,j = 1,..,k. and indices I(T,c) based upon the transition matrix T from the family of transition matrices **T**. Consider:

A3. Continuity: I(T,c) T ε **T**, c ε C : is a continuous function.

A4. Scale Invariance ($I(T_1,c_1) < I(T_2,c_1) \iff I(T_1,c_2) < I(T_2,c_2)$ for all $c_1 \neq c_2$, c_1 , c_2 ε C is invariant to scaling of the index.

A5. Scale Independence ($I(T, c_1) = I(T, c_2)$) for any $c_1, c_2 \in C$

A6. Normalization $0 \leq I(T,c) \leq 1$

These axioms parallel standard axioms used in inequality measurement with A3 requiring continuity, A4 requiring invariance to scale changes and A5 demanding independence of scale changes (Obviously if A5 holds, then so does A4). A6 requires that the index be normalized, i.e. zero is assigned to the lowest valued distribution and one to the highest valued distribution. In terms of mobility the monotonicity property of Shorrocks 1978a, 1978b is relevant taking a view of mobility that sees transition matrix $A=||a_{ij}||$ exhibiting greater mobility than $B=||b_{ij}||$ when $a_{ii} \leq b_{ii}$ for all i with strict inequality holding somewhere and a stronger version in terms of off-diagonals, has $a_{ij} \geq b_{ij}$ for all i≠j with strict inequality for some i≠j, rendering the identity matrix the immobility extreme. Here strong monotonicity implies less chance of remaining in the same position, and greater chance of going elsewhere. Another extreme could be the reversal or oscillation matrix[11] with ones on the counter diagonal and zeros elsewhere. Perfect mobility in the present context is often defined as the identical columns matrix.

For intuition, consider some 3 x 3 examples of transition matrices in the model $x_F = Tx_I$ where x_F and x_I are conformably defined final and initial state vectors of relative class sizes respectively. Note that the initial state could be subject to a sequence of K transitions of the form T in which case $x_F = T^K x_I$, and, in thinking in terms of process where x_F could be next periods x_I, it is necessary to think of the properties of such a sequence in terms of the properties of T. Some examples of Polarizing, Converging, Socially Static (Immobile and Mobile) and Oscillating Class Transition matrices together with their matrix properties for the departure state - arrival state transition and their 10 period sequence counterparts are presented in Tables 1. Arrival state outcomes after 1 and 10 transit periods starting with equi-probable classes, a single middle class and equal sized poor and rich classes are reported in Tables 2a, 2b and 2c.

Table 1.

T^1 *Transition Matrices*

$$\begin{array}{ccccc} \textit{Polarizing} & \textit{Converging} & \textit{Static (IM)} & \textit{Static (MO)} & \textit{Oscillating} \\ \begin{bmatrix} 1 & 0.3 & 0 \\ 0 & 0.4 & 0 \\ 0 & 0.3 & 1 \end{bmatrix} & \begin{bmatrix} 0.5 & 0 & 0 \\ 0.5 & 1 & 0.5 \\ 0 & 0 & 0.5 \end{bmatrix} & \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} & \begin{bmatrix} 1/3 & 1/3 & 1/3 \\ 1/3 & 1/3 & 1/3 \\ 1/3 & 1/3 & 1/3 \end{bmatrix} & \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \end{array}$$

T^{10} *Transition Matrices*

$$\begin{array}{ccccc} \textit{Polarizing} & \textit{Converging} & \textit{Static (IM)} & \textit{Static (MO)} & \textit{Oscillating} \\ \begin{bmatrix} 1 & 0.500 & 0 \\ 0 & 0.000 & 0 \\ 0 & 0.500 & 1 \end{bmatrix} & \begin{bmatrix} 0.001 & 0 & 0 \\ 0.999 & 1 & 0.999 \\ 0 & 0 & 0.001 \end{bmatrix} & \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} & \begin{bmatrix} 1/3 & 1/3 & 1/3 \\ 1/3 & 1/3 & 1/3 \\ 1/3 & 1/3 & 1/3 \end{bmatrix} & \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \end{array}$$

[11] what Atkinson (1981) referred to as complete reversal transitions, a biblical "..that are first shall be last and last shall be first" (Matthew ch19 v30) transition.

Observe that after a 10 period sequence the middle row is vanishing in the non-singular polarizing matrix as is the first and last rows in the non-singular converging matrix, ultimately as k gets large the k'th period polarizing matrix will have rank 2 and the converging matrix will have rank 1. Note also that $|T^k|$-> 0 as k increases for non-static or oscillating transition matrices with the "disappearance" of rows (classes) i.e. $(0<|T|<1)$. Whereas $|T^k|$ does not change with k in the static matrix and the oscillating matrix has $|T^k| =(-1)^k$ so one way of identifying potential for oscillation is $|T| < 0$. Thinking of the outcomes after 1 and 10 periods of the same transition structure observe in table 2a the polarizing matrix engenders rich and poor classes by dissipating the middle class in the long run (the disappearing middle class

Table 2a. Outcomes after 1 and 10 transit periods with initial equi-probable classes

		Polarizing	Converging	Static (Immobile)	Static (Mobile)	Oscillating
1 Transition Period	Poor Class	0.4333	0.1666	0.3333	0.3333	0.3333
	Middle Class	0.1333	0.6666	0.3333	0.3333	0.3333
	Rich Class	0.4333	0.1666	0.3333	0.3333	0.3333
10 Transition Periods.	Poor Class	0.5000	0.0003	0.3333	0.3333	0.3333
	Middle Class	0.0000	0.9993	0.3333	0.3333	0.3333
	Rich Class	0.5000	0.0003	0.3333	0.3333	0.3333

Table 2b. Outcomes after 1 and 10 transit periods starting with an initial single middle class.

		Polarizing	Converging	Static (Immobile)	Static (Mobile)	Oscillating
1 Transition Period	Poor Class	0.3000	0.0000	0.0000	0.3333	0.0000
	Middle Class	0.4000	1.0000	1.0000	0.3333	1.0000
	Rich Class	0.3000	0.0000	0.0000	0.3333	0.0000
10 Transition Periods.	Poor Class	0.5000	0.0000	0.0000	0.3333	0.0000
	Middle Class	0.0000	1.0000	1.0000	0.3333	1.0000
	Rich Class	0.5000	0.0000	0.0000	0.3333	0.0000

Table 2c. Outcomes after 1 and 10 transit periods starting with equal sized rich and poor class

		Polarizing	Converging	Static (Immobile)	Static (Mobile)	Oscillating
1 Transition Period	Poor Class	0.5000	0.2500	1.0000	0.3333	0.5000
	Middle Class	0.0000	0.5000	0.0000	0.3333	0.0000
	Rich Class	0.5000	0.2500	1.0000	0.3333	0.5000
10 Transition Periods.	Poor Class	0.5000	0.0000	1.0000	0.3333	0.5000
	Middle Class	0.0000	1.0000	0.0000	0.3333	0.0000
	Rich Class	0.5000	0.0000	1.0000	0.3333	0.5000

scenario) as it does in table 2b, in table 2c it simply reinforces the existence of the 2 classes. The converging matrix engenders a middle class by dissipating the rich and poor classes in the long run as it does in table 2c, in Table 2b it simply reinforces the existence of the middle class.

In the following consider a general n_F x n_I transition matrix T of the form:

$$T = \begin{bmatrix} t_{11} & t_{12} & . & t_{1n_I} \\ t_{21} & t_{22} & . & t_{2n_I} \\ . & . & t_{ij} & . \\ t_{n_F 1} & t_{n_F 2} & & t_{n_F n_I} \end{bmatrix}$$

Describing the transition from initial n_I x 1 ordered (lowest 1, to highest n_I) vector state probabilities π_I to final n_F x 1 ordered vector state probabilities π_F as the system of equations:

$$\pi_F = T\pi_I$$

Note that t_{ij} is the conditional probability of emerging in Final state “i” given an Initial state “j” as such T is a stochastic, but not necessarily bi-stochastic, matrix with $t_{ij} \geq 0$ and $\Sigma_i\, t_{ij} = 1$ for $j=1,..,n_F$.

An index of Mobility for Square and Non-Square Transition matrices.

Since in much of this work non-square transition matrices are common the Shorrocks (1978a, 1978b) suggestion of a simple trace based index of mobility is not viable. The j'th column of T corresponds to the probability distribution over the Final State outcome space for agents emerging from Initial State "j". As such, perfect mobility (where the Final State is uninfluenced by or independent of the initial state) is characterized by T having common columns which all sum to 1 and in the square transition matrix case complete immobility is characterized by T = I, the identity matrix. Letting $t_{i.}$ be the i'th row of T and MAXR($t_{i.}$) and MINR($t_{i.}$) operators which return the maximum and minimum value in the row vector respectively , TM, an index of mobility, may be written as:

$$TM(T) = 1 - \frac{\sum_{i=1}^{n_F} (MAXR(t_{i.}) - MINR(t_{i.}))}{n_I}$$

TM is one minus an n_c distribution version of Gini's two distribution dissimilarity or "transvariation" index (Gini (1915))[12] rescaled by the number of distributions being compared. When columns of T are identical, the Final State Outcome distributions emerging from the n Initial States will overlap perfectly, the sum of maximums will equal the sum of minimums and TM = 1. If on the other hand the Final State Outcome distributions are orthogonal as in the Perfect Immobility case the intersections of the overlaps will be null (the sum of minimums will be 0) and the sum of maximums will equal n_c, the number of conditional distributions so that

[12] It could be based on the discrete multivariate distribution Overlap measure of Anderson and Leo (2011) the continuous version of which is given in Anderson, Linton and Wang (2012).

TM = 0 so that in the square T case TM(T) will be 0 when T=I. 1-TM(T) has the interpretation of an inequality of distribution index,a distributional GINI index as it were.

Since it may readily be shown that 0 ≤ TM(T) ≤ 1, the index satisfies Normalization, Immobility and Perfect Mobility Axioms, while it satisfies the Strong Perfect Mobility Axiom (TM(T) = 1 if and only if T has common columns), it does not satisfy the Strong Perfect Immobility Axiom (TM(T) =0 if and only if T = I) since TM(T) =0 for any column rearrangement of the identity matrix. Monotonicity axioms require TM(T) > TM(T*) when $t_{ij} \geq t_{ij}{}^*$ for all $i \neq j$ with strict inequality holding somewhere. Period Consistency requires TM(T) ≥ TM(T*) implies TM(T^k) ≥ TM(T^{*k}) for positive integer k > 0. When the initial and outcome states are ordered and associated with some scale the index TM(T) satisfis scale invariance and independence axioms.

Transition Matrix Based Polarizing – Converging Indices.

In the spirit of Esteban and Ray (1994) and Duclos, Esteban and Ray (2004), the class of transitions from and to the center of a distribution with respect to it peripheries are the focus of attention. Working with the notion that there are 3 departure and arrival classes Poor, Middle and Rich[13], the transition matrix T, where T_{ij} corresponds to the conditional probability of transiting from initial period class j to final period class i given an agent is in class j initially, is given by:

$$T = \begin{matrix} T_{11} & T_{12} & T_{13} \\ T_{21} & T_{22} & T_{23} \\ T_{31} & T_{32} & T_{33} \end{matrix}$$

[13] When matrices are more than 3x3 rows and columns can be aggregated to 3x3 form or the following indices can be developed on the basis of bigger dimensioned matrices.

T is related to the joint density matrix P = $||P_{ij}||$ of basic probabilities that an agent is in class j in the initial state and class i in the final state by the equations:

$$T = \begin{bmatrix} P_{11} & P_{12} & P_{13} \\ P_{21} & P_{22} & P_{23} \\ P_{31} & P_{32} & P_{33} \end{bmatrix} x \begin{bmatrix} \sum_{i=1}^{3} P_{i1} & 0 & 0 \\ 0 & \sum_{i=1}^{3} P_{i2} & 0 \\ 0 & 0 & \sum_{i=1}^{3} P_{i3} \end{bmatrix}^{-1}$$

To develop polarization – depolarization indices of class transitions, the net transfer of mass to the final state middle class that is brought about by the transition process is contemplated. This depends upon the balance of (i.e. difference in) probabilities that an agent outside the initial state middle class would move into the final state middle class (a converging transition) versus an agent inside the initial state middle class would move out of it in the final state (a polarizing transition). Negative values of this difference correspond to a Polarizing Transition, positive values correspond to a Converging Transition. When a cardinal class ranking is available these probabilities could be scaled by a distance measure if needed (details from the author on request).

“Balance of Probabilities” Polarization and Net Advancement Measures.

Processes that promote more transitions from the middle class to the peripheries than transitions from the peripheries to the middle class are polarizing. The probability that an agent would move out of the middle class given that she is inside the middle class initially (the divergent component) and the probability that an agent would move in to the middle class given she is outside of it initially (the convergent component) are respectively given by:

$$P(Agent \notin Final\ State\ Middle\ class \mid Agent \in Initial\ State\ Middle\ class) =$$

$$\frac{P(Agent \notin Final\ State\ Middle\ class \cap Agent \in Initial\ State\ Middle\ class)}{P(Agent \in Initial\ State\ Middle\ class)}$$

$$= \frac{P_{12} + P_{32}}{\sum_{i=1}^{3} P_{i2}} = T_{12} + T_{32}$$

$$(Note\ given\ 2\ initial\ middle\ classes\ 2a\ and\ 2b\ this\ becomes:$$

$$= \frac{P_{12a} + P_{32a} + P_{12b} + P_{32b}}{\sum_{i=1}^{3} P_{i2a} + \sum_{i=1}^{3} P_{i2b}} = w(T_{12a} + T_{32a}) + (1-w)(T_{12b} + T_{32b})$$

$$where\ w = P(Agent \in Initial\ State\ class\ 2a \mid Agent \in Initial\ State\ Middle\ class)$$

and

$$P(Agent \in Final\ State\ Middle\ class \mid Agent \notin Initial\ State\ Middle\ class) =$$

$$\frac{P(Agent \in Final\ State\ Middle\ class \cap Agent \notin Initial\ State\ Middle\ class)}{P(Agent \notin Initial\ State\ Middle\ class)}$$

$$\frac{P_{21} + P_{23}}{\sum_{i=1}^{3} P_{i1} + \sum_{i=1}^{3} P_{i3}} = \frac{\sum_{i=1}^{3} P_{i1} T_{21} + \sum_{i=1}^{3} P_{i3} T_{23}}{\sum_{i=1}^{3} P_{i1} + \sum_{i=1}^{3} P_{i3}} = wT_{21} + (1-w)T_{23}$$

$$where\ w = \frac{P(Agent \in Initial\ State\ Low\ class)}{P(Agent \notin Initial\ State\ Middle\ class)}.$$

This yields a Transition Matrix based Polarization/Depolarization index of the form:

$$PT = wT_{21} + (1-w)T_{23} - \left(T_{12} + T_{32}\right)$$

This has the intuition of the net proportion of the population transferring to the peripheries and corresponds to polarization when PT < 0 and convergence when PT > 0. Note that if focus on a “conflict” version of a polarization index (Esteban and Ray 2011) is desired where equilibrium levels of conflict are characterized by equal sized polar distributions then the above index could be modified to:

$$PT = wT_{21} + (1-w)T_{23} - \left(T_{12} + T_{32}\right) + |\, T_{12} - T_{32} \,| - |\, wT_{21} - (1-w)T_{23} \,|$$

This would be maximally negative or positive when the net transfers to the poles are balanced. To satisfy the normalization axiom, consider the transformation PTN = 0.5 + PT/2 so that when net transfers are balanced the index would return 0.5. As a probability measure, on the null hypothesis that PTN = 0.5 it is readily shown that PTN ~ N(0.5,0.25/n) where n is the sample size thus facilitating hypothesis testing – confidence interval interpretations.

The forgoing index focusses on convergence/polarization to and from the center but the transition could be to or from upper or lower classes, asymmetric convergence or polarization as it were. Clearly upwardly mobile societies are preferred to downwardly mobile societies, which is largely what is being captured in Dardanoni (1993), so an index needs to be oriented around that sentiment. Here interest is focused on the balance of probabilities that an agent will end up in a higher category than she started versus the chance that she will end up in a lower category. These probabilities are respectively given by:

$$P(Agent \in Final\ State\ Upper\ class \mid Agent \in Initial\ State\ Lower\ class) =$$

$$\frac{P(Agent \in Final\ State\ Upper\ class \cap Agent \in Initial\ State\ Lower\ class)}{P(Agent \in Initial\ State\ Lower\ class)}$$

$$P(Agent \notin Final\ Low\ State \mid Agent \in Initial\ LowState) * P(Agent \in Initial\ LowState)$$
$$+P(Agent \in Final\ High\ State \mid Agent \in Initial\ Middle\ State) * P(Agent \in Initial\ Middle\ State)$$

$$= \left(1 - T_{11}\right)\sum_{i=1}^{3} P_{i1} + T_{32}\sum_{i=1}^{3} P_{i2}$$

and

$$P(Agent \in Final\ State\ Lower\ class \mid Agent \in Initial\ State\ Upper\ class) =$$

$$\frac{P(Agent \in Final\ State\ Lower\ class \cap Agent \in Initial\ State\ Upper\ class)}{P(Agent \in Initial\ State\ Upper\ class)}$$

$$P(Agent \notin Final\ High\ State \mid Agent \in Initial\ High\ State) * P(Agent \in Initial\ High\ State)$$
$$+P(Agent \in Final\ Low\ State \mid Agent \in Initial\ Middle\ State) * P(Agent \in Initial\ Middle\ State)$$

$$= \left(1 - T_{33}\right)\sum_{i=1}^{3} P_{i3} + T_{12}\sum_{i=1}^{3} P_{i2}$$

When the classes have only an ordinal ranking this yields a Transition Matrix based Upward Advancement index of the form:

$$PUT = (1 - T_{11})w_1 - (1 - T_{33})w_3 - \left(T_{12} - T_{32}\right)w_2$$

$$where\ w_j = \sum_{i=1}^{3} P_{ij}$$

This has the intuition of the net proportion of the population transferring upwards when PT > 0 and downwards when PT < 0. Again these indices may be shown to satisfy most of the axioms outlined above with the transformation PUTN = 0.5(1+PUT) satisfying the normalization criterion. Again as a probability measure, on the null hypothesis that PUTN = 0.5 it is readily shown that PTN ~ N(0.5,0.25/n) where n is the sample size thus facilitating hypothesis testing – confidence interval interpretations.

3. 3 Applications.

Example 1. Generational Relationships in Educational Attainment in Canada

The equality of opportunity social justice imperative can be seen as requiring the elimination of parental circumstance as a source of variation in the outcomes of offspring. One way of doing this is to seek policies which make the outcome distributions of all classes identical, another is to seek policies which reduce variation in circumstances (Ramos and Van de Gaer 2014). When the outcome distribution of one generation is the circumstance distribution of the next generation, transition matrices which are converging in nature can be seen as a means to reducing variation in circumstances. Thus one way of examining progress toward an equality of opportunity goal is to measure whether or not transition matrices are polarizing, i.e. moving a society away from an equality of opportunity state, or converging, i.e. progressing toward an equality of opportunity state.

Anderson, Leo and Muelhaupt (2014) studied alternative versions of the Equality of Opportunity imperative in the context of generational educational relationships of males and females and their respective parents over 5 age cohorts of children in Canada. The generational relationship, represented by a transition matrix, can be seen as a transition from a parentally endowed initial state to the child's educational attainment final state, for a sample of parent – child pairs. Whilst the states are ordered, they are not cardinally ordered. If the transition matrix is constant over cohorts and applied to successive generations it could deliver a polarized, converged or stationary educational structure dependent upon whether it is a polarizing, converging or stationary transition matrix. Essentially the transition matrix delivers information about the progress of educational classes and the indices for such processes tell us whether the process is polarizing or converging.

The data for the empirical analysis on academic achievements of children and their parents in Canada are drawn from Statistics Canada's General Social Survey Cycle 19 (2005). Educational attainment has an ordinal index from 1 to 5 as follows: 1 for some secondary/elementary/no education; 2 for high school diploma; 3 for some university; 4 for Diploma/Certificate in a Trade/Technical skill, and 5 for a university degree. This categorization is for all individuals above the age of 25, including both parents and their children.

A test for the overall commonality of transition structure across all cohorts is given by using the $TM(T)/n_{ic}$ transvariation index where in this case T is formed from the vectorized transition matrices for each cohort being compared and n_{ic} is the number of initial classes. If there are no differences in cohorts, the statistic returns the value 1, if there is no commonality at all the statistic returns the value 0. This yields a test of constancy of transition structure over all 5 cohorts of 0.6277 (0.0075) for boys and 0.6057 (0.0066) for girls establishing substantial differences in the cohort transitions. Table E1.1

presents tests of differences in transitions by gender over the 5 cohorts. Clearly there are significant differences in parent – child transitions by cohort generation by generation.

Table E1.1 Cohort by cohort difference in transitions

	Age Cohort Comparisons			
	25-34 v 35-44	35-44 v 45-54	45-54 v 55-64	55-64 v Over 64
Boys	0.9126 (0.0064)	0.8581 (0.0077)	0.8856 (0.0078)	0.7639 (0.0121)
Girls	0.9001 (0.0060)	0.8726 (0.0066)	0.8147 (0.0085)	0.7489 (0.0103)

To analyze the type of transition structure and how it is progressing across successive generations Table E1.2 presents transition comparisons together with mobility analysis and the Polarization – Convergence Statistics for each cohort. Mobility has generally improved over the last 40 years for both genders with girls being uniformly more mobile than boys across all cohorts. Transition patterns are significantly polarizing for the 3 oldest cohorts in males and significantly convergent i.e. progressing toward and equality of opportunity outcome for the youngest male cohort, the two

Table E1.2 Mobility and Transition Analysis.

	Age Cohort				
	25 – 34	35 – 44	45 – 54	55 – 64	Over 64
Boys					
Sample Size	895	1039	995	659	569
Mobility	0.5719	0.5260	0.5527	0.4706	0.3219
Normalized Polarization Index	0.4682	0.4941	0.5518	0.5870	0.7190
Upward Advancement Index	0.5334	0.7459	0.7843	0.8664	0.8166
Std error (H_0: 0 balance of prob)	0.0167	0.0155	0.0159	0.0195	0.0210
Girls					
Sample Size	1187	1340	1201	884	887
Mobility	0.5910	0.6307	0.5628	0.6047	0.4635
Normalized Polarization Index	0.5202	0.5113	0.5076	0.5594	0.5556
Upward Advancement Index	0.7219	0.7784	0.8357	0.8522	0.7176
Std error (H_0: 0 balance of prob)	0.0145	0.0137	0.0144	0.0168	0.0168

oldest cohorts of females are significantly polarizing whilst the 3 youngest cohorts are neither polarizing nor converging. In all cases the balance of advancement was significantly upward especially so in the case of girls in the more recent cohorts.

Example 2 The Disappearing Middle Class in the World Income Distribution.

Employing a semi-parametric approach to income size distributions Anderson et al. (2016) studied the progress of cardinally ordered Poor, Middle and Rich nation income groupings in the world size distribution of PPP-adjusted real GDP per capita from 1970 – 2010 with a view to allowing group sizes to vary so that classes could potentially emerge or disappear. PPP-adjusted real GDP per capita, chained series at constant 2005 prices, from the Penn World Table (PWT) Version 7.1 (Heston et al., 2012) from 1970 to 2010 were employed in the analysis, the sample consists of data on 155 countries. They compared this to a conventional fixed class size model in which classes cannot emerge or disappear and found very different outcomes. In both analyses 1993 appeared to be a watershed with different transition structures before and after. For the fixed class size analysis final state polarization indices (Esteban and Ray 1994, Anderson 2010, Anderson, Leo and Linton 2012) indicated similar increasing polarization trends between rich and poor and rich and middle classes in the in both pre and post-1993 eras whereas the variable class size analysis polarization indices revealed a slight polarization trend pre1993 with converging trends thereafter resulting in a disappearing middle class in the last 5 years.

To illustrate how the transition analysis can add insight to conventional cardinal polarization / convergence measure the structure of 15 year transitions matrices is studied and reported below in tables E2.1 and E2.2. Note the substantially different behavior patterns revealed in fixed class size and variable class size models. In the variable class size model Mobility is low pre 1993 era and substantially higher post 1993 in the fixed class size model mobility is constant across eras. Indeed, at the 1% significance level, sample size penalized overlap tests (Anderson, et al. 2016) of the difference in transition matrices reject the hypothesis of common transitions in the variable class size model (P(Z>z*) = 0.0009) but fail to reject the hypothesis of common transitions in the fixed class size model (P(Z>z*) = 0.0370). The fixed class size model reveals modest polarizing patterns both pre and post 1993 whereas the variable class size model reveals modest polarizing behavior pre 1993 with significant polarizing behavior in the post 1993 era which is contrary to the Anderson et al. (2016) results. The question is; can the post 1993 convergence in Anderson et al. (2016) be reconciled with the polarization found here? PUT indices were never significantly different from 0.5 at usual significance levels, the most significant being the variable class model (P(Z>z*) = 0.0564 for H_0 PUT≥0.5) in the post 1993 era indicating some downward mobility (the probability of being poor in period t given middle income status in t-15 was 0.691). Here in effect, the middle group is polarizing, breaking up and joining poor and rich groups who Anderson et al.'s distance measures record as getting closer together all combining to produce a two class model in the last 5 years.

Table E2.1. Variable Class Size Transition Matrices.	1970-1993			1993-2010		
	Poor(t)	Mid(t)	Rich(t)	Poor(t)	Mid(t)	Rich(t)
Poor(t-15)	0.993	0.007	0.000	0.990	0.010	0.000
Middle(t-15)	0.095	0.876	0.029	0.691	0.304	0.005
Rich(t-15)	0.000	0.100	0.900	0.000	0.046	0.954
PTN	0.5561 {0.0402}			0.8789 {0.0402}		
PUTN	0.4978 {0.0402}			0.4376 {0.0402}		
Mobility	0.110			0.254		

Table E2.2. Fixed Class Size Transition Matrices.	1970-1993			1993-2010		
	Poor(t)	Mid(t)	Rich(t)	Poor(t)	Mid(t)	Rich(t)
Poor(t-15)	0.918	0.083	0.000	0.944	0.034	0.023
Middle(t-15)	0.199	0.554	0.248	0.307	0.629	0.064
Rich(t-15)	0.001	0.038	0.961	0.000	0.165	0.835
PT	0.6275 {0.0402}			0.6360 {0.0402}		
PUTN	0.5153 {0.0402}			0.4413 {0.0402}		
Mobility	0.201			0.205		

Example 3. Polarizing Grandparent - parent - child generational transition patterns in China.

In the early stage of the Chinese revolution the entire urban and rural population (the "grandparents" in this study) was classified into ordered social groups according to family employment status, income sources, and political loyalties. The Cultural Revolution 1966-76 (the educational period of the "parents" in this study) saw mass school closures (Gregory and Meng 2002, Deng and Treiman 1997) and a "class enemy" purge of "elites", a relatively small portion of the population. When schools reopened children from formerly lower class families were given opportunities for education and occupational attainment, while those from formerly bourgeois families were not (Clark 2014). However Gregory and Meng (2002) suggest that the largest negative impact was faced by children from lower educational achievement and lower social class families with an elevation of outcomes for the middle classes and diminishing outcomes for poor classes. Post 1980 - when the "children" in our study would have been educated - saw the Economic Reforms and the effects of the One Child Policy with increased investment in child education (Anderson and Leo (2009)). It would be of interest to see whether the generational transitions were indeed converging, i.e. favoring equality of opportunity or polarizing.

The Chinese Household Income Project (Li, Luo, Wei, and Yue 2008) is a rich dataset providing information on grandparent's social class designation, parent's educational status and child's (grandchildren's) educational status facilitating measurement of the transition from Grandparents Social class to parent's educational status and ultimately a child's educational status. Grandparent social classification (Chengfen) was C1: Poor Peasant or landless (53.96%), C2: Lower Middle Peasant (14.14%), C3: Upper Middle Peasant (4.81%), C4 : Rich Peasant (2.01%), C5: Landlord (2.82%), C6: Manual Worker (8.21%), C7: Office Worker (3.30%), C8: Enterprise Owner (0.43%), C9 : Petty Proprietor (3.75%), C10: Revolutionary Cadre (1.38%), C11: Revolutionary Army man (1.03%), C12: Other (4.16%). To simplify analysis, and because some cells were very small this categorization was condensed to 5 social classes SC1 = {C1}, SC2 = {C2}, SC3 = {C3, C4, C5}, SC4 = {C6, C7}, SC5 = {C8, C9, C10, C11, C12}. The educational categories were 0 no category, 1 if Never Schooled, 2 if classes for eliminating illiteracy, 3 elementary school, 4 if junior middle school, 5 if senior middle school(including professional middle school), 6 if technical secondary school, 7 if junior college, 8 if college/university, 9 if graduate. Educational categories 0 through 9 were condensed to EDC1 = {0,1,2,3}, EDC2 = {4}, EDC3 = {5}, EDC4 = {6}, EDC5 = {7}, EDC6 = {8}, EDC7 = {9}. Information was available on 9020 parent - grandparent pairings and 1514 parent – child pairings (only children over 22 years old were used under the assumption they would have completed their education).

Table E3.1 presents the grandparent social class -> parent education class transition structure. Note the similarity of the columns of the transition matrix indicating equality of opportunity (the mobility index is 0.9248). However closer inspection reveals a different story. SC2 and SC3 inheritors seem to do somewhat better than their counterparts. SC3 inheritors transit to high class educational outcomes in proportionately greater numbers than all other class inheritors (excluding SC3 increases the mobility index to 0.9327)[14].

Table E3.1 Grand Parent Social Class -> Parent Education Class Transition Structure.

		Grandparents Social Classification				
		Social Class 1	Social Class 2	Social Class3	Social Class 4	Social Class5
Inheritors Class	Marginals	0.4911	0.1399	0.1202	0.1177	0.1310
EDUCATION CLASS 1	0.0520	0.0585	0.0460	0.0295	0.0508	0.0558
EDUCATION CLASS 2	0.2901	0.3140	0.2583	0.2343	0.2825	0.2927
EDUCATION CLASS 3	0.2723	0.2774	0.2742	0.2362	0.3315	0.2310
EDUCATION CLASS 4	0.1134	0.1088	0.1244	0.1347	0.1008	0.1108
EDUCATION CLASS 5	0.1879	0.1713	0.2060	0.2297	0.1676	0.2107
EDUCATION CLASS 6	0.0772	0.0632	0.0864	0.1273	0.0603	0.0905
EDUCATION CLASS 7	0.0069	0.0068	0.0048	0.0083	0.0066	0.0085

The effect is seen more clearly in a simplified analysis where Social classes are amalgamated into Poor {SC1}, Middle {SC2, SC3} and Upper {SC4, SC5} and Education classes are amalgamated into Low {ED1, ED2}, Middle {ED3} and High {ED4, ED5, ED6, ED7}. The transition structure in the 3 Social Class – 3 Education Class model is reported in Table E3.2. The mobility index for this structure is 0.9278 which is very close to the mobility measure on the 5 Social 7 educational class transition structure. The PTN index for this structure is 0.5344 (Z score for H_0 PTN ≤ 0.5 is 6.534) indicating strong polarization in the transition from Grandparents social class to Parental educational class with strong upward mobility, (PUTN is 0.5944 yielding a Z score for H_0 PTN ≤ 0.5 is 18.881) brought about by upward movement by middle class inheritors. Perhaps most interesting result is the propensity for a "first shall be last and the last

[14] Assuming outcome class numbers correspond to a cardinal ordering admits dominance comparisons and reveals several instances of lower class outcome distributions dominating those of higher classes contradicting the equality of opportunity hypothesis of Lefranc, Pistolesi and Trannoy (2008, 2009) see Appendix.

shall be first” paradigm here, a tendency toward the oscillating long run outcome matrix identified in Atkinson (1981) with P(High Edu | Low Class) > P(Middle Edu | Low Class) and P(Low Edu | High Class) > P(Middle Edu | High Class) being statistically significant inequalities (note the determinant of this matrix is marginally negative at -0.000738). This is the type of transition that Kanbur-Stiglitz (2016) suggest could maximize welfare in the context of minimizing dynastic inequality and is probably a consequence of the 1966-76 Cultural Revolution policy of favoring the lower classes over the bourgeoisie (Clark 2014).

Table E3.2. 3 Grand Parent Social Class -> 3 Parent Education Class Transition Structure.

		Poor SC	Middle SC	Upper SC
Inheritor Class		0.4911	0.2601	0.2488
Low Education Class	0.3421	0.3725	0.2856	0.3415
Middle Education Class	0.2722	0.2774	0.2566	0.2785
High Education Class	0.3856	0.3501	0.4578	0.3801

Turning to the Parent - Child educational transitions using the same educational categories for both departure and arrival states a somewhat different story is revealed in Table E3.3. There is much less mobility (0.7387), a greater likelihood of a child being in the same parental class as its parent and very slight evidence of oscillating behavior ($|T| = -0.00000285$). The matrix is really sparse and inferences unreliable with such a fine categorization. Condensing to a 3 departure state – 3 arrival state structure the mobility index is much the same though the polarizing effect of the transitions is stronger (PTN=0.5999 with Z score for H_0 PTN ≤ 0.5 is 7.774) with upward advances continuing to outweigh downward advances in the transition with the PUTN index being 0.6697 (the Z score for H_0 PUTN ≤ 0.5 is 13.210). Note under cell aggregation the transition matrix has lost its oscillating property ($|T| = +0.0000125$).

Table E3.3 Parent Educational Class -> Child Education Class Transition Structures.

		Parent Educational Class						
		1	2	3	4	5	6	7
Childs Edu Class	Marginals	0.0581	0.3336	0.1783	0.1810	0.1744	0.0707	0.0040
1	0.0205	0.0568	0.0158	0.0074	0.0255	0.0189	0.0374	0.0000
2	0.1328	0.3636	0.2178	0.0778	0.0876	0.0341	0.0467	0.0000
3	0.2569	0.3409	0.3109	0.2815	0.2555	0.1477	0.1589	0.0000
4	0.1347	0.1136	0.1168	0.1926	0.1314	0.1402	0.0935	0.0000
5	0.2939	0.1023	0.2416	0.3037	0.3175	0.4015	0.3364	0.5000
6	0.1526	0.0227	0.0950	0.1296	0.1752	0.2386	0.3084	0.3333
7	0.0086	0.0000	0.0020	0.0074	0.0073	0.0189	0.0187	0.1667

Mobility 0.7387

Table E3.4 3 Parent Education Class -> 3 Child Education Class Condensed Transition Structure.

		Parent condensed education class		
		Low	Middle	Upper
Child Condensed Ed Class		0.3917	0.3593	0.2490
Low Education Class	0.5601	0.2614	0.0993	0.0610
Middle Education Class	0.1575	0.4317	0.4301	0.2732
High Education Class	0.2824	0.3069	0.4706	0.6658

PTN 0.5999, Mobility 0.7608

Finally Tables E3.5 and E3.6 report 2 generation grandparent to grandchild transitions. While the mobility index is high, the reduced mobility of the parent – child transition having little effect on grandparent to grandchild mobility, polarizing transitions in successive generations have intensified polarization (PTN=0.7495 with Z score for H_0 PTN ≤ 0.5 is 19.417).

The oscillating property has gone but Middle social class inheritors are transiting to Higher and Lower educational classes in greater proportions than are Low and High class inheritors transiting to middle educational classes, essentially engendering a disappearing middle class in Chinese society. Most transitions are upward with an upward transiting index PUT = 0.7045 (Z score for H_0 PUTN ≤ 0.5 is 15.916).

Table E3.5 Grand Parent Social Class -> Grand Child Education Class Condensed Transition Structure.

		Social Class 1	Social Class 2	Social Class3	Social Class 4	Social Class5
	Marginals	0.4911	0.1399	0.1202	0.1177	0.1310
EDUCATION CLASS 1	0.0205	0.0187	0.0190	0.0197	0.0178	0.0197
EDUCATION CLASS 2	0.1328	0.1296	0.1163	0.1057	0.1232	0.1231
EDUCATION CLASS 3	0.2569	0.2588	0.2491	0.2380	0.2585	0.2489
EDUCATION CLASS 4	0.1347	0.1410	0.1415	0.1380	0.1450	0.1376
EDUCATION CLASS 5	0.2939	0.2941	0.3040	0.3133	0.2970	0.3010
EDUCATION CLASS 6	0.1526	0.1488	0.1603	0.1740	0.1494	0.1595
EDUCATION CLASS 7	0.0086	0.0090	0.0098	0.0113	0.0091	0.0102

Mobility 0.9799

Table E3.6 3 Grand Parent Social Class -> 3 Grand Child Education Class Transition Structure.

		Grand Parent condensed education class		
		Low	Middle	Upper
Grand Child Condensed Ed Class		0.4911	0.2601	0.2488
Low Education Class	0.5601	0.1483	0.1307	0.1420
Middle Education Class	0.1575	0.2588	0.2439	0.2534
High Education Class	0.2824	0.5929	0.6253	0.6046

PTN 0.7495 PUTN 0.7045 Condensed Mobility 0.9784

3. Conclusions.

Notions of Polarization and Convergence, extensively employed in the Social Sciences, are concerned with the realignment, disappearance or emergence of groups, often in the context of variates or classifications devoid of cardinal measure. Inherently dynamic processes they relate to particular types of transition between departure and arrival states defined over groups which are in some sense ordered. Indices have conventionally been developed in terms of distributions of variables which have cardinal measure although in many situations the issues concern variables which can only be compared in an ordinal sense. Here it has been argued that in such situations polarization and convergence are more appropriately studied and understood in the context of indices reflecting the anatomy of transitions between states.

Accordingly indices and tests have been proposed and developed which identify polarization or convergence based upon the nature of an underlying transition process. The indices do not necessarily depend upon a between or within group cardinal ordering and do not depend upon the "square-ness" of the transition matrix, that is to say they can deal with disappearing and emerging groups. 3 examples exemplify their use. The first example, a study of generational dependencies in educational attainments in Canada, revealed considerable heterogeneity across successive cohorts and across genders in generational dependence patterns with polarizing transitions that decline in intensity with younger cohorts with the polarizing effect being more substantive for males. Advancement was upward in all cases. The second example, a study of national mobility in the size distribution of world GDP per capita in the context of a variable class size model revealed polarizing behavior that resulted in a disappearing middle class with downward transiting behavior. The third example studies the anatomy of transitions from the early revolutionary class structure classification in China to the educational class structure of the modern day. Again in this context some polarizing transitional structures are revealed.

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Appendix 1.

1st order dominance comparisons of middle social class outcomes over other classes.

	SC(1v3)	SC(2v3)	SC(4v3)	SC(5v3)	SC(1v4)	SC(2v4)	SC(5v4)
P(A≤ 1)-P(B≤ 1)	0.0289	0.0164	0.0213	0.0263	0.0125	0.0049	0.0099
P(A≤ 2)-P(B≤ 2)	0.1086	0.0404	0.0695	0.0847	0.0682	0.0291	0.0443
P(A≤ 3)-P(B≤ 3)	0.1499	0.0784	0.1648	0.0795	0.0714	0.0863	0.0011
P(A≤ 4)-P(B≤ 4)	0.1240	0.0682	0.1309	0.0557	0.0558	0.0627	-0.0125
P(A≤ 5)-P(B≤ 5)	0.0656	0.0445	0.0688	0.0366	0.0211	0.0243	-0.0079
P(A≤ 6)-P(B≤ 6)	0.0015	0.0035	0.0017	-0.0002	-0.0020	-0.0018	-0.0037
P(A≤ 7)-P(B≤ 7)	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

Most obviously the outcome distribution of SC3 first order dominates that of all other Social Class outcome distributions[15] and since it first order dominates it will second order dominate Similarly SC2 first order dominates the outcome distribution of SC1, and SC4 and second order dominates SC5.

In a similar fashion by endowing the class numbers cardinal measure dominance relations become clear in tables E3.4 and E3.6 with inheritance A dominating inheritance class A-1 for A = 2, 3, reflecting the diminished mobility in the class structure in Table E3.4. In E3.6 and grandchild outcomes of the middle class grand-parents first order dominate inheritor outcomes of both upper and lower grandparent inheritance classes reflecting longer run effects of the Cultural Revolution purges.

[15] The one negative term in the SC5 – SC3 comparison is not significantly different from 0.