Polarization and Convergence: Measurement in the Absence of Cardinality

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Outline

1. Background/Theory
   - Transition Matrices and Mobility
   - Measurement in the Absence of Cardinality
2. Applications
   - Three Empirical Examples
   - Additional Thoughts

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Basic Concepts

- This paper is concerned with the measurement of Polarization/Convergence.
- Do incomes (or other measures of wellbeing) increasingly concentrate in certain social strata?
- Or do advantages from desirable initial conditions slowly erode over time?
- The former is polarization while the latter is convergence.
- Clearly this is an important issue for policy makers.

Basic Concepts

- In this paper polarization/convergence is measured with transition matrices.
- This is a bit different from Esteban and Ray (1994) and Duclos et al (2004) approach.
- There are two time periods and individuals are partitioned into discrete groups.
- Let $x_i^{'} = [x_{i1}, x_{i2}, ..., x_{ik}]$ denote a vector of initial conditions (e.g. k population shares for income groups in a given year).
- Let $x_f^{'} = [x_{f1}, x_{f2}, ..., x_{fk}]$ denote the analogous shares at some latter point in time.
- How did we get from $x_i \to x_f$?

Basic Concepts

- This is studied with transition matrices (Markov Chains).
- The process is given by

$$x_f = T^{\alpha} x_i$$

- Here T is a $k \times k$ right-stochastic transition matrix and α is the number of iterations (an integer?).

Some Examples

- Consider a simple example with three income groups - low, medium and high.
- Define $x_i' = [P_l, P_m, P_h]$ as the proportion of the population in each group.
- Take some different forms for T (assume $\alpha = 1$).

$$\begin{matrix} \textit{Polarizing} & \textit{Converging} & \textit{Static} & \textit{Oscillating} \\ \begin{bmatrix} 1 & 0.3 & 0 \\ 0 & 0.4 & 0 \\ 0 & 0.3 & 1 \end{bmatrix} & \begin{bmatrix} 0.5 & 0 & 0 \\ 0.5 & 1 & 0.5 \\ 0 & 0. & 0.5 \end{bmatrix} & \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} & \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \end{matrix}$$

Some Examples

- Take the generic 3×3 transition matrix

$$T = \begin{bmatrix} t_{11} & t_{12} & t_{13} \\ t_{21} & t_{22} & t_{23} \\ t_{31} & t_{32} & t_{33} \end{bmatrix}$$

- Here t_{21} represents movements from l to m.
- And t_{12} represents movements from m to l.

Outline

Convergence/Polarization

- $P\left(Agent \notin m_f \mid Agent \in m_i\right) = t_{12} + t_{32}$

- This measures transitions from the middle class.
- Can do the same from transfers to the middle class.
- Using this idea an index is built.
- $I = wt_{21} + (1-w)\,t_{23} - \left(t_{12} + t_{32}\right)$ where $w = \frac{P(Agent \in l_i)}{P(Agent \notin m_i)}$
- $I > 0$ implies convergence while $I < 0$ implies polarization.

Upward/Downward movements

- Similarly we can build an index of upward advancement.
- $I_2 = (1 - t_{11})\,w_1 - (1 - t_{33})\,w_3 - (t_{12} - t_{32})\,w_2$
- This measures net upward transitions.
- Again only ordinal relationships are needed.
- This idea can be extended to non-square matrices too.

Hypothesis Testing

- These measures are subjected to linear scaling to normalize.
- After scaling the indices the distributions are found

$$I^* \sim N\left(2^{-1}, (4n)^{-1}\right)$$

- Thus we can test hypotheses and generate interval estimates etc.

Outline

Generational Relationships and Educational Attainments in Canada

- The first example corresponds to the study of inequality of opportunity.
- If states are given by parental characteristics, are transition matrices polarizing (increasing IOP) or converging (decreasing IOP)?
- Data are taken from Canadian General Social Survey (2005) and are stratified by gender.
- Educational attainments are ordinal: scores 1-5 where higher numbers indicate more education.

Generational Relationships and Educational Attainments in Canada

- A transvariation index is applied over different cohorts.
- Mobility, Polarization and Upward Advancement statistics are calculated
- Girls are more mobile and transition matrices are generally polarizing, rather than converging.
- Young males appear to be the exception(?) - cannot reject null of no net polarization/convergence.
- Upward advancement indices are higher for young women. This seems to make sense.

Generational Relationships and Educational Attainments in Canada

Figure: Mobility and Transition Analysis

	Age Cohort				
	25 – 34	35 – 44	45 – 54	55 – 64	Over 64
Boys					
Sample Size	895	1039	995	659	569
Mobility	0.5719	0.5260	0.5527	0.4706	0.3219
Normalized Polarization Index	0.4682	0.4941	0.5518	0.5870	0.7190
Upward Advancement Index	0.5334	0.7459	0.7843	0.8664	0.8166
Std error (H_0: 0 balance of prob)	0.0167	0.0155	0.0159	0.0195	0.0210
Girls					
Sample Size	1187	1340	1201	884	887
Mobility	0.5910	0.6307	0.5628	0.6047	0.4635
Normalized Polarization Index	0.5202	0.5113	0.5076	0.5594	0.5556
Upward Advancement Index	0.7219	0.7784	0.8357	0.8522	0.7176
Std error (H_0: 0 balance of prob)	0.0145	0.0137	0.0144	0.0168	0.0168

The Disappearing Middle Class in the World Income Distribution

- World income distribution data from Anderson et al (2016) are used.
- 15 year transition matrices are calculated.
- The analysis is completed for both fixed and variable classes.
- Results suggest modest polarization after 1993, a finding which appears in conflict with Anderson et al (2016).
- Not much upward advancement?

Polarizing Grandparent - Parent - Child Generational Transition Patterns in China

- Data taken over three generations spanning the Cultural Revolution 1966-76.
- Remarkable social experiment. Were intergenerational transition matrices polarizing or converging over this time?
- Data from The Chinese Household Income Project.
- A number of social classifications are used (e.g. peasant, manual worker, office worker etc). In some cases the are pooled due to limited observations.
- Education classes for subsequent generations are employed (non-square matrix).

Polarizing Grandparent - Parent - Child Generational Transition Patterns in China

- The transition matrix from grandparent social class to parent education is strongly polarizing.
- Also contains a high level of upward mobility.
- Some evidence of oscillating outcomes (!)
- This is interesting as it is anti dynastic inequality.
- Parent to child matrices are similar. Highly polarizing although the oscillating phenomena disappears with aggregation.
- Lastly grandparent to child matrices are also polarizing.

Outline

Additional Thoughts

- The technique developed above is very general and can be applied to a large number of empirical problems.
- I am interested in the α parameter. Suppose we have a 15 year matrix (e.g. as for the world income distribution). If $\alpha = 15$ you could solve for the annual change that, when compounded for 15 years, would give the 15 year change.
- Would the results for $\alpha = 15$ have to look like a scaled down version of the results obtained when $\alpha = 1$?
- I guess a simple way to classify a matrix would be to look at the steady state (i.e. use arbitrarily large value for α).

Additional Thoughts

- It may be possible to reinterpret the problem in terms of critical thresholds.
- Under what conditions will we have a steady state that looks like X?
- Potentially unstable. Small changes in transition matrices could predict very different steady states.
- If converging could calculate the half-life of between group inequality.